Centre Number		Candidate Number	
Surname			
Other Names	WRITTEN SOLUTIONS		
Candidate Signature			

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
TOTAL	

AQA

General Certificate of Education
Advanced Subsidiary Examination
June 2014

Mathematics MPC1

Unit Pure Core 1

Monday 19 May 2014 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You must **not** use a calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The use of calculators is **not** permitted.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

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Answer **all** questions.

Answer each question in the space provided for that question.

1 The point A has coordinates $(-1, 2)$ and the point B has coordinates $(3, -5)$.

(a) (i) Find the gradient of AB.

[2 marks]

(ii) Hence find an equation of the line AB, giving your answer in the form $px + qy = r$, where p, q and r are integers.

[3 marks]

(b) The midpoint of AB is M.

(i) Find the coordinates of M.

[1 mark]

(ii) Find an equation of the line which passes through M and which is perpendicular to AB.

[3 marks]

(c) The point C has coordinates $(k, 2k + 3)$. Given that the distance from A to C is $\sqrt{13}$, find the two possible values of the constant k.

[4 marks]

QUESTION PART REFERENCE

Answer space for question 1

$A(-1, 2)$ $B(3, -5)$

1(a i) gradient $= \dfrac{-5-2}{3-(-1)} = \dfrac{-7}{3+1} = -\dfrac{7}{4}$

ii) use coordinate $\overset{x_1 \; y_1}{(-1, 2)}$ or $(3, -5)$

$m = -\dfrac{7}{4}$

$y - y_1 = m(x - x_1)$

$y - 2 = -\dfrac{7}{4}(x - (-1))$

$4y - 8 = -7(x+1)$

$4y - 8 = -7x - 7 \; (+7x)$

$7x + 4y - 8 = -7 \; (+8)$

$7x + 4y = 1$

Do not write outside the box

QUESTION PART REFERENCE	Answer space for question 1

$$\begin{matrix} x_1 & y_1 \\ (-1, & 2) \end{matrix} \quad \begin{matrix} x_2 & y_2 \\ (3, & -5) \end{matrix}$$

b i)

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$= \left(\frac{-1+3}{2}, \frac{2+(-5)}{2}\right)$$

$$= \left(\frac{2}{2}, \frac{-3}{2}\right) = \underline{\left(1, \frac{-3}{2}\right)}$$

ii)

grad perp to AB $\rightarrow \frac{4}{7}$ $\qquad M\,(\overset{x_1}{1}, \overset{y_1}{-3/2})$

$$y - y_1 = m(x - x_1)$$

$$y - \left(\frac{-3}{2}\right) = \frac{4}{7}(x-1)$$

$$y + \frac{3}{2} = \frac{4}{7}(x-1)$$

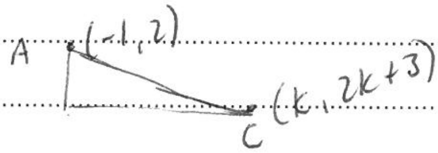
$$7y + \frac{21}{2} = 4(x-1)$$

$$7y + \frac{21}{2} = 4x - 4$$

$$7y + \frac{21}{2} + 4 = 4x \rightarrow 7y + \frac{29}{2} = 4x$$

$$\underline{\underline{7y = 4x - \frac{29}{2}}}$$

c)



$$\sqrt{(k-(-1))^2 + (2k+3-2)^2} = \sqrt{13}$$

$$\sqrt{(k+1)^2 + (2k+1)^2} = \sqrt{13}$$

$$k^2 + 2k + 1 + 4k^2 + 4k + 1 = 13$$

$$5k^2 + 6k - 11 = 0$$

$$(5k + 11)(k - 1) = 0$$

$$5k + 11 = 0 \quad \text{or} \quad k - 1 = 0$$

$$\underline{\underline{k = -11/5}} \qquad \underline{\underline{k = 1}}$$

Turn over ▶

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2 A rectangle has length $(9 + 5\sqrt{3})$ cm and area $(15 + 7\sqrt{3})$ cm^2.

Find the width of the rectangle, giving your answer in the form $(m + n\sqrt{3})$ cm, where m and n are integers.

[4 marks]

QUESTION PART REFERENCE

Answer space for question 2

Area = L x W

$15+7\sqrt{3} = 9+5\sqrt{3} \times W$

$W = \dfrac{15+7\sqrt{3}}{9+5\sqrt{3}}$ → now rationalise

$$\frac{(15+7\sqrt{3})}{(9+5\sqrt{3})} \times \frac{(9-5\sqrt{3})}{(9-5\sqrt{3})}$$

$$= \frac{135-75\sqrt{3}+63\sqrt{3}-35\sqrt{9}}{81-45\sqrt{3}+45\sqrt{3}-25\sqrt{9}}$$

$$= \frac{135-12\sqrt{3}-105}{81-75}$$

$$= \frac{30-12\sqrt{3}}{6}$$

$$= \underline{\underline{5-2\sqrt{3}}}$$

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QUESTION PART REFERENCE

Answer space for question 2

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3 A curve has equation $y = 2x^5 + 5x^4 - 1$.

(a) Find:

(i) $\dfrac{dy}{dx}$ **[2 marks]**

(ii) $\dfrac{d^2y}{dx^2}$ **[1 mark]**

(b) The point on the curve where $x = -1$ is P.

(i) Determine whether y is increasing or decreasing at P, giving a reason for your answer. **[2 marks]**

(ii) Find an equation of the tangent to the curve at P. **[3 marks]**

(c) The point $Q(-2, 15)$ also lies on the curve. Verify that Q is a maximum point of the curve. **[4 marks]**

QUESTION PART REFERENCE	Answer space for question 3
3a i)	$\dfrac{dy}{dx} = 10x^4 + 20x^3$
ii)	$\dfrac{d^2y}{dx^2} = 40x^3 + 60x^2$
b i)	when $x = -1$:-
	$\dfrac{dy}{dx} = 10(-1)^4 + 20(-1)^3$
	$= 10 - 20$
	$= -10$
	$\dfrac{dy}{dx} < 0 \therefore$ decreasing at P.

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QUESTION PART REFERENCE	Answer space for question 3
ii)	when $x = -1$:-
	$y = 2(-1)^5 + 5(-1)^4 - 1$
	$= -2 + 5 - 1$
	$= 2$
	$\therefore (\overset{x_1}{-1}, \overset{y_1}{2}) = P$ grad $m = -10$
	$y - y_1 = m(x - x_1)$
	$y - 2 = -10(x + 1)$
	$y - 2 = -10x - 10$
	$\underline{y = -10x - 8}$
c)	$Q(-2, 15)$
	when $x = -2$:-
	$\frac{dy}{dx} = 10(-2)^4 + 20(-2)^3$
	$= 160 - 160$
	$\frac{dy}{dx} = 0$ $\therefore$ stationary point at Q
	$\frac{d^2y}{dx^2} = 40(-2)^3 + 60(-2)^2$
	$= -320 + 240$
	$= -80$
	$\frac{d^2y}{dx^2} < 0$ $\therefore$ maximum point at Q

Turn over ▶

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4 (a) (i) Express $16 - 6x - x^2$ in the form $p - (x + q)^2$ where p and q are integers.

[2 marks]

(ii) Hence write down the maximum value of $16 - 6x - x^2$.

[1 mark]

(b) (i) Factorise $16 - 6x - x^2$.

[1 mark]

(ii) Sketch the curve with equation $y = 16 - 6x - x^2$, stating the values of x where the curve crosses the x-axis and the value of the y-intercept.

[3 marks]

QUESTION PART REFERENCE

Answer space for question 4

4ai) $16 - 6x - x^2$

$-x^2 - 6x + 16$

$-(x^2 + 6x - 16)$

$-[(x+3)^2 - 9 - 16]$

$= -[(x+3)^2 - 25]$

$= -(x+3)^2 + 25$

$= 25 - (x+3)^2$

ii) Maximum value at 25

bi) $16 - 6x - x^2$

$(8 + x)(2 - x)$

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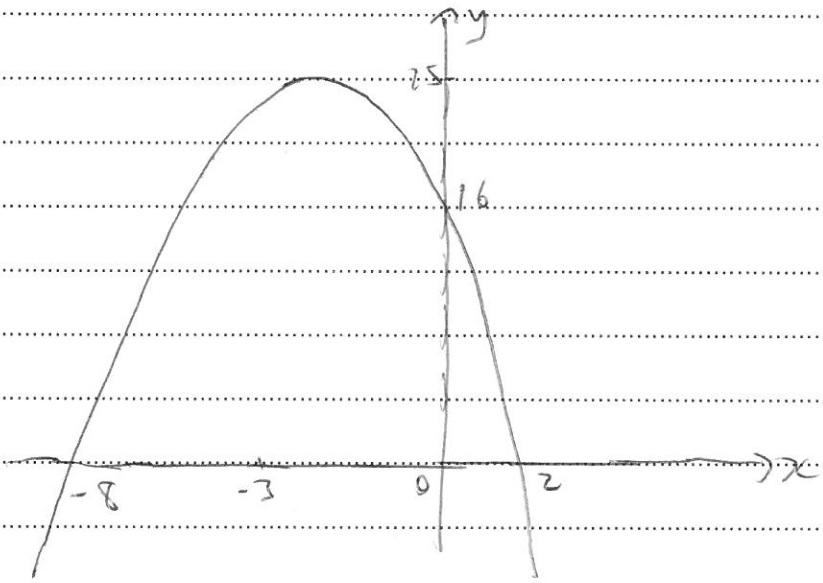
QUESTION PART REFERENCE

Answer space for question 4

ii) Graph crosses x axis at $x=-8$ and $x=2$

y axis at 16

shape as $-x^2$ graph



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5 The polynomial $p(x)$ is given by

$$p(x) = x^3 + cx^2 + dx + 3$$

where c and d are integers.

(a) Given that $x + 3$ is a factor of $p(x)$, show that

$$3c - d = 8$$

[2 marks]

(b) The remainder when $p(x)$ is divided by $x - 2$ is 65.

Obtain a further equation in c and d.

[2 marks]

(c) Use the equations from parts **(a)** and **(b)** to find the value of c and the value of d.

[3 marks]

QUESTION PART REFERENCE

Answer space for question 5

5a) $p(-3) = 0$ if $(x+3)$ is a factor of $p(x)$

$(-3)^3 + c(-3)^2 + d(-3) + 3 = 0$

$-27 + 9c - 3d + 3 = 0$

$-24 + 9c - 3d = 0 \quad (+24)$

$9c - 3d = 24 \quad (\div 3)$

$3c - d = 8$ (as required)

b) $p(2) = 65$

$(2)^3 + c(2)^2 + d(2) + 3 = 65$

$8 + 4c + 2d + 3 = 65$

$11 + 4c + 2d = 65 \quad (-11)$

$4c + 2d = 54 \quad (\div 2)$

$2c + d = 27$

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QUESTION PART REFERENCE

Answer space for question 5

c)

$$3c - d = 8 \quad (1)$$
$$+ \quad 2c + d = 27 \quad (2)$$

DIFFERENT SIGNS ADD

$$5c = 35$$
$$c = 7$$

→ sub in to (2)

$$2(7) + d = 27$$
$$14 + d = 27 \quad (-14)$$
$$d = 13$$

check :-

$$3(7) - 13 = 8$$
$$21 - 13 = 8 \checkmark$$

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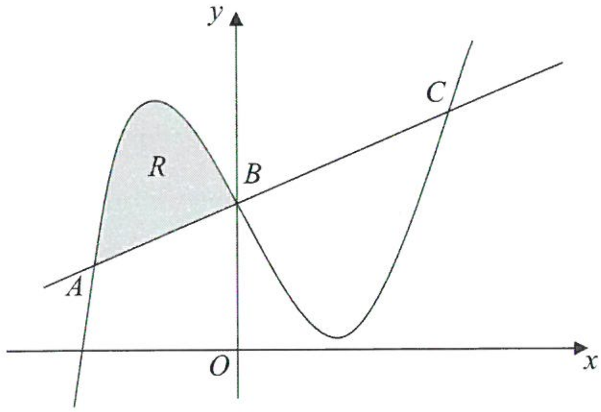
6 The diagram shows a curve and a line which intersect at the points A, B and C.



The curve has equation $y = x^3 - x^2 - 5x + 7$ and the straight line has equation $y = x + 7$. The point B has coordinates $(0, 7)$.

(a) (i) Show that the x-coordinates of the points A and C satisfy the equation

$$x^2 - x - 6 = 0$$

[2 marks]

(ii) Find the coordinates of the points A and C.

[3 marks]

(b) Find $\int (x^3 - x^2 - 5x + 7)\,dx$.

[3 marks]

(c) Find the area of the shaded region R bounded by the curve and the line segment AB.

[4 marks]

QUESTION PART REFERENCE	Answer space for question 6
6(a)(i)	$x^3 - x^2 - 5x + 7 = x + 7 \quad (-x)$
	$x^3 - x^2 - 6x + 7 = 7 \ (-7)$
	$x^3 - x^2 - 6x = 0$
	$x(x^2 - x - 6) = 0$
	$x \neq 0$ (point B)
	$\therefore$ A and C satisfy $x^2 - x - 6 = 0$ (as req)

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QUESTION PART REFERENCE	Answer space for question 6

ii) $x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

$x = 3, \quad x = -2$

At A, $x = -2$:-   At B, $x = 3$

$y = -2 + 7$ $y = 3 + 7$

$y = 5$ $y = 10$

$(-2, 5)$ (A) $(3, 10)$ (B)

b) $\int (x^3 - x^2 - 5x + 7)\, dx$

$= \frac{x^4}{4} - \frac{x^3}{3} - \frac{5x^2}{2} + 7x + c$

c) shaded region R = area under curve − trapezium

area under curve $= \left[\frac{x^4}{4} - \frac{x^3}{3} - \frac{5x^2}{2} + 7x\right]_{-2}^{0}$

curve $= 0 - \left(\frac{(-2)^4}{4} - \frac{(-2)^3}{3} - \frac{5(-2)^2}{2} + 7(-2)\right)$

$= 0 - (4 + 8/3 - 10 + 14) = \frac{52}{3}$

area of trapezium $= \frac{5+7}{2} \times 2 = 12$.

shaded region $= \frac{52}{3} - 12 = \frac{16}{3}$

Turn over ▶

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QUESTION PART REFERENCE

Answer space for question 6

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QUESTION PART REFERENCE

Answer space for question 6

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7 A circle with centre C has equation $x^2 + y^2 - 10x + 12y + 41 = 0$. The point $A(3,\ -2)$ lies on the circle.

(a) Express the equation of the circle in the form

$$(x-a)^2 + (y-b)^2 = k$$

[3 marks]

(b) (i) Write down the coordinates of C.

[1 mark]

(ii) Show that the circle has radius $n\sqrt{5}$, where n is an integer.

[2 marks]

(c) Find the equation of the tangent to the circle at the point A, giving your answer in the form $x + py = q$, where p and q are integers.

[5 marks]

(d) The point B lies on the tangent to the circle at A and the length of BC is 6. Find the length of AB.

[3 marks]

QUESTION PART REFERENCE

Answer space for question 7

7a) $x^2 - 10x + y^2 + 12y + 41 = 0$

$(x-5)^2 - 25 + (y+6)^2 - 36 + 41 = 0$

$\underline{(x-5)^2 + (y+6)^2 = 20}$

bi) $C = \underline{(5, -6)}$

ii) $r^2 = 20$

$r = \sqrt{20} = \sqrt{4}\sqrt{5}$

$= \underline{2\sqrt{5}}$

c) grad of $AC = \frac{-6-(-2)}{5-3} = \frac{-4}{2} = -2$

grad of tangent $= 1/2\ (m)$ $\quad A(\overset{x_1}{3}, \overset{y_1}{-2})$

$y + 2 = \frac{1}{2}(x-3)$

$2y + 4 = x - 3$

$\underline{x - 2y = 7}$

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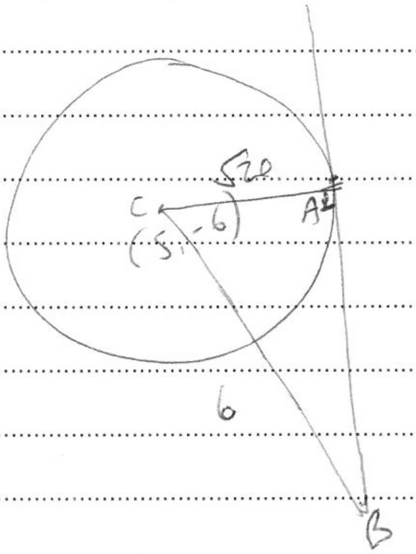
QUESTION PART REFERENCE

Answer space for question 7

d)



length AB :-

$$AB^2 = 6^2 - (\sqrt{20})^2$$
$$= 36 - 20$$
$$AB^2 = 16$$
$$AB = \sqrt{16}$$
$$\underline{AB = 4}$$

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QUESTION PART REFERENCE

Answer space for question 7

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QUESTION PART REFERENCE

Answer space for question 7

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8 Solve the following inequalities:

(a) $3(1-2x)-5(3x+2)>0$

[2 marks]

(b) $6x^2 \leqslant x+12$

[4 marks]

QUESTION PART REFERENCE

Answer space for question 8

8a) $3(1-2x)-5(3x+2)>0$

$3-6x-15x-10>0$

$-7-21x>0$ $(+7)$

$-21x>7$ $(\div 21)$

$-x>\frac{7}{21}$ $(\div -1 \rightarrow$ swap signs$)$

$x<-\frac{1}{3}$

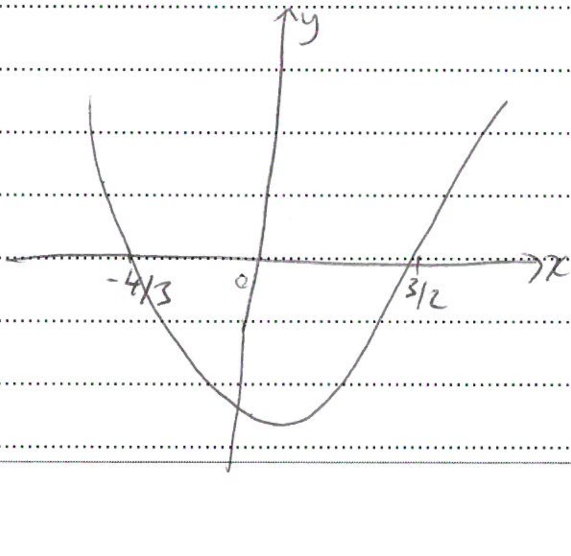
b) $6x^2 \leq x+12$

$6x^2-x-12 \leq 0$

$(3x+4)(2x-3) \leq 0$

$3x+4=0$ $2x-3=0$

$x=\frac{-4}{3}$ $x=\frac{3}{2}$ $\rightarrow$ critical values



$6x^2-x-12 \leq 0$

less than 0

$\therefore$ below x axis

$-4/3 \leq x \leq \frac{3}{2}$

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QUESTION PART REFERENCE

Answer space for question 8

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QUESTION PART REFERENCE	**Answer space for question 8**

END OF QUESTIONS