

Core 2 - January 2006

① $y = 16x + x^{-1}$

$$\frac{dy}{dx} = 16 - x^{-2}$$

$$\frac{dy}{dx} = 0$$

$$16 - x^{-2} = 0$$

$$x^{-2} = 16$$

$$\frac{1}{x^2} = 16$$

$$x^2 = \frac{1}{16}$$

$$x = \pm \sqrt{\frac{1}{16}}$$

$$\underline{x = \pm \frac{1}{4}}$$

② a) $\int_0^4 \frac{1}{x^2+1} dx$

$$n = 4 \text{ (strips)}$$

$$h = \frac{4-0}{4} = 1$$

x	0	1	2	3	4
y	1	$\frac{1}{2}$	$\frac{1}{5}$	$\frac{1}{10}$	$\frac{1}{17}$

$$\int_0^4 \frac{1}{x^2+1} dx = \frac{1}{2} \left(1 + 2 \left(\frac{1}{2} + \frac{1}{5} + \frac{1}{10} \right) + \frac{1}{17} \right)$$

$$= 1.32941 \dots$$

$$= \underline{\underline{1.329}} \text{ (4 s.f.)}$$

b) More ordinates or strips

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③ a) $0.8^x = 0.05$

$$x \log 0.8 = \log 0.05$$

$$x = \frac{\log 0.05}{\log 0.8}$$

$$x = 13.4251348 \dots$$

$$x = \underline{\underline{13.425}} \text{ (3dp)}$$

b) $S_{\infty} = \frac{a}{1-r}$

$$\frac{a}{1-r} = 5a$$

i) $\frac{a}{5a} = 1-r$

$$\frac{1}{5} = 1-r$$

$$r = 4/5 = \underline{0.8} \text{ (as req)}$$

ii) $a = 20, r = 0.8$

$$U_n < 1$$

$$U_n = ar^{n-1}$$

$$U_n = 20 \times 0.8^{n-1}$$

$$\text{so, } 20 \times 0.8^{n-1} < 1 \quad (\div 20)$$

$$0.8^{n-1} < \frac{1}{20}$$

$$(n-1) \log 0.8 < \log 0.05$$

$$n-1 < \frac{\log 0.05}{\log 0.8}$$

$$n-1 < 13.425 \dots$$

$$n < 14.425 \dots$$

$\therefore$ least value of n is 15

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Radians Made

④ a) Area of triangle = $\frac{1}{2} ab \sin C$

$$\therefore \frac{1}{2} ab \sin C = 20$$

$$a = 8, b = 12$$

$$\frac{1}{2} (8)(12) \sin C = 20$$

$$48 \sin C = 20$$

$$\sin C = \frac{20}{48}$$

$$C = \sin^{-1} \left(\frac{20}{48} \right)$$

$$C = 0.42977...$$

$$C = \underline{0.430^\circ} \text{ (3sf) (as req)}$$

b) $AB^2 = 8^2 + 12^2 - (2 \times 8 \times 12 \times \cos 0.42977...)$

$$AB^2 = 64 + 144 - (174.5...) =$$

$$AB^2 = 33.46017...$$

$$AB = \sqrt{33.4601...}$$

$$AB = 5.78447...$$

$$AB = \underline{5.8 \text{ cm}} \text{ (2sf)}$$

ci) arc length = $r\theta$

$$= 8 \times 0.4297...$$

$$= 3.4381...$$

$$= \underline{3.4} \text{ (3sf) cm}$$

ii) Area of shaded = area of triangle - area of sector

$$\text{area of sector} = \frac{1}{2} r^2 \theta$$

$$= \frac{1}{2} (8)^2 \times 0.4297...$$

$$= 13.752...$$

$$\text{shaded area} = 20 - 13.752...$$

$$= 6.248... = \underline{6.2 \text{ cm}^2} \text{ (2sf)}$$

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⑤ $U_{n+1} = pU_n + q$

$U_1 = 200, \quad U_2 = 150, \quad U_3 = 120$

a) $150 = 200p + q$ ①

$- 120 = 150p + q$ ② ① - ②

$30 = 50p \quad (\div 50)$

$p = \frac{30}{50} = \underline{0.6}$ (as req)

$150 = 200 \times 0.6 + q$

$150 = 120 + q \quad (-120)$

$q = \underline{30}$

b) $U_4 = 0.6 \times 120 + 30$

$U_4 = \underline{102}$

c) $n \rightarrow \infty, \quad U_n \rightarrow L$ and $U_{n+1} \rightarrow L$

so, $L = 0.6L + 30$

(sub into $U_{n+1} = pU_n + q$)

$0.4L = 30 \quad (\div 0.4)$

$\underline{L = 75}$

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6ai) $y = \sin x \rightarrow y = 2 \sin x$

stretch scale factor 2 in y direction

bi) $y = \sin x \rightarrow y = -\sin x$ (y changes sign)

reflection in x axis

iii) $y = \sin x \rightarrow y = \sin(x - 30^\circ)$

translation $\begin{pmatrix} 30^\circ \\ 0 \end{pmatrix}$

b) $\sin(\theta - 30^\circ) = 0.7$

$0^\circ \leq \theta \leq 360^\circ$

change range $\rightarrow -30^\circ \leq \theta - 30^\circ \leq 330^\circ$

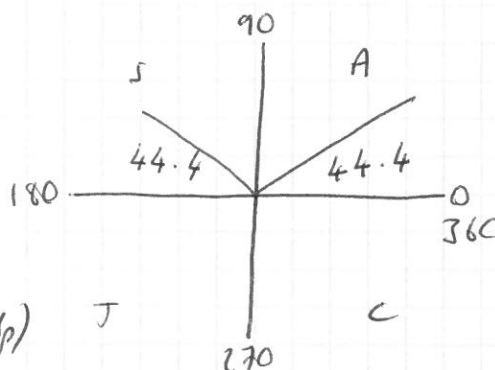
$\theta - 30^\circ = \sin^{-1}(0.7)$

$\theta - 30^\circ = 44.4270^\circ$

135.573°

$\theta = 74.42^\circ, 165.57^\circ$

$\theta = \underline{74.4^\circ}, \underline{165.6^\circ}$ (1dp)



c) $(\cos x + \sin x)^2 + (\cos x - \sin x)^2 = 2$

$(\cos x + \sin x)(\cos x + \sin x) + (\cos x - \sin x)(\cos x - \sin x) = 2$

$(\cos^2 x + \sin x \cos x + \sin x \cos x + \sin^2 x) + (\cos^2 x - \sin x \cos x - \sin x \cos x + \sin^2 x) = 2$

$2\cos^2 x + 2\sin^2 x = 2$

$2(\cos^2 x + \sin^2 x) \rightarrow$

$\boxed{\sin^2 x + \cos^2 x = 1}$

$2(1) = 2$ (as req)

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$$(7) a) \quad 2 \log_a n - \log_a (5n - 24) = \log_a 4$$

$$\log_a n^2 - \log_a 5n - 24 = \log_a 4$$

$$\log_a \left(\frac{n^2}{5n - 24} \right) = \log_a 4 \quad (\text{cancel logs})$$

$$\frac{n^2}{5n - 24} = 4$$

$$n^2 = 4(5n - 24)$$

$$n^2 = 20n - 96$$

$$\underline{n^2 - 20n + 96 = 0} \quad (\text{as required})$$

$$b) \quad n^2 - 20n + 96 = 0$$

$$(n - 8)(n - 12) = 0$$

$$\underline{n = 8} \quad \text{or} \quad \underline{n = 12}$$

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8) $y = x^{3/2} - 3x$

a) $\frac{dy}{dx} = \frac{3}{2} x^{1/2} - 3$

bi) at point O, $x=0$:-

$$\frac{dy}{dx} = \frac{3}{2} (0)^{1/2} - 3$$

$$\frac{dy}{dx} = \underline{\underline{-3}}$$

equation of tangent is $y = \underline{\underline{-3x}}$

ii) gradient at A(9,0), $x=9$:-

$$\frac{dy}{dx} = \frac{3}{2} (9)^{1/2} - 3$$

$$= \frac{3}{2} (m) \quad (9, 0)$$

$$y - y_1 = m(x - x_1)$$

$$y = \frac{3}{2} (x - 9)$$

$$\underline{2y = 3x - 27 \text{ (as req)}}$$

iii) tangents meet at P, solve $y = -3x$ and $2y = 3x - 27$

simultaneously :-

$$2(-3x) = 3x - 27$$

$$-6x = 3x - 27$$

$$9x = 27$$

$$x = 3, \quad y = -3(3)$$

$$= -9$$

$$\therefore P(3, -9)$$

$$\begin{aligned} \text{c) } \int (x^{3/2} - 3x) dx &= \frac{x^{5/2}}{5/2} - \frac{3x^2}{2} + C \\ &= \underline{\underline{\frac{2}{5} x^{5/2} - \frac{3}{2} x^2 + C}} \end{aligned}$$

d) shaded region = area of Δ - area under curve = $\frac{81}{2} - 24 \cdot 3 = \underline{\underline{16.2}}$

$$\text{Area of } \Delta = \frac{9 \times 9}{2} = \underline{\underline{\frac{81}{2}}}$$

$$\begin{aligned} \text{area under curve} &= \left[\frac{2}{5} x^{5/2} - \frac{3}{2} x^2 \right]_0^9 \\ &= 24 \cdot 3 \end{aligned}$$