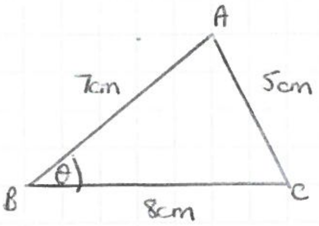
June 2009 - Core 2

1.



(a) Cosine Rule

$a^2 = b^2 + c^2 - 2bc\cos A$

$5^2 = 7^2 + 8^2 - 2 \times 7 \times 8 \times \cos\theta$

$25 = 49 + 64 - 112 \times \cos\theta$

$25 - 49 - 64 = -112 \times \cos\theta$

$-88 = -112 \times \cos\theta$

$\cos\theta = \frac{-88}{-112} = 0.7857\ldots$

$\theta = \cos^{-1} 0.7857$

$= 38.21\ldots$

$= 38.2^\circ$ to the nearest 0.1°

(b) Area

$\text{Area} = \frac{1}{2} ab \sin C$

$= \frac{1}{2} \times 7 \times 8 \times \sin 38.2$

$= 17.3154\ldots$

$= 17.3^\circ$ to 3sf.

2. (a) $\frac{1}{x^4} = x^n \qquad n = -4$

(b) $\left(1 + \frac{3}{x^2}\right)^2 = \left(1 + \frac{3}{x^2}\right)\left(1 + \frac{3}{x^2}\right)$

$= 1 + \frac{3}{x^2} + \frac{3}{x^2} + \frac{9}{x^4}$

$= 1 + \frac{6}{x^2} + \frac{9}{x^4}$

$= 1 + 6x^{-2} + 9x^{-4}$

(c) $\int \left(1 + \frac{3}{x^2}\right)^2 dx = \int 1 + 6x^{-2} + 9x^{-4}\, dx$

$= x + \frac{6x^{-1}}{-1} + \frac{9x^{-3}}{-3} + c$ — A must include +c

$= x - 6x^{-1} - 3x^{-3} + c$

(d) $\int_1^3 \left(1 + \frac{3}{x^2}\right)^2 dx \left[x - 6x^{-1} - 3x^{-3}\right]_1^3$

$= \left[3 - 6(3)^{-1} - 3(3)^{-3}\right] - \left[1 - 6(1)^{-1} - 3(1)^{-3}\right]$

$= \left[3 - 2 - \frac{3}{27}\right] - \left[1 - 6 - 3\right]$

$= \frac{24}{27} - (-8) \qquad = 8\frac{24}{27} = 8\frac{8}{9}$

3. $u_{n+1} = k u_n + 12$

$u_1 = 16 \qquad u_2 = 24$

a) $u_2 = k u_1 + 12$

$24 = k \times 16 + 12$

$24 - 12 = k \times 16$

$12 = k \times 16$

$k = \frac{12}{16} = 0.75$ as required

b) $u_3 = 0.75 u_2 + 12$

$= 0.75 \times 24 + 12$

$= 18 + 12 = 30$

$u_4 = 0.75 u_3 + 12$

$= 0.75 \times 30 + 12$

$= 22.5 + 12 = 34.5$

c) (i) $L = 0.75 L + 12$

(ii) $L - 0.75 L = 12$.

$L(1 - 0.75) = 12$

$L = \frac{12}{0.25} = 48$

4. a)

x	0	2	4	6
y	1	3	$\sqrt{65}$ $= 8.06..$	$\sqrt{217}$ $= 14.73..$

Area $\simeq \frac{h}{2}\left\{y_0 + y_3 + 2(y_1 + y_2)\right\}$

$= \frac{2}{2} \times \left(1 + \sqrt{217} + 2(3 + \sqrt{65})\right)$

$= 1 \times \left(1 + 14.73.. + 2(3 + 8.06..)\right)$

$= 37.85543 \ldots$

$= 37.86$ to 4sf

b) $y = \sqrt{x^3 + 1}$

stretch parallel to the x-axis sf $\frac{1}{2}$

$y = \sqrt{(2x)^3 + 1}$

$y = \sqrt{8x^3 + 1}$

5. $y = 15x^{3/2} - x^{5/2}$

(a) $\frac{dy}{dx} = \frac{3}{2} \times 15x^{1/2} - \frac{5}{2} \times x^{3/2}$

$= \frac{45x^{1/2}}{2} - \frac{5x^{3/2}}{2}$

(b) $\frac{dy}{dx} = 0$ at M

$\frac{45x^{1/2}}{2} - \frac{5x^{3/2}}{2} = 0$

$\frac{5x^{3/2}}{2} = \frac{45x^{1/2}}{2}$

$\frac{x^{3/2}}{x^{1/2}} = \frac{45}{\cancel{2}} \times \frac{\cancel{2}}{5}$

$x = 9$

when $x = 9$, $y = 15x^{3/2} - x^{5/2}$

$= 15 \times (9)^{3/2} - (9)^{5/2}$

$= 15 \times 27 - 243 = 162$ $(9, 162)$

(c) $P(1, 14)$

Eqn of the tangent at P

gradient of tangent, $\frac{dy}{dx} = \frac{45x^{1/2}}{2} - \frac{5x^{3/2}}{2}$

at $x = 1$ $= \frac{45(1)^{1/2}}{2} - \frac{5(1)^{3/2}}{2}$

$= \frac{45}{2} - \frac{5}{2}$

$= \frac{40}{2} = \underline{\underline{20}}$

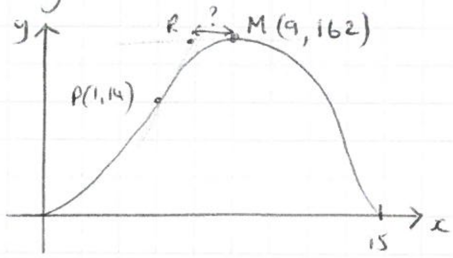
$y - y_1 = m(x - x_1)$

$y - 14 = 20(x - 1)$

$y - 14 = 20x - 20$

$y = 20x - 6$

(d) Tangents at P and M intersect at R.



Tangent at M, $y = 162$ (horizontal line)

Tangent at P, $y = 20x - 6$

Intersect when $20x - 6 = 162$
$+6$

$20x = 168$
$\div 20$

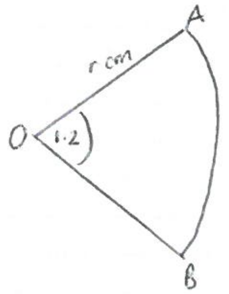
$x = 8.4$

Distance from R to M

$x = 8.4 \rightarrow x = 9$

$= 9 - 8.4 = \underline{\underline{0.6}}$

6.



$$\text{Area} = \frac{1}{2} r^2 \theta \quad (1)$$

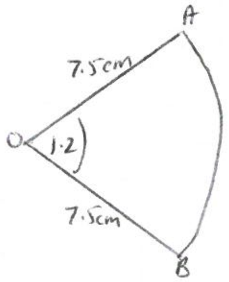
$$33.75 = \frac{1}{2} \times r^2 \times 1.2$$

$$33.75 = 0.6 \times r^2$$

$$r^2 = \frac{33.75}{0.6} = 56.25 \quad (1)$$

$$r = \sqrt{\quad} =$$

$$= 7.5\text{cm} \quad (1)$$



$$\text{Perimete} = OA + OB + AB$$

$$= 7.5 + 7.5 + 9 \quad (1)$$

$$= 24\text{cm} \quad (1)$$

$$AB = r\theta \quad (1)$$

$$= 7.5 \times 1.2$$

$$= 9$$

7. G.P $u_2 = 375$ $u_5 = 81$

(a) (i) n^{th} term $= ar^{n-1}$

(1) both

Second term $a \times r^{2-1} = 375$

$$a \times r = 375$$

$$a = \frac{375}{r}$$

Fifth term $a \times r^{5-1} = 81$

$$a \times r^4 = 81$$

$$a = \frac{81}{r^4}$$

So, $\frac{375}{r} = \frac{81}{r^4}$ (1)

$$\frac{r^4}{r} = \frac{81}{375} \quad (1)$$

$$r^3 = 0.216$$

$$r = \sqrt[3]{0.216} = 0.6$$

(ii) If $r = 0.6$, $a \times r = 375$

$$a \times 0.6 = 375 \quad (1)$$

$$a = \frac{375}{0.6} = 625 \quad (1)$$

(b) $S_\infty = \frac{a}{1-r} = \frac{625}{1-0.6}$ (1) $= \frac{625}{0.4} = 1562.5$ (1)

(c) $\sum_{n=6}^{\infty} u_n = \sum_{n=1}^{\infty} u_n - \sum_{n=1}^{n=5} u_n$ (1)

(1)

$$= 1562.5 - (625 + 375 + 625 \times 0.6^2 + 6.25 \times 0.6^3 + 81)$$

$$= 1562.5 - (625 + 375 + 225 + 135 + 81) \quad (1) \quad = 121.5 \quad (1)$$

8 a)
$$\frac{\sin\theta - \cos\theta}{\cos\theta} = 4$$

$$\frac{\sin\theta}{\cos\theta} - \frac{\cos\theta}{\cos\theta} = 4 \quad (1)$$

$$\tan\theta - 1 = 4$$

$$\tan\theta = 5 \quad (1)$$

b) (i) $2\cos^2 x - \sin x = 1$

$$\sin^2 x + \cos^2 x \equiv 1$$

$$\cos^2 x \equiv 1 - \sin^2 x$$

$$2(1 - \sin^2 x) - \sin x = 1 \quad (1)$$

$$2 - 2\sin^2 x - \sin x = 1$$

$$2\sin^2 x + \sin x - 1 = 0 \quad (1)$$

as required

(ii) $(2\sin x - 1)(\sin x + 1) = 0 \quad (1)$

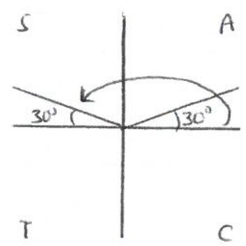
$\sin x = \frac{1}{2}$ $\quad$ $\sin x = -1$

$x = \sin^{-1}\frac{1}{2}$ $\quad$ $x = \sin^{-1} - 1 \quad (1)$

$= 30° \quad (1)$ $\quad$ $= -90$ (from calculator)

$= 270° \quad (1)$

$180 - x = 150° \quad (1)$



9. (a) (i) $\sqrt{125} = 5^p$

$p = 3/2$ ②

(ii) $5^{2x} = \sqrt{125}$

$5^{2x} = 5^{3/2}$

$2x = 3/2$

$x = 3/4$ ①

(b) $3^{2x-1} = 0.05$

$\log 3^{2x-1} = \log 0.05$

$(2x-1)\log 3 = \log 0.05$ ①

$2x - 1 = \dfrac{\log 0.05}{\log 3}$

$2x = -2.7268.. + 1$ ① Method.

$x = \dfrac{-1.7268...}{2}$

$x = -0.8634...$ ① to 4dp

(c) $\log_a x = 2(\log_a 3 + \log_a 2) - 1$

$\log_a x = 2(\log_a 3 \times 2)$ ① $- \log_a a$ ①

$\log_a x = \log_a 6^2$ ① $- \log_a a$

$\log_a x = \log_a \dfrac{36}{a}$

$x = \dfrac{36}{a}$ ①

Log Rules

$\log a + \log b = \log ab$

$\log a - \log b = \log \dfrac{a}{b}$

$n \log a = \log a^n$

9. (a) (i) $\sqrt{125} = 5^p$

$p = \frac{3}{2}$

(ii) $5^{2x} = \sqrt{125}$

$5^{2x} = 5^{3/2}$

$2x = \frac{3}{2}$

$x = \frac{3}{4}$