Centre Number						Candidate Number				
Surname										
Other Names	ANSWERS									
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
TOTAL	

General Certificate of Education
Advanced Subsidiary Examination
June 2012

Mathematics MPC1

Unit Pure Core 1

Wednesday 16 May 2012 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You must **not** use a calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The use of calculators is **not** permitted.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

Do not write outside the box

Answer **all** questions.

Answer each question in the space provided for that question.

1 Express $\dfrac{5\sqrt{3}-6}{2\sqrt{3}+3}$ in the form $m+n\sqrt{3}$, where m and n are integers. *(4 marks)*

QUESTION PART REFERENCE

Answer space for question 1

1) $\dfrac{(5\sqrt{3}-6)}{(2\sqrt{3}+3)} \times \dfrac{(2\sqrt{3}-3)}{(2\sqrt{3}-3)} = \dfrac{10(\sqrt{3})^2-15\sqrt{3}-12\sqrt{3}+18}{4(\sqrt{3})^2-6\sqrt{3}+6\sqrt{3}-9}$

$= \dfrac{30-27\sqrt{3}+18}{12-9}$

$= \dfrac{48-27\sqrt{3}}{3}$

$= 16-9\sqrt{3}$

$m=16, \ n=-9$

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2 The line AB has equation $4x - 3y = 7$.

(a) (i) Find the gradient of AB. *(2 marks)*

(ii) Find an equation of the straight line that is parallel to AB and which passes through the point $C(3, -5)$, giving your answer in the form $px + qy = r$, where p, q and r are integers. *(3 marks)*

(b) The line AB intersects the line with equation $3x - 2y = 4$ at the point D. Find the coordinates of D. *(3 marks)*

(c) The point E with coordinates $(k - 2, 2k - 3)$ lies on the line AB. Find the value of the constant k. *(2 marks)*

QUESTION PART REFERENCE

Answer space for question 2

2ai) $4x - 3y = 7$

$3y = 4x - 7$

$y = \frac{4}{3}x - \frac{7}{3}$ gradient is $4/3$

ii) $y + 5 = \frac{4}{3}(x - 3)$

$3y + 15 = 4x - 12$

$4x - 3y = 27$ (as req)

b) $4x - 3y = 7$ ① $(\times 2)$

$3x - 2y = 4$ ② $(\times 3)$

$8x - 6y = 14$ ③

$9x - 6y = 12$ ④ ④ - ③

$x = -2$, sub in ① to find y:-

$4(-2) - 3y = 7$

$-8 - 3y = 7$ $(-2, -5) \rightarrow D$

$-3y = 15$, $y = -5$

Do not write outside the box

QUESTION PART REFERENCE

Answer space for question 2

c) $x = k - 2$

$y = 2k - 3$

so, $4(k-2) - 3(2k-3) = 7$

$4k - 8 - 6k + 9 = 7$

$-2k + 1 = 7$

$-2k = 6$

$\underline{k = -3}$

Do not write outside the box

3 The polynomial $p(x)$ is given by

$$p(x) = x^3 + 2x^2 - 5x - 6$$

(a) (i) Use the Factor Theorem to show that $x + 1$ is a factor of $p(x)$. *(2 marks)*

(ii) Express $p(x)$ as the product of three linear factors. *(3 marks)*

(b) Verify that $p(0) > p(1)$. *(2 marks)*

(c) Sketch the curve with equation $y = x^3 + 2x^2 - 5x - 6$, indicating the values where the curve crosses the x-axis. *(3 marks)*

QUESTION PART REFERENCE

Answer space for question 3

3ai) $p(x) = x^3 + 2x^2 - 5x - 6$

$p(-1) = (-1)^3 + 2(-1)^2 - 5(-1) - 6$

$= -1 + 2 + 5 - 6$

$= 0$, $\therefore$ $(x+1)$ is a factor of $p(x)$

ii)

$$\begin{array}{r l} & x^2 + x - 6 \\ x+1 & \overline{)\, x^3 + 2x^2 - 5x - 6} \\ & -\underline{x^3 + x^2} \\ & 0 + x^2 - 5x \\ & -\underline{x^2 + x} \\ & 0 \cancel{+} 6x - 6 \\ & \underline{-6x - 6} \\ & 0 + 0 \end{array}$$

$(x+1)(x^2 + x - 6)$

$\underline{(x+1)(x+3)(x-2)}$

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QUESTION PART REFERENCE

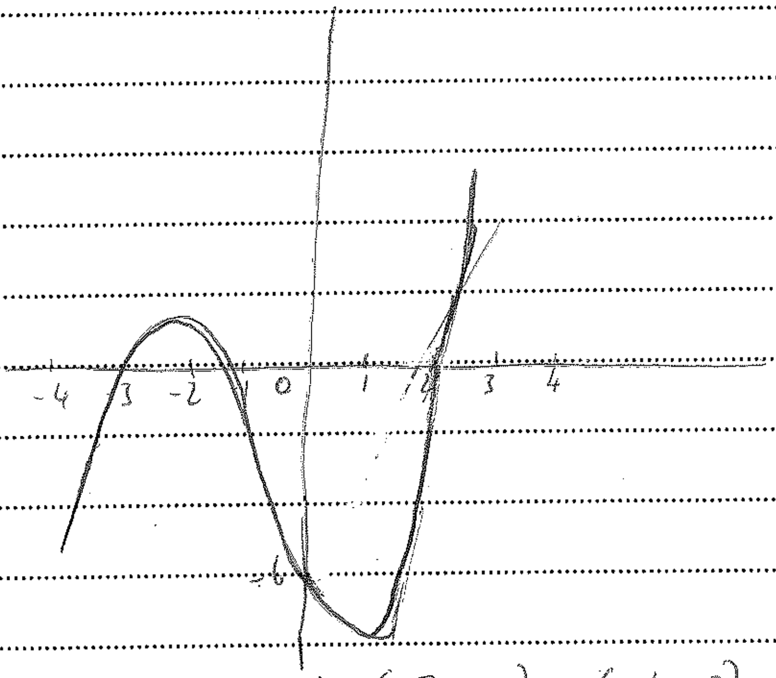
Answer space for question 3

b) $p(0) = (0)^3 + 2(0)^2 - 5(0) - 6$
$= -6$

$p(1) = (1)^3 + 2(1)^2 - 5(1) - 6$
$= 1 + 2 - 5 - 6$
$= -8$

$\therefore p(1) < p(0)$ as req

c)



crosses at $(-3, 0)$, $(-1, 0)$, $(2, 0)$
and $(0, -6)$

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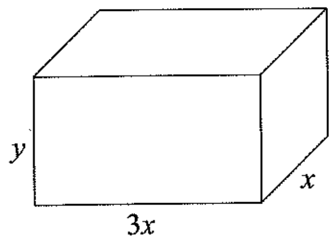
4 The diagram shows a solid cuboid with sides of lengths x cm, $3x$ cm and y cm.



The total surface area of the cuboid is 32 cm^2.

(a) (i) Show that $3x^2 + 4xy = 16$. *(2 marks)*

(ii) Hence show that the volume, $V\text{ cm}^3$, of the cuboid is given by

$$V = 12x - \frac{9x^3}{4}$$ *(2 marks)*

(b) Find $\frac{dV}{dx}$. *(2 marks)*

(c) (i) Verify that a stationary value of V occurs when $x = \frac{4}{3}$. *(2 marks)*

(ii) Find $\frac{d^2V}{dx^2}$ and hence determine whether V has a maximum value or a minimum value when $x = \frac{4}{3}$. *(2 marks)*

QUESTION PART REFERENCE

Answer space for question 4

4ai) $S.A. = 3xy + 3xy + xy + xy + 3x^2 + 3x^2$

$32 = 8xy + 6x^2 \ (\div 2)$

$3x^2 + 4xy = 16$ (as req)

ii) $V = 3x^2y$

$3x^2 + 4xy = 16$

$4xy = 16 - 3x^2$

$\boxed{y = \frac{16 - 3x^2}{4x}}$

$V = 3x^2\left(\frac{16 - 3x^2}{4x}\right)$

$= \frac{48x^2}{4x} - \frac{9x^4}{4x}$

$= 12x - \frac{9x^3}{4}$ (as req)

QUESTION PART REFERENCE

Answer space for question 4

b) $\frac{dV}{dx} = 12 - \frac{27x^2}{4}$

c) i) when $x = 4/3$:-

$\frac{dV}{dx} = 12 - \frac{27(4/3)^2}{4}$

$= 12 - \frac{27(16)}{4(9)}$

$= 12 - 3(4)$

$= 12 - 12$

$= 0$ $\therefore$ stationary point when $\frac{dV}{dx} = 0$

ii) $\frac{d^2V}{dx^2} = \frac{-54x}{4}$

when $x = 4/3$,

$\frac{d^2V}{dx^2} = \frac{-54(4/3)}{4}$

$= \frac{-54(4)}{12}$

$= -18$

$\frac{d^2y}{dx^2} < 0$ $\therefore$ maximum point when $x = 4/3$

Turn over ▶

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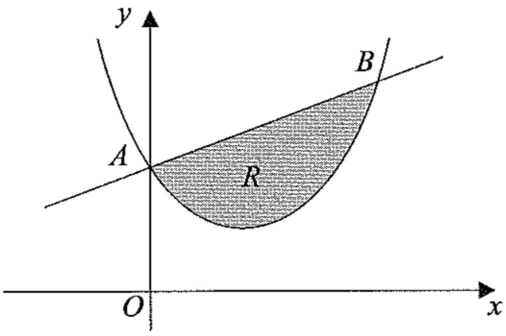
5 (a) (i) Express $x^2 - 3x + 5$ in the form $(x - p)^2 + q$. *(2 marks)*

(ii) Hence write down the equation of the line of symmetry of the curve with equation $y = x^2 - 3x + 5$. *(1 mark)*

(b) The curve C with equation $y = x^2 - 3x + 5$ and the straight line $y = x + 5$ intersect at the point $A(0, 5)$ and at the point B, as shown in the diagram below.



(i) Find the coordinates of the point B. *(3 marks)*

(ii) Find $\int (x^2 - 3x + 5)\,dx$. *(3 marks)*

(iii) Find the area of the shaded region R bounded by the curve C and the line segment AB. *(4 marks)*

QUESTION PART REFERENCE	Answer space for question 5
5ai)	$x^2 - 3x + 5 = \left(x - \frac{3}{2}\right)^2 + \frac{11}{4}$
ii)	$x = \frac{3}{2}$
bi)	$x + 5 = x^2 - 3x + 5$
	$x^2 - 4x = 0$
	$x(x - 4) = 0$
	$x = 0,\quad x = 4,\quad y = 4 + 5$
	(A) $\qquad y = 9 \qquad \underline{(4, 9) \to B}$

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QUESTION PART REFERENCE

Answer space for question 5

ii) $\int (x^2 - 3x + 5)\,dx = \frac{x^3}{3} - \frac{3x^2}{2} + 5x + c$

iii) area under curve :-

$\int_0^4 (x^2 - 3x + 5)\,dx = \left[\frac{x^3}{3} - \frac{3x^2}{2} + 5x\right]_0^4$

$= \left(\frac{4^3}{3} - \frac{3(4)^2}{2} + 5(4)\right) - 0$

$= \left(\frac{64}{3} - 24 + 20\right)$

$= \frac{64}{3} - \frac{72}{3} + \frac{60}{3}$

$= \frac{52}{3} = 17\tfrac{1}{3}$

area under trapezium :- $(0, 5)$ $(4, 9)$

$\left(\frac{5+9}{2}\right) \times 4 = 28$

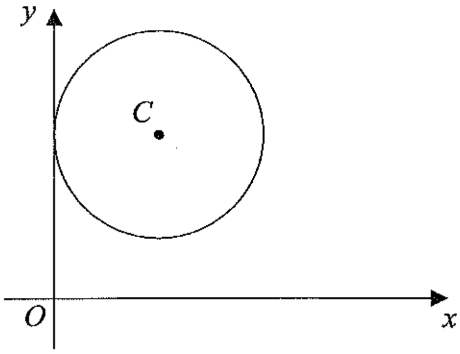
shaded region = trapezium - area under curve

$= 28 - \frac{52}{3}$

$= \frac{84}{3} - \frac{52}{3} = \frac{32}{3}$ OR $10\frac{2}{3}$

Turn over ▶

6 The circle with centre $C(5, 8)$ touches the y-axis, as shown in the diagram.



(a) Express the equation of the circle in the form

$$(x-a)^2 + (y-b)^2 = k$$ *(2 marks)*

(b) (i) Verify that the point $A(2, 12)$ lies on the circle. *(1 mark)*

(ii) Find an equation of the tangent to the circle at the point A, giving your answer in the form $sx + ty + u = 0$, where s, t and u are integers. *(5 marks)*

(c) The points P and Q lie on the circle, and the mid-point of PQ is $M(7, 12)$.

(i) Show that the length of CM is $n\sqrt{5}$, where n is an integer. *(2 marks)*

(ii) Hence find the area of triangle PCQ. *(3 marks)*

QUESTION PART REFERENCE	**Answer space for question 6**
5a)	$C:(5,8)$, $r=5$
	$(x-5)^2+(y-8)^2=25$
b i)	$(2,12)$:-
	$(2-5)^2+(12-8)^2=25$
	$(-3)^2+(4)^2=25$
	$9+16=25$
	$\therefore (2,12)$ lies on circle

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QUESTION PART REFERENCE	Answer space for question 6
ii)	gradient of normal AC :-
	A(2, 12) , C(5, 8)
	$\frac{12-8}{2-5} = \frac{-4}{3}$
	gradient of tangent is $\frac{3}{4}$
	$y-12 = \frac{3}{4}(x-2)$
	$4y-48 = 3x-6$
	$3x-4y+42=0$ (as req)
c) i)	$CM = \sqrt{(7-5)^2+(12-8)^2}$
	$= \sqrt{2^2+4^2}$
	$= \sqrt{20}$
	$= \sqrt{4}\sqrt{5}$
	$= 2\sqrt{5}$, $n=2$
ii)	$PM^2 = PC^2 - CM^2$
	$= 25-20$
	$PM = \sqrt{5}$
	area $= \frac{1}{2} \times \sqrt{5} \times \sqrt{20}$
	$= 10$

Turn over ▶

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7 The gradient, $\frac{dy}{dx}$, of a curve C at the point (x, y) is given by

$$\frac{dy}{dx} = 20x - 6x^2 - 16$$

(a) (i) Show that y is increasing when $3x^2 - 10x + 8 < 0$. *(2 marks)*

(ii) Solve the inequality $3x^2 - 10x + 8 < 0$. *(4 marks)*

(b) The curve C passes through the point $P(2, 3)$.

(i) Verify that the tangent to the curve at P is parallel to the x-axis. *(2 marks)*

(ii) The point $Q(3, -1)$ also lies on the curve. The normal to the curve at Q and the tangent to the curve at P intersect at the point R. Find the coordinates of R. *(7 marks)*

QUESTION PART REFERENCE	Answer space for question 7

7a i) y is increasing when $\frac{dy}{dx} > 0$

$20x - 6x^2 - 16 > 0$

$6x^2 - 20x + 16 < 0 \quad (\div 2)$

$3x^2 - 10x + 8 < 0$ (as req)

ii) $(3x + 4)(x - 2)$

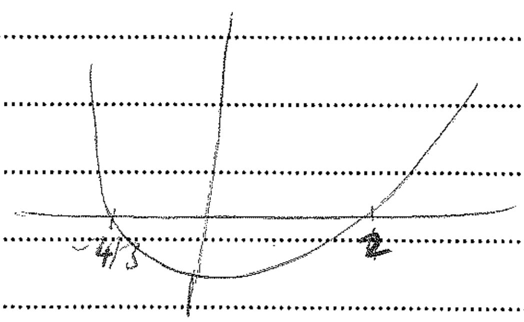
$3x + 4 = 0 \qquad x - 2 = 0$

$x = -4/3 \qquad x = 2$

when $3x^2 - 10x + 8 < 0$ graph is below x axis

so,

$-4/3 < x < 2$



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QUESTION PART REFERENCE	Answer space for question 7
b i)	when $x = 2$
	$\frac{dy}{dx} = 20(2) - 6(2)^2 - 16$
	$= 40 - 24 - 16$
	$= 0$
	$\therefore$ gradient is 0 and tangent parallel to x axis
ii)	gradient of tangent at Q :–
	$x = 3$, $\frac{dy}{dx} = 20(3) - 6(3)^2 - 16$
	$= 60 - 54 - 16$
	$= -10$
	gradient of normal is $1/10$
	$y + 1 = \frac{1}{10}(x - 3)$
	$10y + 10 = x - 3$
	$10y = x - 13$ (equation of Q)
	$y = 3$ (equation of P)
	$\therefore$ R is intersept :–
	$y = 3$ sub in to $10y = x - 13$.
	$30 = x - 13$
	$x = 43$
	R $(43, 3)$

Turn over ▶