Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature	WRITTEN SOLUTIONS									

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
TOTAL	

AQA

General Certificate of Education
Advanced Subsidiary Examination
January 2011

Mathematics MPC1

Unit Pure Core 1

Monday 10 January 2011 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You must **not** use a calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The use of calculators is **not** permitted.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Do not write outside the box

Answer **all** questions in the spaces provided.

1 The curve with equation $y = 13 + 18x + 3x^2 - 4x^3$ passes through the point P where $x = -1$.

(a) Find $\frac{dy}{dx}$. *(3 marks)*

(b) Show that the point P is a stationary point of the curve and find the other value of x where the curve has a stationary point. *(3 marks)*

(c) (i) Find the value of $\frac{d^2y}{dx^2}$ at the point P. *(3 marks)*

(ii) Hence, or otherwise, determine whether P is a maximum point or a minimum point. *(1 mark)*

QUESTION PART REFERENCE

1a) $y = 13 + 18x + 3x^2 - 4x^3$

$\frac{dy}{dx} = 18 + 6x - 12x^2$

b) when $x = -1$,

$\frac{dy}{dx} = 18 + 6(-1) - 12(-1)^2$

$= 18 - 6 - 12$

$= 0$ ∴ stationary point at P as $\frac{dy}{dx} = 0$

$12x^2 + 6x - 18 = 0$ $(\div 6)$

$2x^2 - x - 3 = 0$

$(2x - 3)(x + 1) = 0$

$2x - 3 = 0$ $\quad$ $x + 1 = 0$

$x = 3/2$ $\quad$ $x = -1$ (point P)

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QUESTION PART REFERENCE

c i) $\frac{d^2y}{dx^2} = 6 - 24x$

when $x = -1$,

$\frac{d^2y}{dx^2} = 6 - 24(-1)$

$= \underline{\underline{30}}$

ii) $\frac{d^2y}{dx^2} > 30$, $\therefore$ a minimum point

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2 (a) Simplify $(3\sqrt{3})^2$. *(1 mark)*

(b) Express $\dfrac{4\sqrt{3}+3\sqrt{7}}{3\sqrt{3}+\sqrt{7}}$ in the form $\dfrac{m+\sqrt{21}}{n}$, where m and n are integers. *(4 marks)*

QUESTION PART REFERENCE

2a) $(3\sqrt{3})^2 = 3\sqrt{3} \times 3\sqrt{3}$

$= 9 \times 3$

$= \underline{27}$

b) $\dfrac{(4\sqrt{3}+3\sqrt{7})}{(3\sqrt{3}+\sqrt{7})} \times \dfrac{(3\sqrt{3}-\sqrt{7})}{(3\sqrt{3}-\sqrt{7})}$

$\dfrac{12\sqrt{9} - 4\sqrt{21} + 9\sqrt{21} - 3\sqrt{49}}{9\sqrt{9} - 3\sqrt{21} + 3\sqrt{21} - \sqrt{49}}$

$\dfrac{36 + 5\sqrt{21} - 21}{27 - 7} = \dfrac{15 + 5\sqrt{21}}{20}$

$= \dfrac{3 + \sqrt{21}}{4}$

QUESTION PART REFERENCE

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3 The line AB has equation $3x + 2y = 7$. The point C has coordinates $(2, -7)$.

(a) (i) Find the gradient of AB. *(2 marks)*

(ii) The line which passes through C and which is parallel to AB crosses the y-axis at the point D. Find the y-coordinate of D. *(3 marks)*

(b) The line with equation $y = 1 - 4x$ intersects the line AB at the point A. Find the coordinates of A. *(3 marks)*

(c) The point E has coordinates $(5, k)$. Given that CE has length 5, find the two possible values of the constant k. *(3 marks)*

QUESTION PART REFERENCE

3ai) $3x + 2y = 7$

$2y = -3x + 7$

$y = -\frac{3}{2}x + \frac{7}{2}$

gradient $-\frac{3}{2}$

ii) $C(2, -7)$ gradient $= -\frac{3}{2}$

$y + 7 = -\frac{3}{2}(x - 2)$

$2y + 14 = -3x + 6$

$2y + 3x = -8$ (when $x = 0$, crosses y axis)

$2y = -8$

$\underline{\underline{y = -4}}$

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QUESTION PART REFERENCE

b) $3x + 2y = 7$ ① rearrange $y = 1 - 4x$

$4x + y = 1$ ② $(\times 2)$ $\rightarrow 4x + y = 1$

$8x + 2y = 2$

$-\ 3x + 2y = 7$

$5x = -5$

$\underline{x = -1}$ $\rightarrow$ sub in ②

$4(-1) + y = 1$

$-4 + y = 1$ $(+4)$ $A\underline{(-1, 5)}$

$\underline{y = 5}$

c) $C(2, -7)$ $E(5, k)$

$CE = \sqrt{3^2 + (k-(-7))^2}$

$5 = \sqrt{9 + (k+7)^2}$ $(^2)$

$25 = 9 + (k+7)^2$

$25 = 9 + (k+7)(k+7)$

$25 = 9 + k^2 + 7k + 7k + 49$

$25 = 9 + k^2 + 14k + 49$

$k^2 + 14k + 33 = 0$

$(k+11)(k+3) = 0$

$\underline{k = -11}$ or $\underline{k = -3}$

OR $(k+7)^2 = 16$

$k + 7 = \pm\sqrt{16}$

$k = -7 \pm 4$

$\underline{k = -3}$ $\underline{\text{OR } -11}$

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QUESTION PART REFERENCE

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QUESTION PART REFERENCE	

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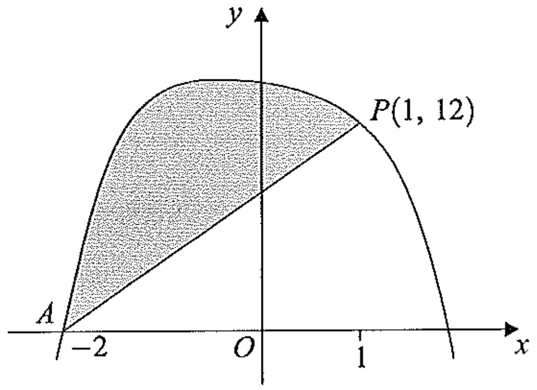
4 The curve sketched below passes through the point $A(-2, 0)$.



The curve has equation $y = 14 - x - x^4$ and the point $P(1, 12)$ lies on the curve.

(a) (i) Find the gradient of the curve at the point P. *(3 marks)*

(ii) Hence find the equation of the tangent to the curve at the point P, giving your answer in the form $y = mx + c$. *(2 marks)*

(b) (i) Find $\int_{-2}^{1} (14 - x - x^4)\, dx$. *(5 marks)*

(ii) Hence find the area of the shaded region bounded by the curve $y = 14 - x - x^4$ and the line AP. *(2 marks)*

QUESTION PART REFERENCE	
4ai)	$y = 14 - x - x^4$ when $x = 1$,
	$\frac{dy}{dx} = -1 - 4x^3$ $\frac{dy}{dx} = -1 - 4(1)^3$
	$= -1 - 4$
	$= -5$
	gradient at point P is -5
ii)	$m = -5$, $P(1, 12)$
	$y - 12 = -5(x - 1)$
	$y - 12 = -5x + 5$
	$y = -5x + 17$

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QUESTION PART REFERENCE

b i) $\int_{-2}^{1}(14-x-x^4)\,dx$

$\left[14x-\frac{x^2}{2}-\frac{x^5}{5}\right]_{-2}^{1}$

$=\left(14(1)-\frac{(1)^2}{2}-\frac{(1)^5}{5}\right)-\left(14(-2)-\frac{(-2)^2}{2}-\frac{(-2)^5}{5}\right)$

$=\left(14-\frac{1}{2}-\frac{1}{5}\right)-\left(-28-2+\frac{32}{5}\right)$

$=\left(14-\frac{1}{2}-\frac{1}{5}\right)-\left(-30+\frac{32}{5}\right)$

$=14+30-\frac{1}{2}-\frac{1}{5}-\frac{32}{5}$

$=44-1/2-33/5$

$=43\,1/2-33/5$

$=\frac{87}{2}-\frac{33}{5}$

$=\frac{435}{10}-\frac{66}{10}=\frac{369}{10}=\underline{36.9}$

ii) Area of shaded = Area under curve − Area of Δ

$=36.9-\left(\frac{3\times 12}{2}\right)$

$=36.9-18$

$=\underline{18.9}$

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QUESTION PART REFERENCE

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QUESTION PART REFERENCE

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5 (a) (i) Sketch the curve with equation $y = x(x-2)^2$. *(3 marks)*

(ii) Show that the equation $x(x-2)^2 = 3$ can be expressed as

$$x^3 - 4x^2 + 4x - 3 = 0$$ *(1 mark)*

(b) The polynomial $\mathrm{p}(x)$ is given by $\mathrm{p}(x) = x^3 - 4x^2 + 4x - 3$.

(i) Find the remainder when $\mathrm{p}(x)$ is divided by $x + 1$. *(2 marks)*

(ii) Use the Factor Theorem to show that $x - 3$ is a factor of $\mathrm{p}(x)$. *(2 marks)*

(iii) Express $\mathrm{p}(x)$ in the form $(x-3)(x^2 + bx + c)$, where b and c are integers. *(2 marks)*

(c) Hence show that the equation $x(x-2)^2 = 3$ has only one real root and state the value of this root. *(3 marks)*

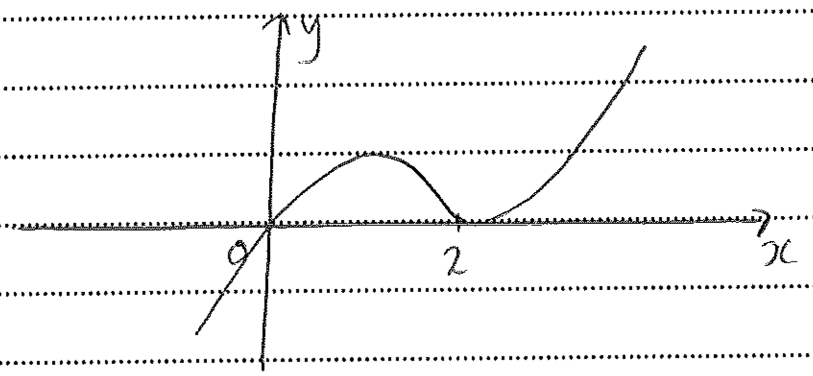
QUESTION PART REFERENCE

5ai) $y = x(x-2)^2$ crosses x axis at :-

$x(x-2)^2 = 0$ $(0,0)$ $(2,0)$

$x = 0$, $(x-2)^2 = 0$ touches at

$x = 2$



ii) $x(x-2)^2 = 3$

$x(x-2)(x-2) = 3$

$x(x^2 - 4x + 4) = 3$

$x^3 - 4x^2 + 4x - 3 = 0$ (as req)

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QUESTION PART REFERENCE

b) $p(x) = x^3 - 4x^2 + 4x - 3$

i) $p(-1) = (-1)^3 - 4(-1)^2 + 4(-1) - 3$

$= -1 - 4 - 4 - 3$

$= \underline{-12}$ remainder is -12

ii) $p(3) = (3)^3 - 4(3)^2 + 4(3) - 3$

$= 27 - 36 + 12 - 3$

$= 0$ $\therefore (x-3)$ is a factor of $p(x)$

iii)

$$\begin{array}{r} x^2 - x + 1 \\ x-3 \overline{) x^3 - 4x^2 + 4x - 3} \\ -\underline{x^3 - 3x^2} \\ -x^2 + 4x \\ -\underline{-x^2 + 3x} \\ x - 3 \\ -\underline{x - 3} \\ 0 \end{array}$$

$\underline{(x-3)(x^2-x+1)}$

c) $(x-3)(x^2-x+1) = 0$ since $x(x-2)^2 = 3$

$\to (x-3)(x^2-x+1) = 0$

$x - 3 = 0$ $\underline{OR}$

$\underline{x = 3}$ $x^2 - x + 1 = 0$

$a = 1,\ b = -1,\ c = 1,$ $b^2 - 4ac < 0$

$\therefore$ only one root at $\underline{x = 3}$ $(-1)^2 - 4(1)(1) = 1 - 4 = -3$ $\therefore$ no solution

Turn over ▶

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QUESTION PART REFERENCE

QUESTION PART REFERENCE

Do not write outside the box

6 A circle has centre $C(-3, 1)$ and radius $\sqrt{13}$.

(a) (i) Express the equation of the circle in the form

$$(x-a)^2 + (y-b)^2 = k$$ *(2 marks)*

(ii) Hence find the equation of the circle in the form

$$x^2 + y^2 + mx + ny + p = 0$$

where m, n and p are integers. *(3 marks)*

(b) The circle cuts the y-axis at the points A and B. Find the distance AB. *(3 marks)*

(c) (i) Verify that the point $D(-5, -2)$ lies on the circle. *(1 mark)*

(ii) Find the gradient of CD. *(2 marks)*

(iii) Hence find an equation of the tangent to the circle at the point D. *(2 marks)*

QUESTION PART REFERENCE

6ai) $C(-3, 1)$ $r = \sqrt{13}$

$(x+3)^2 + (y-1)^2 = 13$

ii) $(x+3)^2 + (y-1)^2 = 13$

$(x+3)(x+3) + (y-1)(y-1) = 13$

$x^2 + 3x + 3x + 9 + y^2 - y - y + 1 = 13$

$x^2 + 6x + 9 + y^2 - 2y + 1 = 13$

$x^2 + y^2 + 6x - 2y - 3 = 0$

b) crosses y axis when $x = 0$,

$y^2 - 2y - 3 = 0$

$(y+1)(y-3) = 0$

$y = -1$, $y = 3$ AB distance is 4

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QUESTION PART REFERENCE

c i) $D(-5,-2)$ sub in :-

$(x+3)^2 + (y-1)^2 = 13$

$(-5+3)^2 + (-2-1)^2 = 13$

$2^2 + (-3)^2 = 13$

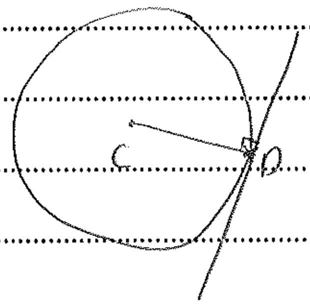
$4 + 9 = 13$ ✓

$\therefore$ point D lies on the circle

ii) $C(-3,1)$ $D(-5,-2)$

gradient CD $= \dfrac{-2-1}{-5-(-3)} = \dfrac{-3}{-2} = \dfrac{3}{2}$

iii)



gradient of CD perpendicular to tangent at D

(tangent meets radius at 90°)

$\therefore$ gradient is $^{-2}/_3$, $D(-5,-2)$

$y+2 = {}^{-2}/_3(x+5)$

$3y+6 = -2(x+5)$

$3y+6 = -2x-10$

$2x+3y+16 = 0$

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QUESTION PART REFERENCE

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QUESTION PART REFERENCE

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7 (a) (i) Express $4 - 10x - x^2$ in the form $p - (x+q)^2$. *(2 marks)*

(ii) Hence write down the equation of the line of symmetry of the curve with equation $y = 4 - 10x - x^2$. *(1 mark)*

(b) The curve C has equation $y = 4 - 10x - x^2$ and the line L has equation $y = k(4x - 13)$, where k is a constant.

(i) Show that the x-coordinates of any points of intersection of the curve C with the line L satisfy the equation

$$x^2 + 2(2k+5)x - (13k+4) = 0$$ *(1 mark)*

(ii) Given that the curve C and the line L intersect in two distinct points, show that

$$4k^2 + 33k + 29 > 0$$ *(3 marks)*

(iii) Solve the inequality $4k^2 + 33k + 29 > 0$. *(4 marks)*

QUESTION PART REFERENCE

7a i) $4 - 10x - x^2$

$4 - (10 + x^2)$

$4 - (x^2 + 10)$

$4 - [(x+5)^2 - 25]$

$4 - (x+5)^2 + 25$

$\underline{29 - (x+5)^2}$

ii) $x = -5$

b i) $4 - 10x - x^2 = k(4x - 13)$

$4 - 10x - x^2 = 4kx - 13k$

$x^2 + 4kx + 10x - 13k - 4 = 0$

$x^2 + 2x(2k+5) - (13k+4) = 0$

$x^2 + 2(2k+5)x - (13k+4) = 0$ (as req)

QUESTION PART REFERENCE

ii) $a = 1, \quad b = 2(2k+5), \quad c = -(13k+4)$

if two distinct points, $b^2 - 4ac > 0$

$(4k+10)^2 - 4(1)(-13k-4) > 0$

$16k^2 + 80k + 100 - 4(-13k-4) > 0$

$16k^2 + 80k + 100 + 52k + 16 > 0$

$16k^2 + 132k + 116 > 0 \quad (\div 4)$

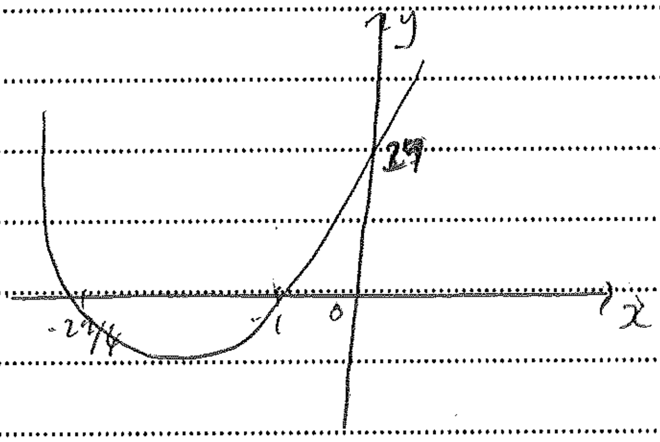
$4k^2 + 33k + 29 > 0$ (as req)

iii) $4k^2 + 33k + 29 > 0$

$(4k+29)(k+1) = 0$

$4k = -29 \qquad k = -1$

$k = -\frac{29}{4}$



$4k^2 + 33k + 29 > 0$

$\therefore k < -29/4 \quad$ OR $\quad k > -1$

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QUESTION PART REFERENCE

END OF QUESTIONS