Centre Number						Candidate Number				
Surname										
Other Names	WRITTEN									
Candidate Signature	SOLUTIONS									

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
TOTAL	

General Certificate of Education
Advanced Subsidiary Examination
January 2012

Mathematics MPC1

Unit Pure Core 1

Friday 13 January 2012 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You must **not** use a calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The use of calculators is **not** permitted.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

Answer **all** questions in the spaces provided.

1 The point A has coordinates $(6, -4)$ and the point B has coordinates $(-2, 7)$.

(a) Given that the point O has coordinates $(0, 0)$, show that the length of OA is less than the length of OB. *(3 marks)*

(b) (i) Find the gradient of AB. *(2 marks)*

(ii) Find an equation of the line AB in the form $px + qy = r$, where p, q and r are integers. *(3 marks)*

(c) The point C has coordinates $(k, 0)$. The line AC is perpendicular to the line AB. Find the value of the constant k. *(3 marks)*

QUESTION PART REFERENCE

$O(0,0) \quad A(6,-4) \quad B(-2,7)$

1a) $OA = \sqrt{6^2+4^2} \qquad OB = \sqrt{2^2+7^2}$

$= \sqrt{52} \qquad = \sqrt{53}$

$\sqrt{52} < \sqrt{53}$, therefore $OA < OB$

bi) gradient $AB = \frac{7-(-4)}{-2-6} = \frac{11}{-8} = -\frac{11}{8}$

ii) $m = -\frac{11}{8} \quad (6,-4)$

$y + 4 = -\frac{11}{8}(x-6)$

$8y + 32 = -11(x-6)$

$8y + 32 = -11x + 66$

$11x + 8y = 34$

Do not write outside the box

QUESTION PART REFERENCE

c) gradient $AC = \frac{8}{11}$ (m)

$A(6, -4)$ $C(k, 0)$

$m = \frac{8}{11} = \frac{0+4}{k-6}$

$\frac{8}{11} = \frac{4}{k-6}$

$8(k-6) = 44$

$8k - 48 = 44$

$8k = 92$

$k = \frac{92}{8} = \frac{46}{4} = \underline{\frac{23}{2}}$

2 (a) Factorise $x^2 - 4x - 12$. *(1 mark)*

(b) Sketch the graph with equation $y = x^2 - 4x - 12$, stating the values where the curve crosses the coordinate axes. *(4 marks)*

(c) (i) Express $x^2 - 4x - 12$ in the form $(x-p)^2 - q$, where p and q are positive integers. *(2 marks)*

(ii) Hence find the minimum value of $x^2 - 4x - 12$. *(1 mark)*

(d) The curve with equation $y = x^2 - 4x - 12$ is translated by the vector $\begin{bmatrix} -3 \\ 2 \end{bmatrix}$.

Find an equation of the new curve. You need not simplify your answer. *(2 marks)*

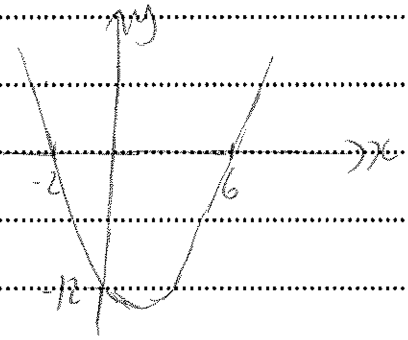
QUESTION PART REFERENCE

2a) $x^2 - 4x - 12$

$(x-6)(x+2)$

b) crosses x axis at $(6,0)$, $(-2,0)$

y axis at $(0,-12)$



ci) $x^2 - 4x - 12 = (x-2)^2 - 4 - 12$

$= (x-2)^2 - 16$

ii) minimum value of $x^2 - 4x - 12$ (y) is -16

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QUESTION PART REFERENCE

d) translation by $\begin{pmatrix}-3\\2\end{pmatrix}$, $f(x) \to f(x+3)+2$

replace 'x' with $x+3$', then add 2

$x^2-4x-12 \to \boxed{(x+3)^2-4(x+3)-12}+2$

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3 (a) (i) Simplify $(3\sqrt{2})^2$. *(1 mark)*

(ii) Show that $(3\sqrt{2}-1)^2+(3+\sqrt{2})^2$ is an integer and find its value. *(4 marks)*

(b) Express $\dfrac{4\sqrt{5}-7\sqrt{2}}{2\sqrt{5}+\sqrt{2}}$ in the form $m-\sqrt{n}$, where m and n are integers. *(4 marks)*

QUESTION PART REFERENCE

3ai) $(3\sqrt{2})^2 = 3\sqrt{2} \times 3\sqrt{2}$

$= 9\sqrt{4} = 9 \times 2$

$= \underline{18}$

ii) $(3\sqrt{2}-1)^2 + (3+\sqrt{2})^2$

$(3\sqrt{2}-1)(3\sqrt{2}-1) = 18 - 3\sqrt{2} - 3\sqrt{2} + 1$

$= 19 - 6\sqrt{2}$

$(3+\sqrt{2})(3+\sqrt{2}) = 9 + 3\sqrt{2} + 3\sqrt{2} + \sqrt{4}$

$= 11 + 6\sqrt{2}$

$(19 - 6\sqrt{2}) + (11 + 6\sqrt{2}) = \underline{30}$

b) $\dfrac{(4\sqrt{5} - 7\sqrt{2})}{(2\sqrt{5} + \sqrt{2})} \times \dfrac{(2\sqrt{5} - \sqrt{2})}{(2\sqrt{5} - \sqrt{2})} = \dfrac{8\sqrt{25} - 4\sqrt{10} - 14\sqrt{10} + 7\sqrt{4}}{4\sqrt{25} - 2\sqrt{10} + 2\sqrt{10} - \sqrt{4}}$

$= \dfrac{8(5) - 18\sqrt{10} + 7(2)}{4(5) - 2}$

$= \dfrac{40 - 18\sqrt{10} + 14}{20 - 2}$

$= \dfrac{54 - 18\sqrt{10}}{18} = \underline{3 - \sqrt{10}}$

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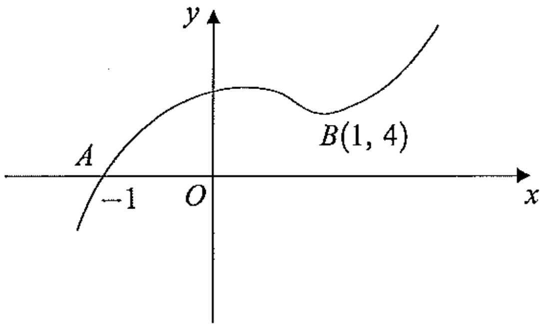
QUESTION PART REFERENCE

4 The curve with equation $y = x^5 - 3x^2 + x + 5$ is sketched below. The point O is at the origin and the curve passes through the points $A(-1, 0)$ and $B(1, 4)$.



(a) Given that $y = x^5 - 3x^2 + x + 5$, find:

(i) $\dfrac{dy}{dx}$; *(3 marks)*

(ii) $\dfrac{d^2y}{dx^2}$. *(1 mark)*

(b) Find an equation of the tangent to the curve at the point $A(-1, 0)$. *(2 marks)*

(c) Verify that the point B, where $x = 1$, is a minimum point of the curve. *(3 marks)*

QUESTION PART REFERENCE

4a) $y = x^5 - 3x^2 + x + 5$

i) $\frac{dy}{dx} = 5x^4 - 6x + 1$

ii) $\frac{d^2y}{dx^2} = 20x^3 - 6$

b) gradient when $x = -1$, $\frac{dy}{dx} = 5(-1)^4 - 6(-1) + 1$

$A(-1, 0)$ $= 5 + 6 + 1 = 12$

$y = 12(x + 1)$

$y = 12x + 12$

Do not write outside the box

QUESTION PART REFERENCE

c) when $x = 1$, $\frac{dy}{dx} = 5(1)^4 - 6(1) + 1$

$= 5 - 6 + 1$

$= 0$, therefore a stationary point

when $x = 1$, $\frac{d^2y}{dx^2} = 20(1)^3 - 6$

$= 14$

$\frac{d^2y}{dx^2} > 0$, therefore a minimum point where $x = 1$

Question 4 continues on the next page

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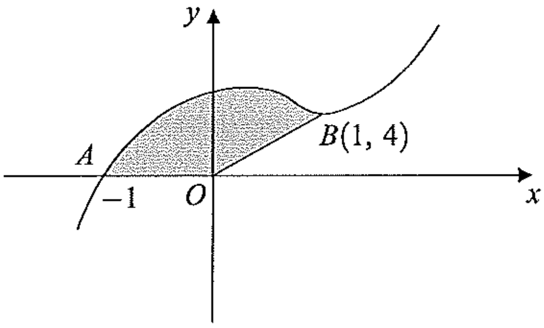
4 (d) The curve with equation $y = x^5 - 3x^2 + x + 5$ is sketched below. The point O is at the origin and the curve passes through the points $A(-1, 0)$ and $B(1, 4)$.



(i) Find $\int_{-1}^{1} (x^5 - 3x^2 + x + 5)\,dx$. *(5 marks)*

(ii) Hence find the area of the shaded region bounded by the curve between A and B and the line segments AO and OB. *(2 marks)*

QUESTION PART REFERENCE

4(d)i) $\int_{-1}^{1} (x^5 - 3x^2 + x + 5)dx = \left[\frac{x^6}{6} - \frac{3x^3}{3} + \frac{x^2}{2} + 5x\right]_{-1}^{1}$

$= \left[\frac{x^6}{6} - x^3 + \frac{x^2}{2} + 5x\right]_{-1}^{1}$

$= \left(\frac{(1)^6}{6} - (1)^3 + \frac{(1)^2}{2} + 5(1)\right) - \left(\frac{(-1)^6}{6} - (-1)^3 + \frac{(-1)^2}{2} + 5(-1)\right)$

$= \left(\frac{1}{6} - 1 + \frac{1}{2} + 5\right) - \left(\frac{1}{6} + 1 + \frac{1}{2} - 5\right)$

$= \frac{1}{6} - \frac{1}{6} - 1 - 1 + 5 + 5 + \frac{1}{2} - \frac{1}{2} = \underline{8}$

ii) shaded region = area under curve − area of $\triangle$

Area of $\triangle$ = $\frac{1 \times 4}{2} = 2$, area under curve = 8

shaded = 8 − 2 = $\underline{6}$

1 0

QUESTION PART REFERENCE

Do not write outside the box

5 The polynomial $p(x)$ is given by $p(x) = x^3 + cx^2 + dx - 12$, where c and d are constants.

(a) When $p(x)$ is divided by $x + 2$, the remainder is -150.

Show that $2c - d + 65 = 0$. *(3 marks)*

(b) Given that $x - 3$ is a factor of $p(x)$, find another equation involving c and d. *(2 marks)*

(c) By solving these two equations, find the value of c and the value of d. *(3 marks)*

QUESTION PART REFERENCE

5a) $p(x) = x^3 + cx^2 + dx - 12$

$p(-2) = -150$

$-150 = (-2)^3 + c(-2)^2 + d(-2) - 12$

$-150 = -8 + 4c - 2d - 12$

$0 = 130 + 4c - 2d \quad (\div 2)$

$2c - d + 65 = 0$ (as req)

b) $p(3) = 0$ if $(x-3)$ is a factor of $p(x)$

$(3)^3 + c(3)^2 + d(3) - 12 = 0$

$27 + 9c + 3d - 12 = 0$

$9c + 3d + 15 = 0 \quad (\div 3)$

$3c + d + 5 = 0$ (another eq)

c) $2c - d = -65$

$+ \; 3c + d = -5$

$5c = -70$

$c = -14$

$3(-14) + d = -5$

$-42 + d = -5$

$d = 37$

QUESTION PART REFERENCE

6 A rectangular garden is to have width x metres and length $(x+4)$ metres.

(a) The perimeter of the garden needs to be greater than 30 metres.

Show that $2x > 11$. *(1 mark)*

(b) The area of the garden needs to be less than 96 square metres.

Show that $x^2 + 4x - 96 < 0$. *(1 mark)*

(c) Solve the inequality $x^2 + 4x - 96 < 0$. *(4 marks)*

(d) Hence determine the possible values of the width of the garden. *(1 mark)*

QUESTION PART REFERENCE

6a) perimeter $= x + x + x + 4 + x + 4$

$= 4x + 8$

$4x + 8 > 30$ (perimeter greater than 30)

$4x > 22$

$2x > 11$ (as req)

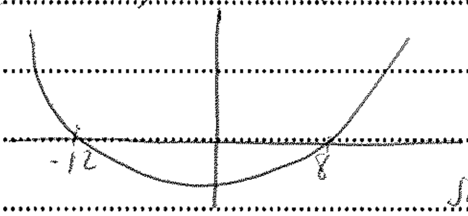
b) area $= x(x+4)$

$x(x+4) < 96$ (area less than 96)

$x^2 + 4x < 96$

$x^2 + 4x - 96 < 0$ (as req)

c) $(x+12)(x-8) = 0$ (cross x axis)



$x^2 + 4x - 96 < 0$

so, $-12 < x < 8$

d) $2x > 11$, $x > 5.5$, so $5.5 < x < 8$

QUESTION PART REFERENCE

Do not write outside the box

7 A circle with centre C has equation $x^2 + y^2 + 14x - 10y + 49 = 0$.

(a) Express this equation in the form

$$(x-a)^2 + (y-b)^2 = r^2$$

(3 marks)

(b) Write down:

(i) the coordinates of C;

(ii) the radius of the circle. *(2 marks)*

(c) Sketch the circle. *(2 marks)*

(d) A line has equation $y = kx + 6$, where k is a constant.

(i) Show that the x-coordinates of any points of intersection of the line and the circle satisfy the equation $(k^2+1)x^2 + 2(k+7)x + 25 = 0$. *(2 marks)*

(ii) The equation $(k^2+1)x^2 + 2(k+7)x + 25 = 0$ has equal roots. Show that

$$12k^2 - 7k - 12 = 0$$

(3 marks)

(iii) Hence find the values of k for which the line is a tangent to the circle. *(2 marks)*

QUESTION PART REFERENCE

7a) $x^2 + 14x + y^2 - 10y + 49 = 0$

$(x+7)^2 - 49 + (y-5)^2 - 25 + 49 = 0$

$(x+7)^2 + (y-5)^2 = 25$

$(x+7)^2 + (y-5)^2 = 5^2$

bi) $C = (-7, 5)$

ii) $r^2 = 5^2$

$r = 5$

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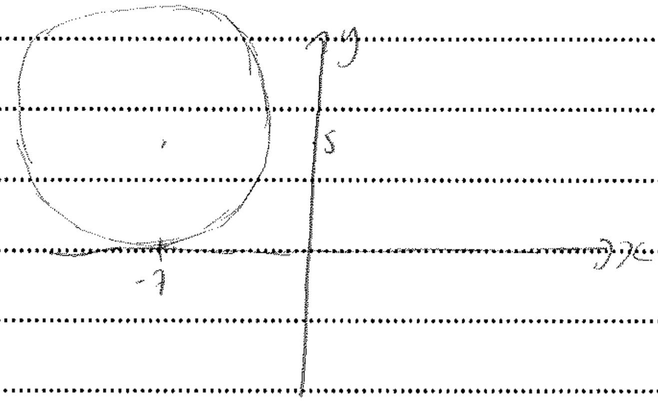
QUESTION PART REFERENCE	
c)	
d)	$y = kx + 6$
i)	$kx + 6 = y \rightarrow$ sub into circle
	$x^2 + (kx+6)^2 + 14x - 10(kx+6) + 49 = 0$
	$x^2 + (kx+6)(kx+6) + 14x - 10kx - 60 + 49 = 0$
	$x^2 + k^2x^2 + 6kx + 6kx + 36 + 14x - 10kx - 60 + 49 = 0$
	$(k^2+1)x^2 + 2kx + 14x + 25 = 0$
	$(k^2+1)x^2 + 2(k+7)x + 25 = 0$ (as req)
ii)	equal roots $\rightarrow b^2 - 4ac = 0$, $a = k^2+1$, $b = 2(k+7)$, $c = 25$
	$(2k+14)^2 - 4(k^2+1)(25) = 0$
	$4k^2 + 56k + 196 - (4k^2+4)(25) = 0$
	$4k^2 + 56k + 196 - 100k^2 - 100 = 0$
	$-96k^2 + 56k + 96 = 0$
	$96k^2 - 56k - 96 = 0$ $(\div 8)$
	$12k^2 - 7k - 12 = 0$ (as req)
iii)	$(4k+3)(3k-4) = 0$, $k = -3/4$ $k = 4/3$

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QUESTION PART REFERENCE

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END OF QUESTIONS

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