Centre Number						Candidate Number				
Surname	ANSWERS									
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
TOTAL	

General Certificate of Education
Advanced Subsidiary Examination
June 2011

Mathematics MPC1

Unit Pure Core 1

Wednesday 18 May 2011 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You must **not** use a calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The use of calculators is **not** permitted.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Do not write outside the box

Answer **all** questions in the spaces provided.

1 The line AB has equation $7x + 3y = 13$.

(a) Find the gradient of AB. *(2 marks)*

(b) The point C has coordinates $(-1, 3)$.

(i) Find an equation of the line which passes through the point C and which is parallel to AB. *(2 marks)*

(ii) The point $(1\frac{1}{2}, -1)$ is the mid-point of AC. Find the coordinates of the point A. *(2 marks)*

(c) The line AB intersects the line with equation $3x + 2y = 12$ at the point B. Find the coordinates of B. *(3 marks)*

QUESTION PART REFERENCE

1a) $7x + 3y = 13$ gradient $= -\frac{7}{3}$

$3y = -7x + 13$

$y = -\frac{7}{3}x + \frac{13}{3}$

bi) $y - 3 = -\frac{7}{3}(x+1)$

$3y - 9 = -7x - 7$

$3y + 7x = 2$

ii) $-1 \rightarrow 1\frac{1}{2} = 2\frac{1}{2}$, $x = 4$ A $(4, -5)$

$3 \rightarrow -1 = -4$, $y = -5$

c) $7x + 3y = 13 \ (\times 2)$ $14x + 6y = 26$ $7(-2) + 3y = 13$

$3x + 2y = 12 \ (\times 3)$ $9x + 6y = 36$ $-14 + 3y = 13$

$5x = -10$ $3y = 27$

$(-2, 9)$ $x = -2$ $y = 9$

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2 (a) (i) Express $\sqrt{48}$ in the form $k\sqrt{3}$, where k is an integer. *(1 mark)*

(ii) Simplify $\dfrac{\sqrt{48}+2\sqrt{27}}{\sqrt{12}}$, giving your answer as an integer. *(3 marks)*

(b) Express $\dfrac{1-5\sqrt{5}}{3+\sqrt{5}}$ in the form $m+n\sqrt{5}$, where m and n are integers. *(4 marks)*

QUESTION PART REFERENCE

2(a)i) $\sqrt{48} = \sqrt{16} \times \sqrt{3} = 4\sqrt{3}$

ii) $\dfrac{\sqrt{48}+2\sqrt{27}}{\sqrt{12}} = \dfrac{4\sqrt{3}+2\sqrt{9}\times\sqrt{3}}{\sqrt{12}}$

$= \dfrac{4\sqrt{3}+6\sqrt{3}}{\sqrt{12}} = \dfrac{10\sqrt{3}}{\sqrt{12}} = \dfrac{10\sqrt{3}}{\sqrt{4}\sqrt{3}}$

$= \dfrac{10\sqrt{3}}{2\sqrt{3}} = \underline{5}$

b) $\dfrac{(1-5\sqrt{5})}{(3+\sqrt{5})} \times \dfrac{(3-\sqrt{5})}{(3-\sqrt{5})} = \dfrac{3-\sqrt{5}-15\sqrt{5}+5\sqrt{25}}{9-5}$

$= \dfrac{3-16\sqrt{5}+25}{4}$

$= \dfrac{28-16\sqrt{5}}{4} = \underline{7-4\sqrt{5}}$

3 The volume, $V\,\text{m}^3$, of water in a tank after time t seconds is given by

$$V = \frac{t^3}{4} - 3t + 5$$

(a) Find $\dfrac{dV}{dt}$. *(2 marks)*

(b) (i) Find the rate of change of volume, in $\text{m}^3\,\text{s}^{-1}$, when $t = 1$. *(2 marks)*

(ii) Hence determine, with a reason, whether the volume is increasing or decreasing when $t = 1$. *(1 mark)*

(c) (i) Find the positive value of t for which V has a stationary value. *(3 marks)*

(ii) Find $\dfrac{d^2V}{dt^2}$, and hence determine whether this stationary value is a maximum value or a minimum value. *(3 marks)*

QUESTION PART REFERENCE

3a) $V = \frac{t^3}{4} - 3t + 5$

$\frac{dV}{dt} = \frac{3t^2}{4} - 3$

bi) when $t = 1$ $\frac{dV}{dt} = \frac{3(1)^2}{4} - 3 = -2\frac{1}{4}$

ii) $\frac{dV}{dt} < 0 \therefore$ decreasing

c i) $\frac{3t^2}{4} - 3 = 0$ $3t^2 = 12$ $t^2 = 4$

$t = \pm 2 \therefore t = 2$

ii) $\frac{d^2V}{dt^2} = \frac{6t}{4} = \frac{3t}{2}$ when $\cancel{t} = 2$,

$\frac{d^2V}{dt^2} = \frac{3(2)}{2} = 3$

$\frac{d^2V}{dt^2} > 0 \therefore$ minimum

Do not write outside the box

4 (a) Express $x^2 + 5x + 7$ in the form $(x + p)^2 + q$, where p and q are rational numbers. *(3 marks)*

(b) A curve has equation $y = x^2 + 5x + 7$.

(i) Find the coordinates of the vertex of the curve. *(2 marks)*

(ii) State the equation of the line of symmetry of the curve. *(1 mark)*

(iii) Sketch the curve, stating the value of the intercept on the y-axis. *(3 marks)*

(c) Describe the geometrical transformation that maps the graph of $y = x^2$ onto the graph of $y = x^2 + 5x + 7$. *(3 marks)*

QUESTION PART REFERENCE

$\left(\frac{5}{2}\right)^2 = \frac{25}{4}$ $\quad 7 = \frac{28}{4}$

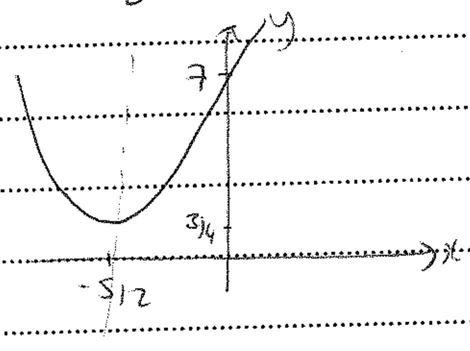
4a) $x^2 + 5x + 7$

$\left(x + \frac{5}{2}\right)^2 + \frac{3}{4}$

i) vertex is $\left(-\frac{5}{2}, \frac{3}{4}\right)$

ii) $x = -\frac{5}{2}$

iii)



c) $y = x^2 \rightarrow y = \left(x + \frac{5}{2}\right)^2 + \frac{3}{4}$

Translation $\begin{pmatrix} -5/2 \\ 3/4 \end{pmatrix}$

5 The polynomial p(x) is given by $p(x) = x^3 - 2x^2 + 3$.

(a) Use the Remainder Theorem to find the remainder when p(x) is divided by $x - 3$. *(2 marks)*

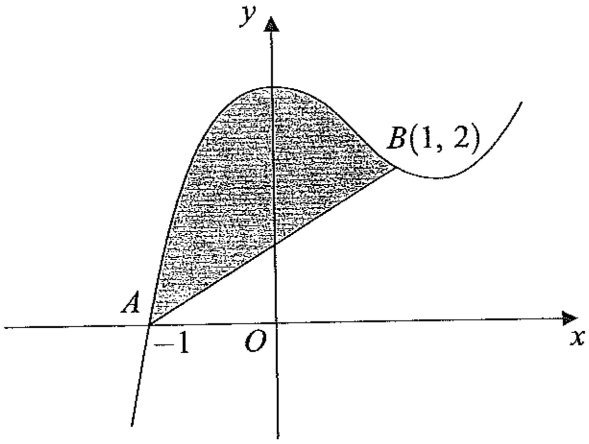
(b) Use the Factor Theorem to show that $x + 1$ is a factor of p(x). *(2 marks)*

(c) (i) Express $p(x) = x^3 - 2x^2 + 3$ in the form $(x+1)(x^2 + bx + c)$, where b and c are integers. *(2 marks)*

(ii) Hence show that the equation $p(x) = 0$ has exactly one real root. *(2 marks)*

QUESTION PART REFERENCE	
5a)	$p(3) = 3^3 - 2(3)^2 + 3$
	$= 27 - 18 + 3 = 12$ ∴ remainder is 12.
b)	$p(-1) = (-1)^3 - 2(-1)^2 + 3$
	$= -1 - 2 + 3 = 0$ ∴ $(x+1)$ is a factor
c)i)	$x^2 - 3x + 3$
	$x+1 \overline{) x^3 - 2x^2 + 0x + 3}$
	$x^3 + x^2$
	$0 - 3x^2 + 0x$
	$-3x^2 - 3x$
	$0 + 3x + 3$
	$3x + 3$
	0
	$(x+1)(x^2 - 3x + 3)$
ii)	$x = -1 \rightarrow$ one root
	$x^2 - 3x + 3 \rightarrow b^2 - 4ac < 0$ if no roots.
	$(-3)^2 - 4(1)(3) < 0$
	$9 - 12 < 0$ ✓ ∴ no real roots.

6 The curve with equation $y = x^3 - 2x^2 + 3$ is sketched below.



The curve cuts the x-axis at the point $A(-1, 0)$ and passes through the point $B(1, 2)$.

(a) Find $\int_{-1}^{1} (x^3 - 2x^2 + 3)\,\mathrm{d}x$. *(5 marks)*

(b) Hence find the area of the shaded region bounded by the curve $y = x^3 - 2x^2 + 3$ and the line AB. *(3 marks)*

QUESTION PART REFERENCE

5a) $\int_{-1}^{1} (x^3 - 2x^2 + 3)\,dx = \left[\frac{x^4}{4} - \frac{2x^3}{3} + 3x\right]_{-1}^{1}$

$= \left(\frac{(1)^4}{4} - \frac{2(1)^3}{3} + 3(1)\right) - \left(\frac{(-1)^4}{4} - \frac{2(-1)^3}{3} + 3(-1)\right)$

$= \left(\cancel{\frac{1}{4}} - \cancel{\frac{2}{3}} + 3\right) - \left(\cancel{\frac{1}{4}} + \cancel{\frac{2}{3}} - 3\right)$

$= \underline{4\,{}^2/_3}$

b) shaded = area under curve − area under $\triangle$

$= 4\,{}^2/_3 - \left(\frac{2 \times 2}{2}\right) = \underline{\underline{2\,{}^2/_3}}$

7 Solve each of the following inequalities:

(a) $2(4-3x) > 5-4(x+2)$; *(2 marks)*

(b) $2x^2+5x \geqslant 12$. *(4 marks)*

QUESTION PART REFERENCE

7a) $2(4-3x) > 5-4(x+2)$

$8-6x > 5-4x-8$

$8 > -3+2x$

$2x < 11$

$x < 5\frac{1}{2}$

b) $2x^2+5x-12 \geqslant 0$

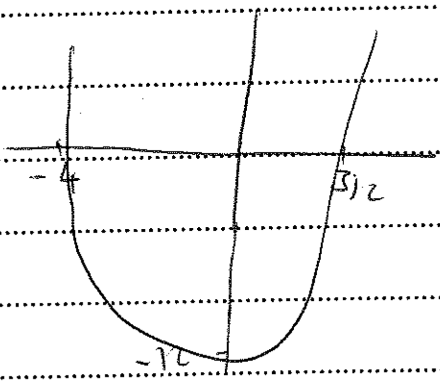
$(2x-3)(x+4) \geqslant 0$

$2x = 3$ $\quad x = -4$

$x = 3/2$

$x \leqslant -4$

$x \geqslant 3/2$



8 A circle has centre $C(3, -8)$ and radius 10.

(a) Express the equation of the circle in the form

$$(x-a)^2 + (y-b)^2 = k$$ *(2 marks)*

(b) Find the x-coordinates of the points where the circle crosses the x-axis. *(3 marks)*

(c) The tangent to the circle at the point A has gradient $\frac{5}{2}$. Find an equation of the line CA, giving your answer in the form $rx + sy + t = 0$, where r, s and t are integers. *(3 marks)*

(d) The line with equation $y = 2x + 1$ intersects the circle.

(i) Show that the x-coordinates of the points of intersection satisfy the equation

$$x^2 + 6x - 2 = 0$$ *(3 marks)*

(ii) Hence show that the x-coordinates of the points of intersection are of the form $m \pm \sqrt{n}$, where m and n are integers. *(2 marks)*

QUESTION PART REFERENCE

8a) $(x-3)^2 + (y+8)^2 = 100$

b) when $y = 0$

$(x-3)^2 + 8^2 = 100$

$(x-3)^2 = 36$

$x - 3 = \pm 6$

$x = 9$ or $x = -3$

c) $y + 8 = -\frac{2}{5}(x-3)$ grad normal $= -\frac{2}{5}$

$5y + 40 = -2x + 6$

$2x + 5y + 34 = 0$

di) $y = 2x + 1$ $(x-3)^2 + (2x+9)^2 = 100$

$x^2 - 6x + 9 + 4x^2 + 36x + 81 = 100$

$5x^2 + 30x - 10 = 0$

$x^2 + 6x - 2 = 0$ (as req.)

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QUESTION PART REFERENCE

ii) $(x+3)^2 - 11 = 0$

$(x+3)^2 = 11$

$x + 3 = \pm\sqrt{11}$

$x = -3 \pm \sqrt{11}$

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