Centre Number		Candidate Number	
Surname			
Other Names			
Candidate Signature	WRITTEN SOLUTIONS		

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
TOTAL	

AQA

General Certificate of Education
Advanced Subsidiary Examination
June 2015

Mathematics MPC1

Unit Pure Core 1

Wednesday 13 May 2015 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You must **not** use a calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The use of calculators is **not** permitted.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

Do not write outside the box

Answer **all** questions.

Answer each question in the space provided for that question.

1 The line AB has equation $3x + 5y = 7$.

(a) Find the gradient of AB.

[2 marks]

(b) Find an equation of the line that is perpendicular to the line AB and which passes through the point $(-2, -3)$. Express your answer in the form $px + qy + r = 0$, where p, q and r are integers.

[3 marks]

(c) The line AC has equation $2x - 3y = 30$. Find the coordinates of A.

[3 marks]

QUESTION PART REFERENCE

Answer space for question 1

1a) needs to be in form

$y = mx + c$

$m \rightarrow$ gradient

$c \rightarrow$ y intercept

$3x + 5y = 7$

$5y = -3x + 7$

$y = -\frac{3}{5}x + \frac{7}{5}$

gradient is $-\frac{3}{5}$

b) perpendicular gradient $\rightarrow \frac{5}{3}$ (m)

coordinate $\rightarrow (-2, -3)$ (x_1, y_1)

$y - y_1 = m(x - x_1)$

$y + 3 = 5/3\,(x + 2)$

$3y + 9 = 5x + 10$

$5x - 3y + 1 = 0$

Do not write outside the box

QUESTION PART REFERENCE

Answer space for question 1

c) AC intersects with AB at point A

$3x + 5y = 7$ ① $(\times 3)$

$2x - 3y = 30$ ② $(\times 5)$

$9x + 15y = 21$

$+$ $10x - 15y = 150$

$19x = 171$ $(\div 19)$

$x = 9$ → sub in to find y into equation ①

$3(9) + 5y = 7$

$27 + 5y = 7$

$5y = -20$

$y = -4$

$\therefore$ point A is $(9, -4)$

Turn over ▶

Do not write outside the box

2 The point P has coordinates $(\sqrt{3}, 2\sqrt{3})$ and the point Q has coordinates $(\sqrt{5}, 4\sqrt{5})$. Show that the gradient of PQ can be expressed as $n + \sqrt{15}$, stating the value of the integer n.

[5 marks]

QUESTION PART REFERENCE

Answer space for question 2

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad \begin{matrix} x_1 & y_1 \\ (\sqrt{3}, & 2\sqrt{3}) \\ x_2 & y_2 \\ (\sqrt{5}, & 4\sqrt{5}) \end{matrix}$$

$$m = \frac{4\sqrt{5} - 2\sqrt{3}}{\sqrt{5} - \sqrt{3}}$$

$$= \frac{(4\sqrt{5} - 2\sqrt{3})}{(\sqrt{5} - \sqrt{3})} \times \frac{(\sqrt{5} + \sqrt{3})}{(\sqrt{5} + \sqrt{3})}$$

$$= \frac{4\sqrt{25} + 4\sqrt{15} - 2\sqrt{15} - 2\sqrt{9}}{\sqrt{25} - \sqrt{15} + \sqrt{15} - \sqrt{9}}$$

$$= \frac{5 \times 4 + 2\sqrt{15} - 2 \times 3}{5 - 3}$$

$$= \frac{20 + 2\sqrt{15} - 6}{2}$$

$$= \frac{14 + 2\sqrt{15}}{2}$$

$$= 7 + \sqrt{15}$$

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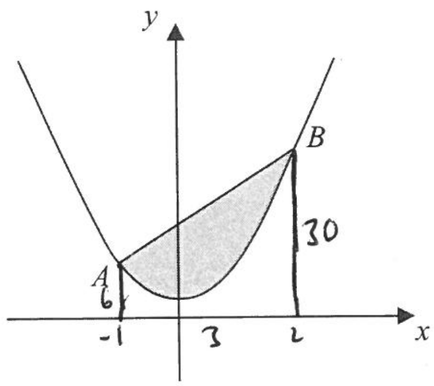
3 The diagram shows a sketch of a curve and a line.



The curve has equation $y = x^4 + 3x^2 + 2$. The points $A(-1,\ 6)$ and $B(2,\ 30)$ lie on the curve.

(a) Find an equation of the tangent to the curve at the point A.

[4 marks]

(b) (i) Find $\int_{-1}^{2} (x^4 + 3x^2 + 2)\, \mathrm{d}x$.

[5 marks]

(ii) Calculate the area of the shaded region bounded by the curve and the line AB.

[3 marks]

QUESTION PART REFERENCE	Answer space for question 3
3a)	equation of tangent :- need → gradient $\left(\frac{dy}{dx}\right)$
	→ coordinate (A)
	$y = x^4 + 3x^2 + 2$
	$\frac{dy}{dx} = 4x^3 + 6x$
	when $x = -1$,
	$\frac{dy}{dx} = 4(-1)^3 + 6(-1)$
	$= -4 - 6$
	$= -10$ (m) $\quad$ $x_1\ y_1$ $(-1, 6)$

Do not write outside the box

QUESTION PART REFERENCE

Answer space for question 3

$y - y_1 = m(x - x_1)$

$y - 6 = -10(x + 1)$

$y - 6 = -10x - 10$

$\underline{y = -10x - 4}$

bi) $\int_{-1}^{2} (x^4 + 3x^2 + 2)\,dx$

$\left[\frac{x^5}{5} + \frac{3x^3}{3} + 2x\right]_{-1}^{2}$

$\left[\frac{x^5}{5} + x^3 + 2x\right]_{-1}^{2}$

$\left(\frac{(2)^5}{5} + (2)^3 + 2(2)\right) - \left(\frac{(-1)^5}{5} + (-1)^3 + 2(-1)\right)$

$\left(\frac{32}{5} + 8 + 4\right) - \left(-\frac{1}{5} + -1 - 2\right)$

$\left(\frac{32}{5} + 12\right) - \left(-3\frac{1}{5}\right) = \frac{32}{5} + 12 + 3\frac{1}{5}$

$= 6\,^2/_5 + 12 + 3\,^1/_5$

$= \underline{\underline{21\,^3/_5}}$

ii) shaded area = area of trapezium - area under curve

area of trapezium $= \frac{6 + 30}{2} \times 3 = \underline{54}$

shaded area $= 54 - 21\,^3/_5$

$= \underline{\underline{32\,^2/_5}}$

Turn over ▶

Do not write outside the box

4 A circle with centre C has equation $x^2 + y^2 + 2x - 6y - 40 = 0$.

(a) Express this equation in the form

$$(x-a)^2 + (y-b)^2 = d$$

[3 marks]

(b) (i) State the coordinates of C. **[1 mark]**

(ii) Find the radius of the circle, giving your answer in the form $n\sqrt{2}$. **[2 marks]**

(c) The point P with coordinates $(4, k)$ lies on the circle. Find the possible values of k. **[3 marks]**

(d) The points Q and R also lie on the circle, and the length of the chord QR is 2. Calculate the shortest distance from C to the chord QR. **[2 marks]**

QUESTION PART REFERENCE	Answer space for question 4
4a)	$x^2 + y^2 + 2x - 6y - 40 = 0$
	$x^2 + 2x + y^2 - 6y - 40 = 0$
	$(x+1)^2 - 1 + (y-3)^2 - 9 - 40 = 0$
	$(x+1)^2 + (y-3)^2 - 50 = 0$
	$\underline{(x+1)^2 + (y-3)^2 = 50}$
bi)	$C = \underline{(-1, 3)}$
ii)	$r^2 = 50$
	$r = \sqrt{50}$
	$r = \sqrt{25} \times \sqrt{2}$
	$r = \underline{5\sqrt{2}}$

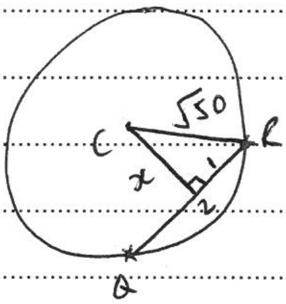
Do not write outside the box

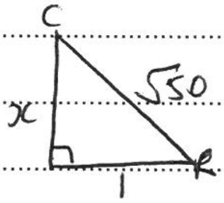
QUESTION PART REFERENCE	Answer space for question 4

c) $P(4, k)$ $\quad x = 4 \quad y = k \rightarrow$ sub into circle

$$(4)^2 + k^2 + 2(4) - 6k - 40 = 0$$
$$16 + k^2 + 8 - 6k - 40 = 0$$
$$k^2 - 6k - 16 = 0$$
$$(k-8)(k+2) = 0$$
$$\underline{k = 8} \text{ OR } \underline{k = -2}$$

d)





$$x^2 = (\sqrt{50})^2 - 1^2$$
$$x^2 = 50 - 1$$
$$x = \sqrt{49}$$
$$\underline{\underline{x = 7}}$$

Turn over ▶

5 (a) Express $x^2 + 3x + 2$ in the form $(x+p)^2 + q$, where p and q are rational numbers. **[2 marks]**

(b) A curve has equation $y = x^2 + 3x + 2$.

(i) Use the result from part **(a)** to write down the coordinates of the vertex of the curve. **[2 marks]**

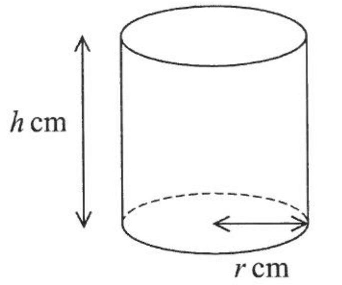
(ii) State the equation of the line of symmetry of the curve. **[1 mark]**

(c) The curve with equation $y = x^2 + 3x + 2$ is translated by the vector $\begin{bmatrix} 2 \\ 4 \end{bmatrix}$.

Find the equation of the resulting curve in the form $y = x^2 + bx + c$. **[3 marks]**

QUESTION PART REFERENCE	**Answer space for question 5**
5a)	$x^2 + 3x + 2$
	$\left(x + \frac{3}{2}\right)^2 - \frac{9}{4} + 2$ $\quad\left(2 = \frac{8}{4}\right)$
	$\left(x + \frac{3}{2}\right)^2 - \frac{1}{4}$ OR $(x+1.5)^2 - 0.25$
b i)	$\left(-3/2, -1/4\right)$ OR $(-1.5, -0.25)$
ii)	$x = -3/2$ OR $x = -1.5$
c)	translation $\begin{pmatrix} 2 \\ 4 \end{pmatrix}$ means 2 right, 4 up
	Replace 'x' with '$x-2$', then '$+4$'
	$y = (x-2)^2 + 3(x-2) + 2 + 4$
	$y = x^2 - 4x + 4 + 3x - 6 + 6$
	$y = x^2 - x + 4$

6 The diagram shows a cylindrical container of radius r cm and height h cm. The container has an **open** top and a circular base.



The **external** surface area of the container's curved surface and base is 48π cm^2.

When the radius of the base is r cm, the volume of the container is V cm^3.

(a) (i) Find an expression for h in terms of r.

[3 marks]

(ii) Show that $V = 24\pi r - \frac{\pi}{2}r^3$.

[2 marks]

(b) (i) Find $\frac{dV}{dr}$.

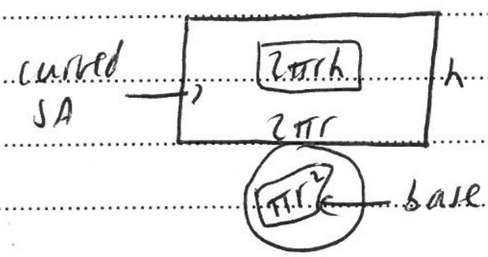
[2 marks]

(ii) Find the positive value of r for which V is stationary, and determine whether this stationary value is a maximum value or a minimum value.

[4 marks]

QUESTION PART REFERENCE	Answer space for question 6

6) external surface area = base + curved S.A



$= \pi r^2 + 2\pi r h$

so $\pi r^2 + 2\pi r h = 48\pi$

$2\pi r h = 48 - \pi r^2$

$h = \frac{48\pi - \pi r^2}{2\pi r} = \frac{48 - r^2}{2r}$

Do not write outside the box

QUESTION PART REFERENCE	Answer space for question 6

ii) $V = \pi r^2 h \longrightarrow \boxed{h = \frac{48 - r^2}{2r}}$ (from part a))

$V = \pi r^2 \left(\frac{48 - r^2}{2r}\right)$

$= \pi r \left(\frac{48 - r^2}{2}\right)$

$= \pi r \left(24 - \frac{r^2}{2}\right)$

$V = 24\pi r - \frac{\pi r^3}{2}$ (as required)

b i) $\frac{dV}{dr} = 24\pi - \frac{3\pi}{2} r^2$

ii) when V is stationary $\rightarrow$ $\frac{dV}{dr} = 0$

$24\pi - \frac{3\pi}{2} r^2 = 0$

$24\pi = \frac{3}{2}\pi r^2$ $(\div \frac{3}{2}\pi)$

$\frac{48\pi}{3\pi} = r^2$

$r^2 = 16$

$r = 4$

$\frac{d^2V}{dr^2} = \frac{-6}{2}\pi r$, when $r = 4$, $\frac{d^2V}{dr^2} = \frac{-6}{2} \times \pi \times 4$

$= -12\pi$

$\frac{d^2V}{dr^2} < 0 \therefore$ V is a maximum value

Turn over ▶

7 (a) Sketch the curve with equation $y = x^2(x-3)$.

[3 marks]

(b) The polynomial $p(x)$ is given by $p(x) = x^2(x-3) + 20$.

(i) Find the remainder when $p(x)$ is divided by $x - 4$.

[2 marks]

(ii) Use the Factor Theorem to show that $x + 2$ is a factor of $p(x)$.

[2 marks]

(iii) Express $p(x)$ in the form $(x+2)(x^2 + bx + c)$, where b and c are integers.

[2 marks]

(iv) Hence show that the equation $p(x) = 0$ has exactly one real root and state its value.

[3 marks]

QUESTION PART REFERENCE

Answer space for question 7

7a) crosses y axis when $x = 0$:-

$y = 0^2(0-3)$

$y = 0 \quad (0, 0)$

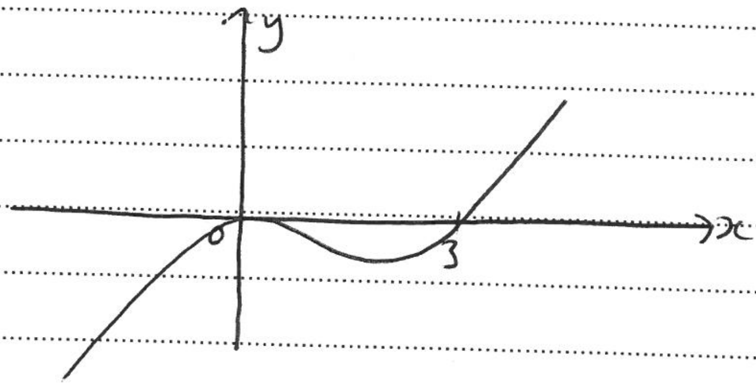
crosses x axis when $y = 0$:-

$0 = x^2(x-3)$

$x^2 = 0$ (touches)

$x - 3 = 0$

$x = 3$ (crosses)



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QUESTION PART REFERENCE	Answer space for question 7
b i)	$p(x) = x^2(x-3) + 20$
	$p(4) = 4^2(4-3) + 20$
	$= 16(1) + 20$
	$= 36$
	$\therefore$ remainder is 36 when divided by $(x-4)$
ii)	$p(-2) = (-2)^2(-2-3) + 20$
	$= 4(-5) + 20$
	$= -20 + 20 = 0$
	$\therefore$ $(x+2)$ is a factor of $p(x)$
iii)	$x^2(x-3) + 20 \rightarrow x^3 - 3x^2 + 20$
	$(x+2)(ax^2 + bx + c) = x^3 - 3x^2 + 20$
	$(x+2)(x^2 + bx + c)$
	$2c = 20$, so $\underline{c = 10}$
	$(x+2)(x^2 + bx + 10)$
	$2x^2 + bx^2 = -3x$
	$2 + b = -3$
	$\underline{b = -5}$
	$\underline{(x+2)(x^2 - 5x + 10)}$ (OR division)
iv)	$(x+2)(x^2 - 5x + 10) = 0$
	$\underline{x = -2}$ OR $x^2 - 5x + 10 = 0$
	$a = 1$, $b = -5$, $c = 10$
	if no solution, $b^2 - 4ac < 0$
	$(-5)^2 - 4(1)(10) = 25 - 40 = -15$

$\therefore$ only real root is $x = -2$

Turn over ▶

Do not write outside the box

8 A curve has equation $y = x^2 + (3k - 4)x + 13$ and a line has equation $y = 2x + k$, where k is a constant.

(a) Show that the x-coordinate of any point of intersection of the line and curve satisfies the equation

$$x^2 + 3(k-2)x + 13 - k = 0$$

[1 mark]

(b) Given that the line and the curve do not intersect:

(i) show that $9k^2 - 32k - 16 < 0$;

[3 marks]

(ii) find the possible values of k.

[4 marks]

QUESTION PART REFERENCE

Answer space for question 8

8a) $y = x^2 + (3k-4)x + 13$ intersects $y = 2x + k$

$2x + k = x^2 + (3k-4)x + 13$

$x^2 + (3k-4)x - 2x + 13 - k = 0$

$x^2 + 3(k-2)x + 13 - k = 0$ (as required)

from $3kx - 6x \rightarrow (3k-6)x$

b) no intersection = no real roots

so $b^2 - 4ac < 0$

$a = 1$, $b = 3(k-2)$, $c = 13 - k$

$(3k-6)^2 - 4(1)(13-k) < 0$

$(3k-6)(3k-6) - 4(13-k) < 0$

$9k^2 - 18k - 18k + 36 - 52 + 4k < 0$

$9k^2 - 32k - 16 < 0$ (as required)

Do not write outside the box

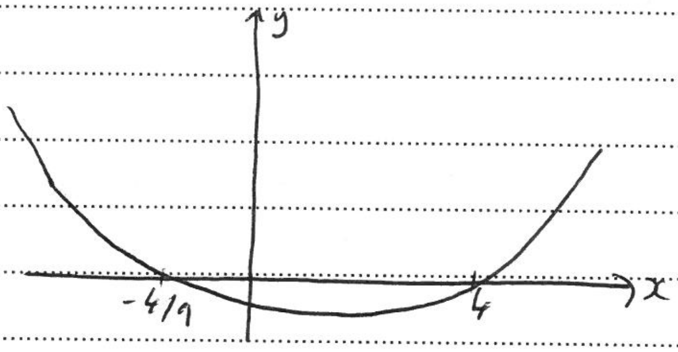
QUESTION PART REFERENCE	Answer space for question 8

ii) $9k^2 - 32k - 16$

$(9k + 4)(k - 4)$

$9k + 4 = 0 \qquad k - 4 = 0$

$k = -4/9 \qquad k = 4$ (crosses x axis)



$9k^2 - 32k - 16 < 0$ $\therefore$ graph below x axis

$-4/9 < k < 4$