June 2004

1. (a) $\sqrt{x^3} = x^{3/2}$ $k = 3/2$

(b) $y = x^2 - \sqrt{x^3}$

$= x^2 - x^{3/2}$

(i) $\frac{dy}{dx} = 2x - \frac{3}{2}x^{1/2}$

(ii) gradient of tangent at $x = 4$

$2(4) - \frac{3}{2}(4)^{1/2}$

$8 - \frac{3}{2} \times 2$

$8 - 3 = 5$

y-coordinate at $x = 4$

$y = x^2 - x^{3/2}$

$= (4)^2 - (4)^{3/2}$

$= 16 - 8$

$= 8$

Eqn of tangent,

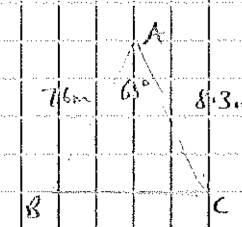
$y = mx + c$

$8 = 5(4) + c$

$c = -12$

$y = 5x - 12$

4.



(a) $BC^2 = 7.6^2 + 8.3^2 - 2(7.6)(8.3) \times \cos 65$

$BC^2 =$

$BC = \sqrt{}$

$BC =$

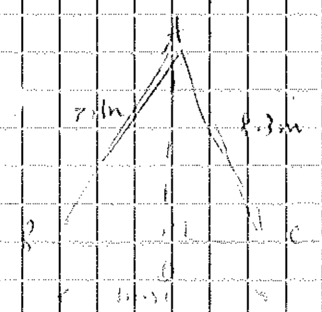
$= 8.56\,m$ to 3sf as required

(b) Area $= \frac{1}{2} ab \sin C$

$= \frac{1}{2}(7.6)(8.3) \times \sin 65$

$=$

$= 28.6$ to 3sf.



(c) Area $= \frac{1}{2} \times \underset{(BC)}{\text{base}} \times \text{height}_{(AD)}$

Area $= \frac{1}{2} \times 8.56 \times AD = 28.6$

$4.28 \times AD = 28.6$

$AD = \frac{28.6}{4.28}$

$= 6.6822\ldots$

$= 6.7\,m$ to 1dp

5. (a) (i) $\log_a 1 = 0$ since $a^0 = 1$

(ii) $\log_a a = 1$ since $a^1 = a$

(b) $\log_a x = \log_a 5 + \log_a 6 - \log_a 1.5$

$\log_a x = \log_a \left(\frac{5 \times 6}{1.5}\right)$

$\log_a x = \log_a \left(\frac{30}{1.5}\right)$

$\log_a x = \log_a 20$

$x = 20.$

6. (a) $u_{n+1} = pu_n + q$

$u_1 = -8$

$u_2 = p(-8) + q = 8$

$u_3 = p(8) + q = 4$

Simultaneous Eqn, $-8p + q = 8$

$q = 8 + 8p$

$8p + q = 4$

$q = 4 - 8p$

$8 + 8p = 4 - 8p$

$4 = -16p$

$-\frac{1}{4} = p$

$q = 4 - 8p$

$= 4 - 8(-\frac{1}{4})$

$= 4 + 2$

$= 6$ as required.

(b) $u_{n+1} = -\frac{1}{4} u_n + 6$

$u_4 = -\frac{1}{4}(4) + 6$

$= -1 + 6$

$= 5$

(c) (i) As $n \to \infty$, $u_n \to L$

$u_\infty \Rightarrow -\frac{1}{4}L + 6 = L$

(ii) $-\frac{1}{4}L - L = -6$

$-\frac{5}{4}L = -6$

$L = \frac{-6 \times 4}{-5} = \frac{24}{5} = 4.8$

7. (a) $\left(1+\frac{4}{x^2}\right)^3 = {}^3C_0 \times 1^3 \times \left(\frac{4}{x^2}\right)^0 = 1 \times 1 \times 1 = 1$

${}^3C_1 \times 1^2 \times \left(\frac{4}{x^2}\right)^1 = 3 \times 1 \times \frac{4}{x^2} = \frac{12}{x^2}$

${}^3C_2 \times 1^1 \times \left(\frac{4}{x^2}\right)^2 = 3 \times 1 \times \frac{16}{x^4} = \frac{48}{x^4}$

${}^3C_3 \times 1^0 \times \left(\frac{4}{x^2}\right)^3 = 1 \times 1 \times \frac{64}{x^6} = \frac{64}{x^6}$

$1 + \frac{12}{x^2} + \frac{48}{x^4} + \frac{64}{x^6}$

as required, where $p=12$ and $q=48$

(b) (i) $\int\left(1+\frac{4}{x^2}\right)^3 dx$

$= \int 1 + \frac{12}{x^2} + \frac{48}{x^4} + \frac{64}{x^6} \; dx$

$= \int 1 + 12x^{-2} + 48x^{-4} + 64x^{-6} \; dx$

$= x + \frac{12x^{-1}}{-1} + \frac{48x^{-3}}{-3} + \frac{64x^{-5}}{-5} + c$

$= x - \frac{12}{x} - \frac{16}{x^3} - \frac{64}{5x^5} + c$

(ii) $\int_1^2 \left(1+\frac{4}{x^2}\right)^3 dx$

$= \left[x - \frac{12}{x} - \frac{16}{x^3} - \frac{64}{3x^5}\right]_1^2$

$= \left[\left((2) - \frac{12}{(2)} - \frac{16}{(2)^3} - \frac{64}{5(2)^5}\right) - \left((1) - \frac{12}{(1)} - \frac{16}{(1)^3} - \frac{64}{5(1)^5}\right)\right]$

$= \left[\left(2 - 6 - 2 - \frac{2}{5}\right) - \left(1 - 12 - 16 - \frac{64}{5}\right)\right]$

$= \left[\left(-6\tfrac{2}{5}\right) - \left(-27 - \frac{64}{5}\right)\right]$

$= \left[\left(-\frac{32}{5}\right) - \left(-\frac{199}{5}\right)\right]$

$= \frac{167}{5}$ or 33.4

8. (a) (i) Trapezium Rule, 5 ordinates

$$A = \frac{h}{2}\{y_0 + y_4 + 2(y_1 + y_2 + y_3)\}$$

x	0	0.5	1	1.5	2
y	1	$\sqrt{6}$	6	$\sqrt{216}$	36

$h = 0.5$

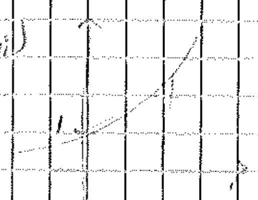
$$A = \frac{0.5}{2}\{1 + 36 + 2(\sqrt{6} + 6 + \sqrt{216})\}$$

$$= 0.25 \times \{1 + 36 + 2(23.1466\ldots)\}$$

$$= 0.25 \times \{83.2928\ldots\}$$

$$= 20.823\ldots$$

$$= 20.8 \text{ to } 3sf$$

(ii) 

This is an over estimate.

(b) (i) $y = 6^x \rightarrow y = 6^{3x}$

Stretch parallel to the x axis of sf $1/3$

(ii) $y = 84$ intersects $y = 6^{3x}$

$$6^{3x} = 84$$

$$\log 6^{3x} = \log 84$$

$$3x \log 6 = \log 84$$

$$x = \frac{\log 84}{3 \log 6}$$

$$x = 0.82429\ldots$$

$$x = 0.824 \text{ to } 3dp.$$

(c) $y = 6^x$ is translated $\begin{bmatrix} 1 \\ -2 \end{bmatrix}$ 1 right 2 down

$$y = 6^{(x-1)} - 2$$

1. (a) $\sin 2x = \sin 48$ $\quad 0 \le x < 360°$

$2x = 48 \quad 0 \le 2x < 720°$

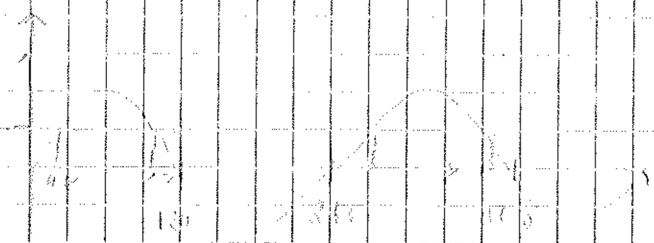
$x = 24°$

or $2x = 180 - 48$
$= 132°$, $x = 66°$

$2x = 360 + 48$
$= 408°$,

$x = 204°$

or $2x = 540 - 48$
$= 492°$, $x = 246°$



(b) $2\sin\theta - 3\cos\theta = 0$

$2\sin\theta = 3\cos\theta$

$\frac{\sin\theta}{\cos\theta} = \frac{3}{2}$

$\tan\theta = \frac{3}{2}$

$\theta = \tan^{-1}\frac{3}{2}$

$= 56.$

$= 56.3°$ to 1dp

$\theta = 180 + 56.3°$

$= 236.3°$ to 1dp