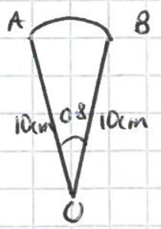
January 2009

1.



(a) $A = \frac{1}{2} r^2 \theta$ 1

$= \frac{1}{2} \times 10^2 \times 0.8$

$= 40 cm^2$ 1

(b) (i) Perimeter $= AO + BO + AB$

$= r + r + r\theta$ 1

$= 10 + 10 + 8$ 1

$= 28 cm$ 1

$\underline{AB} = r\theta$

$= 10 \times 0.8$

$= 8 cm$

(ii) Perimeter of Sector = Perimeter of Square

28 = 4 × length of one side

length $= \frac{28}{4} = 7$

Area of Square $= 7^2 = 49 cm^2$ 1 1

2. (a) $\int_{1.5}^{6} x^2 \sqrt{x^2 - 1} \, dx$

x	1.5	3	4.5	6
y	2.51	25.4	88.8	212.9

1 method
1 answer

Area $= \frac{1.5}{2} \left\{ 2.51 + 212.9 + 2\left[25.4 + 88.8 \right] \right\}$ 1

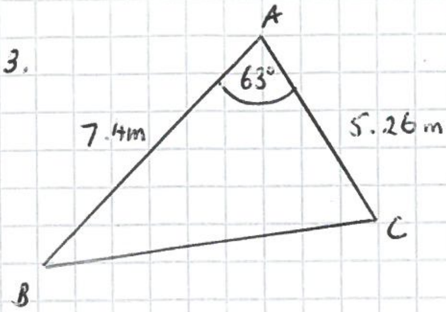
$= 0.75 \times 444.3960893$

$= 333.297069$

$= 333$ to 3sf. 1

(b) Increase the number of ordinates. 1

3.



(a) $\text{Area} = \frac{1}{2} ab \sin C$.

$= \frac{1}{2} \times 7.4 \times 5.26 \times \sin 63$ (1)

$= 17.34076897$

$= 17.3 \ cm^2$ (1) to 2sf.

(b) Cosine Rule

$a^2 = b^2 + c^2 - 2bc \cos A$

$a^2 = 7.4^2 + 5.26^2 - 2 \times 7.4 \times 5.26 \times \cos 63$ (1)

(1) Correct Order of Evaluation

$a^2 = 47.08534758$

$a = \sqrt{47.085}$

$= 6.861876389$

$= 6.86 \ m$ (1) to 3sf.

(c) Sine Rule

$\frac{\sin 63}{6.86} = \frac{\sin B}{5.26}$ (1)

$\sin B = 5.26 \times \frac{\sin 63}{6.86}$

$= 0.6831915914$

$= 0.68$ (1) to 2sf.

4. (a) (i) $y = 2x^{3/2}$

$$\frac{dy}{dx} = \frac{3}{2} \times 2x^{1/2} = 3x^{1/2}$$

When $x = 4$

$$3(4)^{1/2} = 3 \times 2 = 6.$$

(ii) Eqn of normal at A.

x-coordinate of A, $x = 4$

y-coordinate of A $\quad y = 2x^{3/2} = 2 \times (4)^{3/2}$

$$= 2 \times 8 = 16$$

Gradient of tangent at A $= 6$ (from (a)(i))

Gradient of normal at A $= -\frac{1}{6}$ $\qquad m_1 m_2 = -1$

Eqn of normal at A $\Rightarrow$ $\quad y - 16 = -\frac{1}{6}(x - 4)$

$$y - 16 = -\frac{x}{6} + \frac{4}{6}$$

$$y = -\frac{x}{6} + 16\frac{4}{6}$$

(b) (i) $\int 8x^{1/2}\,dx$

$$\frac{8x^{3/2}}{3/2} + c = \frac{2}{3} \times 8x^{3/2} = \frac{16x^{3/2}}{3}$$

(ii) $\int_0^4 8x^{1/2}\,dx - \int_0^4 2x^{3/2}\,dx$

$$\left[\frac{16x^{3/2}}{3}\right]_0^4 - \left[\frac{4x^{5/2}}{5}\right]_0^4$$

$$\left[\frac{16}{3}(4)^{3/2} - 0\right] - \left[\frac{4}{5}(4)^{5/2} - 0\right]$$

$$\frac{16}{3} \times 8 - \frac{4}{5} \times 32 = \frac{256}{15}$$

(c) $y = 2x^{3/2} \Rightarrow y = 2(x+3)^{3/2}$

↑

Translation 3 left $\qquad \begin{bmatrix} -3 \\ 0 \end{bmatrix}$

5. (a) $(1+2x)^4 = {}^4C_0 1^4 (2x)^0 + {}^4C_1 1^3 (2x)^1 + {}^4C_2 1^2 (2x)^2 + {}^4C_3 1^1 (2x)^3 + {}^4C_4 1^0 (2x)^4$

$= 1 \times 1 \times 1 + 4 \times 1 \times 2x + 6 \times 1 \times 4x^2 + 4 \times 1 \times 8x^3 + 1 \times 1 \times 16x^4$

$= 1 + 8x + 24x^2 + 32x^3 + 16x^4$

$a = 8 \quad b = 24 \quad c = 32$

(b) $(1+2x)^4 + (1-2x)^4$

$(1 + \cancel{8x} + 24x^2 + \cancel{32x^3} + 16x^4) + (1 - \cancel{8x} + 24x^2 - \cancel{32x^3} + 16x^4)$

$= 2 + 48x^2 + 32x^4$

(c) $\frac{dy}{dx} = 2(48)x^1 + 4(32)x^3$

$= 96x + 128x^3$

At stationary point, $96x + 128x^3 = 0$

$x(96 + 128x^2) = 0$

$x = 0 \qquad 96 + 128x^2 = 0$ has no solutions

$\therefore$ There is only one stationary point at $x = 0$.

$y = (1+2x)^4 + (1-2x)^4$

$y = (1+2(0))^4 + (1-2(0))^4$

$y = 1 + 1 = 2 \qquad (0, 2)$

6. (a) (i) $\log_a 4 + \log_a 10 = \log_a 4 \times 10 = \log_a 40$ (1)

(ii) $\log_a 16 - \log_a 2 = \log_a \frac{16}{2} = \log_a 8$ (1)

(iii) $3 \log_a 5 = \log_a 5^3 = \log_a 125$ (1)

(b) $(1.5)^{3x} = 7.5$

$3x \log 1.5$ (1) $= \log 7.5$ (1)

$3x = \frac{\log 7.5}{\log 1.5}$

$x = \frac{\log 7.5}{3 \log 1.5} = 1.65645\ldots = 1.656$ (1) to 3dp

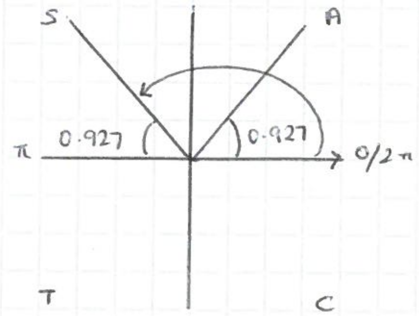
(c) $\log_2 p = m \qquad \log_8 q = n$

$p = 2^m$ (1) $\qquad q = 8^n$ (1) $= 2^{3n}$

$pq = (2^m)(2^{3n})$

$= 2^{m+3n}$ (1)

7. (a) $\sin x = 0.8 \qquad 0 \le x \le 2\pi$



$x = \sin^{-1} 0.8$ (1)

$= 0.927295\ldots = 0.93$ to 3sf

$x = \pi - 0.927$ (1)

$= 2.21429\ldots = 2.21$ to 3sf. (1)

(b) (i) At minimum point, M, $y = -1$

$\sin x = -1$

$x = \sin^{-1} -1$

$= -1.57079\ldots$

$= \frac{3}{2}\pi \qquad \left(\frac{3}{2}\pi, -1\right)$ (1)

(ii) At P, the x-coordinate is α.

At Q, the x-coordinate is $\pi - \alpha$ (1) (since graph is symmetrical)

(ii) At R, the x-coordinate is $\pi + \alpha$

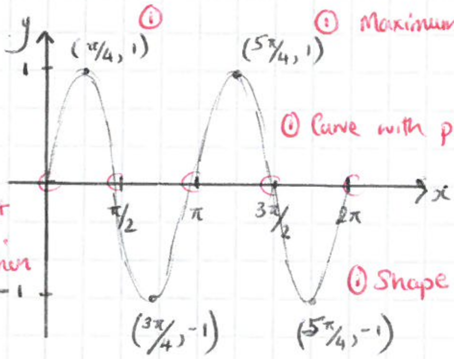
At S, the x-coordinate is $2\pi - \alpha$

Since RS is a horizontal line the length $= (2\pi - \alpha) - (\pi + \alpha)$ (1)

$= \pi - 2\alpha$ (1)

(c)

(1) Maximum Points



(1) Curve with positive gradient at O and at least 3 stationary points

(1) All 5 correct points of intersection

(1) Shape

8. $u_{25} = 38 \qquad S_{40} = 1250$

(a) $u_{25} = a + (25-1)d$ (1)

$= a + 24d = 38$ (1)

$S_{40} = \frac{40}{2}\left[2a + (40-1)d\right]$ (1)

$= 20\left[2a + 39d\right]$

$= 40a + 780d = 1250$ (1)

$40a + 780d = 1250$

$a + 24d = 38 \quad \times 40$

$40a + 780d = 1250$

$-\ 40a + 960d = 1520$

$180d = 270$

$d = 1.5$ (1) as required

$a + 24(1.5) = 38$

$a + 36 = 38$

$a = 2$

(b) If $d = 1.5$

and $a + 24d = 38$

$a + 24(1.5) = 38$

$a + 36 = 38$

$a = 2$ (1)

$u_n = a + (n-1)d$

$2 + (n-1) \times 1.5 < 100$ (1)

$2 + 1.5n - 1.5 < 100$

$1.5n + 0.5 < 100$

$1.5n < 99.5$

$n < 66.333...$

OR

$n < \frac{100-2}{1.5} + 1$

$n < \frac{98}{1.5} + 1$

$n < 66.333...$

(1)

$\therefore$ The number of terms less than 100 is 66