Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature	WRITTEN SOLUTIONS									

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
TOTAL	

General Certificate of Education
Advanced Subsidiary Examination
June 2013

Mathematics MPC1

Unit Pure Core 1

Monday 13 May 2013 1.30 pm to 3.00 pm

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You must **not** use a calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The use of calculators is **not** permitted.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

Do not write outside the box

Answer **all** questions.

Answer each question in the space provided for that question.

1 The line AB has equation $3x - 4y + 5 = 0$.

(a) The point with coordinates $(p, p+2)$ lies on the line AB. Find the value of the constant p. *(2 marks)*

(b) Find the gradient of AB. *(2 marks)*

(c) The point A has coordinates $(1, 2)$. The point $C(-5, k)$ is such that AC is perpendicular to AB. Find the value of k. *(3 marks)*

(d) The line AB intersects the line with equation $2x - 5y = 6$ at the point D. Find the coordinates of D. *(3 marks)*

QUESTION PART REFERENCE

Answer space for question 1

1a) $3x - 4y + 5 = 0$

$x = p, \quad y = p + 2$ → sub in

$3p - 4(p+2) + 5 = 0$

$3p - 4p - 8 + 5 = 0$

$-p - 3 = 0$

$\underline{\underline{p = -3}}$

b) $3x - 4y + 5 = 0$

$4y = 3x + 5 \quad (\div 4)$

$y = \frac{3}{4}x + \frac{5}{4}$

gradient is $\underline{\frac{3}{4}}$

Do not write outside the box

QUESTION PART REFERENCE

Answer space for question 1

c) gradient $AB = \frac{3}{4}$

perpendicular gradient $= -\frac{4}{3}$

gradient $= \frac{y_2 - y_1}{x_2 - x_1}$ $\quad \overset{x_1\ y_1}{(1, 2)} \quad \overset{x_2\ y_2}{(-5, k)}$

$-\frac{4}{3} = \frac{k-2}{-5-1}$

$-\frac{4}{3} = \frac{k-2}{-6}$

$24 = 3(k-2)$

$24 = 3k - 6$

$3k = 30$

$\underline{\underline{k = 10}}$

d) $AB \rightarrow 3x - 4y + 5 = 0$

$3x - 4y = -5 \quad (\times 5)$

$2x - 5y = 6 \quad (\times 4)$

$15x - 20y = -25$

$- \quad 8x - 20y = 24$

$7x = -49$

$\underline{x = -7}$, $\rightarrow$ sub in $2(-7) - 5y = 6$

$-14 - 5y = 6$

$-5y = 20$

$\underline{y = -4}$

$\underline{(-7, -4)}$ - D.

Do not write outside the box

2 (a) (i) Express $\sqrt{48}$ in the form $n\sqrt{3}$, where n is an integer. *(1 mark)*

(ii) Solve the equation

$$x\sqrt{12} = 7\sqrt{3} - \sqrt{48}$$

giving your answer in its simplest form. *(3 marks)*

(b) Express $\dfrac{11\sqrt{3} + 2\sqrt{5}}{2\sqrt{3} + \sqrt{5}}$ in the form $m - \sqrt{15}$, where m is an integer. *(4 marks)*

QUESTION PART REFERENCE

Answer space for question 2

2ai) $\sqrt{48} = \sqrt{16} \times \sqrt{3}$

$= \underline{\underline{4\sqrt{3}}}$

ii) $x\sqrt{12} = 7\sqrt{3} - \sqrt{48}$

$x\sqrt{12} = 7\sqrt{3} - 4\sqrt{3}$

$x\sqrt{12} = 3\sqrt{3}$

$x \times \sqrt{4} \times \sqrt{3} = 3\sqrt{3}$

$x \times 2 \times \sqrt{3} = 3\sqrt{3}$

$2x\cancel{\sqrt{3}} = 3\cancel{\sqrt{3}}$

$2x = 3$

$\underline{x = 3/2}$

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QUESTION PART REFERENCE	Answer space for question 2
b)	$\dfrac{(11\sqrt{3} + 2\sqrt{5})}{(2\sqrt{3} + \sqrt{5})} \times \dfrac{(2\sqrt{3} - \sqrt{5})}{(2\sqrt{3} - \sqrt{5})}$
	$\dfrac{22\sqrt{9} - 11\sqrt{15} + 4\sqrt{15} - 2\sqrt{25}}{4\sqrt{9} - 2\sqrt{15} + 2\sqrt{15} - \sqrt{25}}$
	$\dfrac{22 \times 3 - 7\sqrt{15} - 2 \times 5}{4 \times 3 - 5}$
	$\dfrac{66 - 7\sqrt{15} - 10}{12 - 5}$
	$\dfrac{56 - 7\sqrt{15}}{7}$
	$\underline{\underline{8 - \sqrt{15}}}$ $\quad$ $\underline{m = 8}$

Turn over ▶

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3 A circle C has equation

$$x^2 + y^2 - 10x + 14y + 25 = 0$$

(a) Write the equation of C in the form

$$(x - a)^2 + (y - b)^2 = k$$

where a, b and k are integers. *(3 marks)*

(b) Hence, for the circle C, write down:

(i) the coordinates of its centre; *(1 mark)*

(ii) its radius. *(1 mark)*

(c) (i) Sketch the circle C. *(2 marks)*

(ii) Write down the coordinates of the point on C that is furthest away from the x-axis. *(2 marks)*

(d) Given that k has the same value as in part **(a)**, describe geometrically the transformation which maps the circle with equation $(x+1)^2 + y^2 = k$ onto the circle C. *(3 marks)*

QUESTION PART REFERENCE

Answer space for question 3

3a) $x^2 - 10x + y^2 + 14y + 25 = 0$

$(x-5)^2 - 25 + (y+7)^2 - 49 + 25 = 0$

$(x-5)^2 + (y+7)^2 - 49 = 0$

$(x-5)^2 + (y+7)^2 = 49$

bi) centre is $(5, -7)$

ii) $r^2 = 49$

$r = 7$

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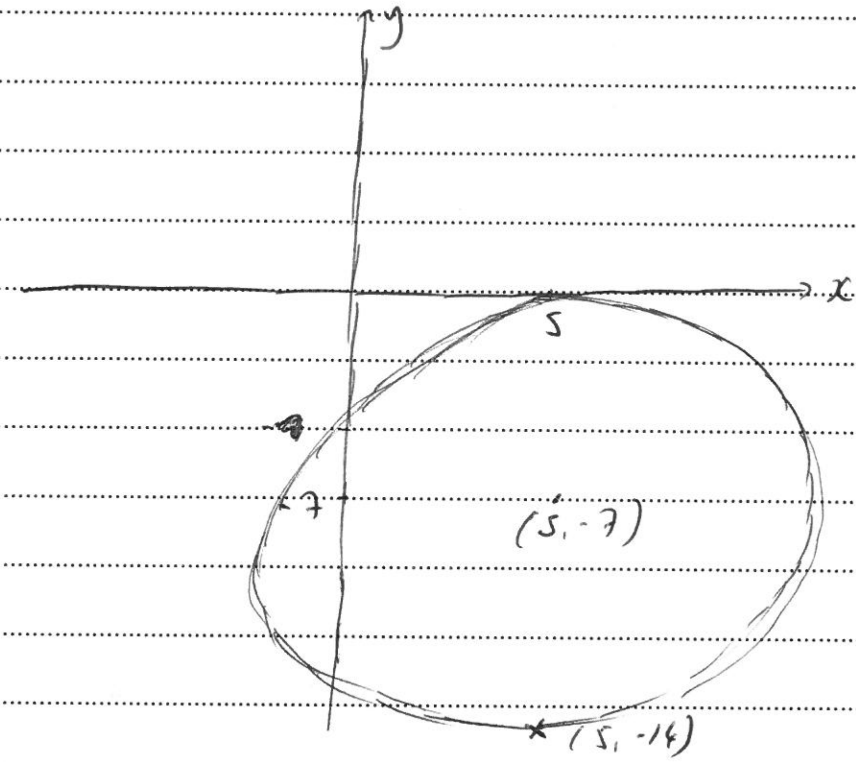
QUESTION PART REFERENCE	**Answer space for question 3**

c) i)



ii) Furthest point $(5, -14)$

d) $K = 49$

$\therefore$ same size circle of radius 7

centre is $(-1, 0)$ to centre $(5, -7)$

Translation $\begin{pmatrix} 6 \\ -7 \end{pmatrix}$

Do not write outside the box

4 (a) The polynomial $f(x)$ is given by $f(x) = x^3 - 4x + 15$.

(i) Use the Factor Theorem to show that $x + 3$ is a factor of $f(x)$. *(2 marks)*

(ii) Express $f(x)$ in the form $(x + 3)(x^2 + px + q)$, where p and q are integers. *(2 marks)*

(b) A curve has equation $y = x^4 - 8x^2 + 60x + 7$.

(i) Find $\frac{dy}{dx}$. *(3 marks)*

(ii) Show that the x-coordinates of any stationary points of the curve satisfy the equation

$$x^3 - 4x + 15 = 0$$ *(1 mark)*

(iii) Use the results above to show that the only stationary point of the curve occurs when $x = -3$. *(2 marks)*

(iv) Find the value of $\frac{d^2y}{dx^2}$ when $x = -3$. *(3 marks)*

(v) Hence determine, with a reason, whether the curve has a maximum point or a minimum point when $x = -3$. *(1 mark)*

QUESTION PART REFERENCE

Answer space for question 4

4ai) $f(x) = x^3 - 4x + 15$

$f(-3) = (-3)^3 - 4(-3) + 15$

$= -27 + 12 + 15$

$= 0$ $\therefore$ $(x+3)$ is a factor of $f(x)$

ii)

$$\begin{array}{r} x^2 - 3x + 5 \\ x+3 \overline{) x^3 + 0x^2 - 4x + 15} \\ -\underline{x^3 + 3x^2} \\ 0 - 3x^2 - 4x \\ -\underline{-3x^2 - 9x} \\ 0 + 5x + 15 \\ \underline{5x + 15} \\ 0 \end{array}$$

$(x+3)(x^2 - 3x + 5)$

Do not write outside the box

QUESTION PART REFERENCE

Answer space for question 4

b i) $y = x^4 - 8x^2 + 60x + 7$

$\frac{dy}{dx} = 4x^3 - 16x + 60$

ii) $\frac{dy}{dx} = 0$ if stationary point

$4x^3 - 16x + 60 = 0 \quad (\div 4)$

$x^3 - 4x + 15 = 0$ (as required)

iii) $x^3 - 4x + 15 = 0$

$(x+3)(x^2 - 3x + 5) = 0$

$x = -3$ or $x^2 - 3x + 5 = 0 \quad a = 1, b = -3, c = 5$

$b^2 - 4ac \rightarrow (-3)^2 - 4(1)(5)$

$9 - 20 = -11$

$b^2 - 4ac < 0 \therefore$ no real root

So only solution is $x = -3$

iv) $\frac{dy}{dx} = 4x^3 - 16x + 60$

$\frac{d^2y}{dx^2} = 12x^2 - 16$, when $x = -3$:-

$\frac{d^2y}{dx^2} = 12(-3)^2 - 16$

$= 108 - 16 = 92$

v) $\frac{d^2y}{dx^2} > 0 \therefore$ a minimum point

QUESTION PART REFERENCE

Answer space for question 4

Do not write outside the box

QUESTION PART REFERENCE

Answer space for question 4

Turn over ▶

5 (a) (i) Express $2x^2 + 6x + 5$ in the form $2(x+p)^2 + q$, where p and q are rational numbers. *(2 marks)*

(ii) Hence write down the minimum value of $2x^2 + 6x + 5$. *(1 mark)*

(b) The point A has coordinates $(-3, 5)$ and the point B has coordinates $(x, 3x+9)$.

(i) Show that $AB^2 = 5(2x^2 + 6x + 5)$. *(3 marks)*

(ii) Use your result from part **(a)(ii)** to find the minimum value of the length AB as x varies, giving your answer in the form $\frac{1}{2}\sqrt{n}$, where n is an integer. *(2 marks)*

QUESTION PART REFERENCE

Answer space for question 5

5ai) $2x^2 + 6x + 5$

$2\left[x^2 + 3x + \frac{5}{2}\right]$

$2\left[\left(x+\frac{3}{2}\right)^2 - \frac{9}{4} + \frac{5}{2}\right]$

$2\left[\left(x+\frac{3}{2}\right)^2 + \frac{1}{4}\right]$

$2\left(x+\frac{3}{2}\right)^2 + \frac{1}{2}$

ii) minimum value is $1/2$

b) $A(\underset{x_1}{-3}, \underset{y_1}{5})$ $\quad B(\underset{x_2}{x}, \underset{y_2}{3x+9})$ (pythagoras formula)

$AB^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

$AB^2 = (x - -3)^2 + (3x + 9 - 5)^2$

$AB^2 = (x+3)^2 + (3x+4)^2$ $\quad$ $(3x+4)(3x+4)$ $\quad 9x^2 + 12x + 12x + 16$

$AB^2 = x^2 + 6x + 9 + 9x^2 + 24x + 16$

$AB^2 = 10x^2 + 30x + 25$

$AB^2 = 5(2x^2 + 6x + 5)$ (as req)

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QUESTION PART REFERENCE

Answer space for question 5

ii) minimum value of $2x^2 + 6x + 5$ is $1/2$

$AB^2 = 5(1/2)$

$AB^2 = \frac{5}{2}$

$AB = \sqrt{5/2} = \frac{\sqrt{5}}{\sqrt{2}} \quad \left(\frac{\sqrt{5}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}\right)$

$= \frac{\sqrt{10}}{2} = \frac{1}{2}\sqrt{10}$

6 A curve has equation $y = x^5 - 2x^2 + 9$. The point P with coordinates $(-1, 6)$ lies on the curve.

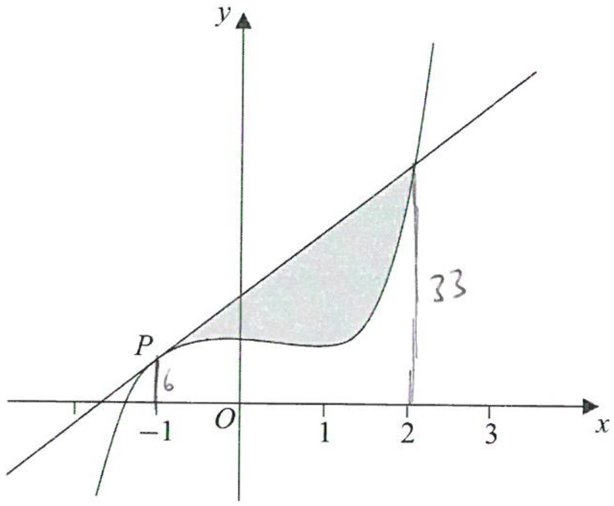
(a) Find the equation of the tangent to the curve at the point P, giving your answer in the form $y = mx + c$. *(5 marks)*

(b) The point Q with coordinates $(2, k)$ lies on the curve.

(i) Find the value of k. *(1 mark)*

(ii) Verify that Q also lies on the tangent to the curve at the point P. *(1 mark)*

(c) The curve and the tangent to the curve at P are sketched below.



(i) Find $\int_{-1}^{2} (x^5 - 2x^2 + 9)\,dx$. *(5 marks)*

(ii) Hence find the area of the shaded region bounded by the curve and the tangent to the curve at P. *(3 marks)*

QUESTION PART REFERENCE	**Answer space for question 6**
6a)	$y = x^5 - 2x^2 + 9 \qquad P(-1, 6)$
	$\frac{dy}{dx} = 5x^4 - 4x$, when $x = -1$ $\frac{dy}{dx} = 5(-1)^4 - 4(-1)$
	$\frac{dy}{dx} = 9$
	$y - 6 = 9(x+1)$
	$y - 6 = 9x + 9$
	$\underline{y = 9x + 15}$

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QUESTION PART REFERENCE	Answer space for question 6
bi)	$Q(2, k)$, $x=2$, $y=k$
	~~y~~ ~~k~~ $= 2^5 - 2(2)^2 + 9$
	$= 32 - 8 + 9 = \underline{\underline{33}}$
ii)	sub in $x=2$ and $y=33$ into
	$y = 9x + 15$
	$33 = 9(2) + 15$
	$33 = 33$ ✓
	$\therefore$ Q lies on the tangent
ci)	$\int_{-1}^{2} (x^5 - 2x^2 + 9)\,dx$
	$\left[\frac{x^6}{6} - \frac{2x^3}{3} + 9x\right]_{-1}^{2}$
	$\left(\frac{2^6}{6} - \frac{2(2)^3}{3} + 9(2)\right) - \left(\frac{(-1)^6}{6} - \frac{2(-1)^3}{3} + 9(-1)\right)$
	$\left(\frac{64}{6} - \frac{16}{3} + 18\right) - \left(\frac{1}{6} + \frac{2}{3} - 9\right) = \underline{\underline{31.5}}$
ii)	shaded region = trapezium - curve
	trapezium $= \frac{6+33}{2} \times 3 = \frac{117}{2}$
	shaded $= \frac{117}{2} - \frac{63}{2} = \frac{54}{2} = \underline{\underline{27}}$

Turn over ▶

7 The quadratic equation

$$(2k-7)x^2-(k-2)x+(k-3)=0$$

has real roots.

(a) Show that $7k^2-48k+80 \leqslant 0$. *(4 marks)*

(b) Find the possible values of k. *(4 marks)*

QUESTION PART REFERENCE

Answer space for question 7

7) $a = 2k-7 \quad b = -(k-2) \quad c = k-3$

$= -k+2$

a) $b^2-4ac \geq 0$ if real roots

$(-k+2)^2 - 4(2k-7)(k-3) \geq 0$

$(-k+2)(-k+2) - 4(2k^2-6k-7k+21) \geq 0$

$k^2-2k-2k+4-4(2k^2-13k+21) \geq 0$

$k^2-4k+4-8k^2+52k-84 \geq 0$

$-7k^2+48k-80 \geq 0$

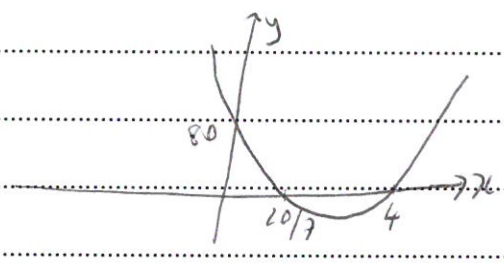
$7k^2-48k+80 \leq 0$ (as req.)

b) $7k^2-48k+80 \leq 0$

$(7k-20)(k-4) \leq 0$

$7k-20=0 \qquad k-4=0$

$k = 20/7 \qquad k=4$



graph is less than 0 (below x axis)

$\therefore 20/7 \leq k \leq 4$

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QUESTION PART REFERENCE

Answer space for question 7

Turn over ▶

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QUESTION PART REFERENCE

Answer space for question 7

END OF QUESTIONS