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| Other Names | ANSWERS | | | | | | | | | |
| Candidate Signature | | | | | | | | | | |

| For Examiner's Use | |
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| Examiner's Initials | |
| Question | Mark |
| 1 | |
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| TOTAL | |

General Certificate of Education
Advanced Subsidiary Examination
June 2010

# Mathematics MPC1

**Unit Pure Core 1**

**Monday 24 May 2010 1.30 pm to 3.00 pm**

**For this paper you must have:**
- the blue AQA booklet of formulae and statistical tables.

You must **not** use a calculator.

**Time allowed**
- 1 hour 30 minutes

**Instructions**
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The use of calculators is **not** permitted.

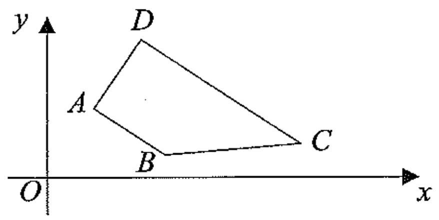
**Information**
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

**Advice**
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Do not write outside the box

Answer **all** questions in the spaces provided.

**1** The trapezium $ABCD$ is shown below.




The line $AB$ has equation $2x + 3y = 14$ and $DC$ is parallel to $AB$.

**(a)** Find the gradient of $AB$. *(2 marks)*

**(b)** The point $D$ has coordinates $(3, 7)$.

**(i)** Find an equation of the line $DC$. *(2 marks)*

**(ii)** The angle $BAD$ is a right angle. Find an equation of the line $AD$, giving your answer in the form $mx + ny + p = 0$, where $m$, $n$ and $p$ are integers. *(4 marks)*

**(c)** The line $BC$ has equation $5y - x = 6$. Find the coordinates of $B$. *(3 marks)*

QUESTION PART REFERENCE

1a) $2x + 3y = 14$

$3y = -2x + 14$

$y = -\frac{2}{3}x + \frac{14}{3}$

gradient is $-2/3$

bi) gradient of DC is $-2/3$ (parallel to AB)

coordinate is $(3, 7)$

$y - 7 = -2/3(x - 3)$

$3y - 21 = -2(x-3)$

$3y - 21 = -2x + 6 \rightarrow 2x + 3y = 27$ (or equivalent)

Do not write outside the box

QUESTION PART REFERENCE

ii) AD is perpendicular to AB

gradient of AD is $3/2$, coordinate is $(3, 7)$

$y - 7 = \frac{3}{2}(x - 3)$

$2y - 14 = 3(x - 3)$

$2y - 14 = 3x - 9$

$\underline{3x - 2y + 5 = 0}$ (as required)

c) BC intersects AB at point B, solving simultaneously :-

$2x + 3y = 14$ ① (AB)

$-x + 5y = 6$ ② (BC) $(\times 2)$

$-2x + 10y = 12$ ③ add ① and ③

$+\ \underline{2x + 3y = 14}$

$13y = 26$

$\underline{y = 2}$ sub $y = 2$ into ①

$2x + 6 = 14$

$2x = 8$

$\underline{x = 4}$, $\therefore$ $\underline{B(4, 2)}$

Turn over ▶

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**2 (a)** Express $\left(3-\sqrt{5}\right)^2$ in the form $m+n\sqrt{5}$, where $m$ and $n$ are integers. *(2 marks)*

**(b)** Hence express $\dfrac{\left(3-\sqrt{5}\right)^2}{1+\sqrt{5}}$ in the form $p+q\sqrt{5}$, where $p$ and $q$ are integers. *(4 marks)*

QUESTION PART REFERENCE

2a) $(3-\sqrt{5})^2 = (3-\sqrt{5})(3-\sqrt{5})$

$= 9 - 3\sqrt{5} - 3\sqrt{5} + \sqrt{25}$

$= 9 - 6\sqrt{5} + 5$

$= \underline{14 - 6\sqrt{5}}$

b) $\dfrac{(3-\sqrt{5})^2}{1+\sqrt{5}} = \dfrac{14-6\sqrt{5}}{1+\sqrt{5}}$

$\dfrac{14-6\sqrt{5}}{1+\sqrt{5}} \times \dfrac{1-\sqrt{5}}{1-\sqrt{5}} = \dfrac{14-14\sqrt{5}-6\sqrt{5}+30}{1+\sqrt{5}-\sqrt{5}-5}$

$= \dfrac{44-20\sqrt{5}}{-4}$

$= \underline{-11+5\sqrt{5}}$

Do not write outside the box

**3** The polynomial $\mathrm{p}(x)$ is given by

$$\mathrm{p}(x) = x^3 + 7x^2 + 7x - 15$$

**(a) (i)** Use the Factor Theorem to show that $x + 3$ is a factor of $\mathrm{p}(x)$. *(2 marks)*

**(ii)** Express $\mathrm{p}(x)$ as the product of three linear factors. *(3 marks)*

**(b)** Use the Remainder Theorem to find the remainder when $\mathrm{p}(x)$ is divided by $x - 2$. *(2 marks)*

**(c) (i)** Verify that $\mathrm{p}(-1) < \mathrm{p}(0)$. *(1 mark)*

**(ii)** Sketch the curve with equation $y = x^3 + 7x^2 + 7x - 15$, indicating the values where the curve crosses the coordinate axes. *(4 marks)*

QUESTION PART REFERENCE

3(a)(i) $p(-3) = (-3)^3 + 7(-3)^2 + 7(-3) - 15$

$= -27 + 63 - 21 - 15$

$= 0$

$\therefore (x+3)$ is a factor of $p(x)$

(ii)

$$\begin{array}{r} x^2 + 4x - 5 \\ x+3 \overline{\smash{)}\, x^3 + 7x^2 + 7x - 15} \\ -\ x^3 + 3x^2 \\ \hline 0 + 4x^2 + 7x \\ -\ 4x^2 + 12x \\ \hline 0 - 5x - 15 \\ -\ -5x - 15 \\ \hline 0 \quad 0 \end{array}$$

$(x+3)(x^2 + 4x - 5)$

$(x+3)(x+5)(x-1)$

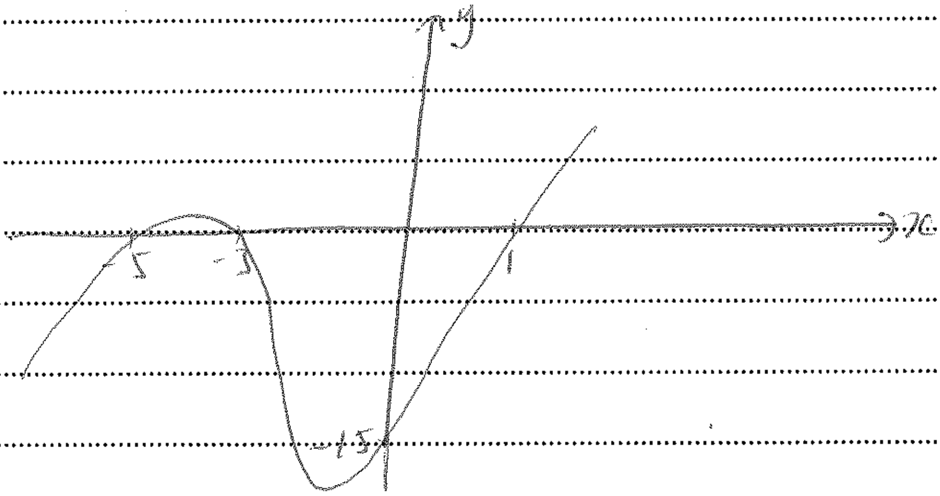
Do not write outside the box

QUESTION PART REFERENCE

b) $p(2) = (2)^3 + 7(2)^2 + 7(2) - 15$
$= 8 + 28 + 14 - 15$
$= 35$
remainder is $\underline{35}$

a) $p(-1) = (-1)^3 + 7(-1)^2 + 7(-1) - 15$
$= -1 + 7 - 7 - 15$
$= -16$
$p(0) = -15$
$\therefore \underline{p(-1) < p(0)}$ (as required)

ii) intersect x axis at $(-3, 0)$ $(-5, 0)$ $(1, 0)$
y axis at $(0, -15)$



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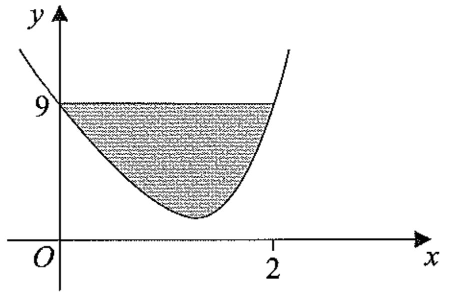
**4** The curve with equation $y = x^4 - 8x + 9$ is sketched below.




The point $(2, 9)$ lies on the curve.

**(a) (i)** Find $\int_0^2 (x^4 - 8x + 9)\,dx$. *(5 marks)*

**(ii)** Hence find the area of the shaded region bounded by the curve and the line $y = 9$. *(2 marks)*

**(b)** The point $A(1, 2)$ lies on the curve with equation $y = x^4 - 8x + 9$.

**(i)** Find the gradient of the curve at the point $A$. *(4 marks)*

**(ii)** Hence find an equation of the tangent to the curve at the point $A$. *(1 mark)*

QUESTION PART REFERENCE

4ai)

$$\int_0^2 (x^4 - 8x + 9)\,dx = \left[\frac{x^5}{5} - \frac{8x^2}{2} + 9x\right]_0^2$$

$$= \left[\frac{x^5}{5} - 4x^2 + 9x\right]_0^2$$

$$= \left(\frac{2^5}{5} - 4(2)^2 + 9(2)\right) - 0$$

$$= \left(\frac{32}{5} - 16 + 18\right) - 0$$

$$= \frac{42}{5} \text{ or } 8^2/5$$

Do not write outside the box

QUESTION PART REFERENCE

ii) Area of shaded = rectangle − area under curve

$= (9 \times 2) - \frac{42}{5}$

$= 18 - \frac{42}{5}$

$= \underline{\frac{48}{5}}$ or $\underline{9^{3}/_{5}}$

b i) $\frac{dy}{dx} = 4x^3 - 8$ when $x = 1$.

$\frac{dy}{dx} = 4(1)^3 - 8$

$\underline{= -4}$

ii) gradient is $-4$, coordinate is $(1, 2)$

$y - 2 = (-4)(x - 1)$

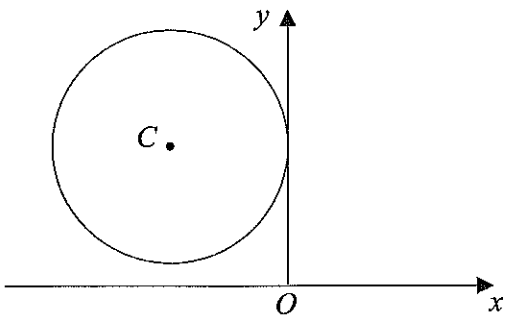
$y - 2 = -4x + 4$

$\underline{y = -4x + 6}$ OR $\underline{4x + y = 6}$

Turn over ▶

Do not write outside the box

**5** A circle with centre $C(-5, 6)$ touches the $y$-axis, as shown in the diagram.




**(a)** Find the equation of the circle in the form

$$(x-a)^2 + (y-b)^2 = r^2$$ *(3 marks)*

**(b) (i)** Verify that the point $P(-2, 2)$ lies on the circle. *(1 mark)*

**(ii)** Find an equation of the normal to the circle at the point $P$. *(3 marks)*

**(iii)** The mid-point of $PC$ is $M$. Determine whether the point $P$ is closer to the point $M$ or to the origin $O$. *(4 marks)*

QUESTION PART REFERENCE

5a) $(x+5)^2 + (y-6)^2 = 5^2$ $r = 5$ (touches y axis)

bi) sub in $x = -2$, $y = 2$

$(-2+5)^2 + (2-6)^2 = 5^2$

$(-3)^2 + (-4)^2 = 25$

$9 + 16 = 25$ ✓

$\therefore$ $(-2, 2)$ lies on the circle

ii) gradient of PC $= \dfrac{6-2}{-5(-2)} = \dfrac{4}{-3}$ coordinate $(-2, 2)$

$y - 2 = -\frac{4}{3}(x+2)$

$3y - 6 = -4x - 8$

$4x + 3y + 2 = 0$ (or equivalent)

Do not write outside the box

QUESTION PART REFERENCE

iii) length of PM $= \frac{1}{2} \times$ radius (midpoint)

$= \frac{1}{2} \times 5 = \underline{2.5}$

length PO $= \sqrt{2^2 + 2^2}$

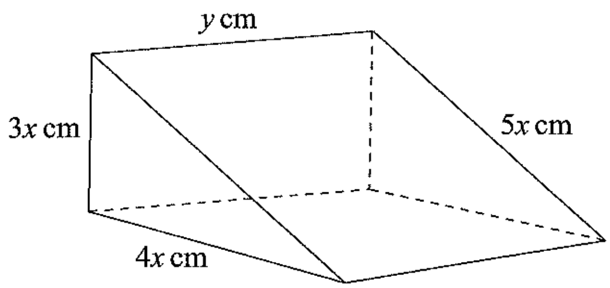
$= \sqrt{8}$

$\sqrt{8} > 2.5$

$\therefore$ closer to point A

Do not write outside the box

**6** The diagram shows a block of wood in the shape of a prism with triangular cross-section. The end faces are right-angled triangles with sides of lengths $3x$ cm, $4x$ cm and $5x$ cm, and the length of the prism is $y$ cm, as shown in the diagram.




The total surface area of the five faces is $144\text{ cm}^2$.

**(a) (i)** Show that $xy + x^2 = 12$. *(3 marks)*

**(ii)** Hence show that the volume of the block, $V\text{ cm}^3$, is given by

$$V = 72x - 6x^3$$ *(2 marks)*

**(b) (i)** Find $\dfrac{dV}{dx}$. *(2 marks)*

**(ii)** Show that $V$ has a stationary value when $x = 2$. *(2 marks)*

**(c)** Find $\dfrac{d^2V}{dx^2}$ and hence determine whether $V$ has a maximum value or a minimum value when $x = 2$. *(2 marks)*

QUESTION PART REFERENCE

6a)i) S.A = 2 triangles + 3 rectangles

$= 2\left(\dfrac{3x \times 4x}{2}\right) + 4xy + 3xy + 5xy$

$S.A = 12x^2 + 12xy$

$144 = 12x^2 + 12xy \quad (\div 12)$

$xy + x^2 = 12$ (as req)

Do not write outside the box

| QUESTION PART REFERENCE | |
|---|---|
| ii) | $V = \frac{1}{2}(3x \times 4x) \times y$ |
| | $V = 6x^2y$ $\qquad x^2 + xy = 12$ |
| | $xy = 12 - x^2$ |
| | $\boxed{y = \frac{12 - x^2}{x}}$ sub in |
| | $V = 6x^2\left(\frac{12 - x^2}{x}\right)$ |
| | $V = 6x(12 - x^2)$ |
| | $V = 72x - 6x^3$ (as req) |
| b i) | $\frac{dV}{dx} = 72 - 18x^2$ |
| ii) | when $x = 2$, |
| | $\frac{dV}{dx} = 72 - 18(2)^2$ |
| | $= 0$ $\therefore$ stationary value at $x = 2$ |
| c) | $\frac{d^2V}{dx^2} = -36x$ |
| | when $x = 2$, |
| | $\frac{d^2V}{dx^2} = -36(2)$ |
| | $= -72$ $\qquad \frac{d^2V}{dx^2} < 0$ $\therefore$ maximum |

Turn over ▶

Do not write outside the box

**7 (a) (i)** Express $2x^2 - 20x + 53$ in the form $2(x-p)^2 + q$, where $p$ and $q$ are integers. *(2 marks)*

**(ii)** Use your result from part **(a)(i)** to explain why the equation $2x^2 - 20x + 53 = 0$ has no real roots. *(2 marks)*

**(b)** The quadratic equation $(2k-1)x^2 + (k+1)x + k = 0$ has real roots.

**(i)** Show that $7k^2 - 6k - 1 \leqslant 0$. *(4 marks)*

**(ii)** Hence find the possible values of $k$. *(4 marks)*

QUESTION PART REFERENCE

7ai) $2(x^2 - 10x) + 53$

$2((x-5)^2 - 25) + 53$

$2(x-5)^2 - 50 + 53$

$2(x-5)^2 + 3$

ii) $2(x-5)^2 \geqslant 0$

$3 > 0$

$\therefore 2(x-5)^2 + 3 > 0$

which lies above the x axis and no

real solution or roots

bi) real roots means $b^2 - 4ac \geqslant 0$

$a = 2k-1 \quad b = k+1 \quad c = k$

$(k+1)^2 - 4(2k-1)(k) \geqslant 0$

$k^2 + 2k + 1 - 4k(2k-1) \geqslant 0$

$k^2 + 2k + 1 - 8k^2 + 4k \geqslant 0$

$-7k^2 + 6k + 1 \geqslant 0$

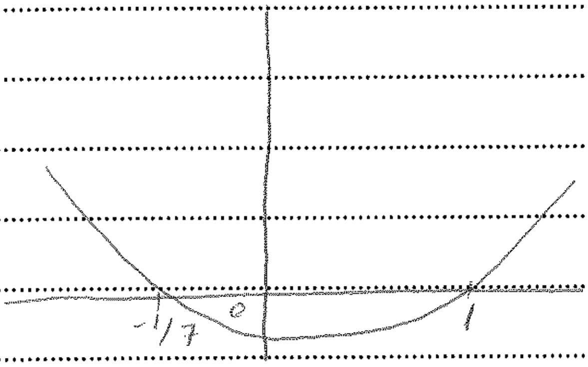
$7k^2 - 6k - 1 \leqslant 0$ (as req.)

QUESTION PART REFERENCE

ii) $7k^2 - 6k - 1 = 0$

$(7k+1)(k-1) = 0$

$k = -1/7 \quad k = 1$




when $7k^2 - 6k - 1 \leq 0$ (below x axis)

$\underline{-1/7 \leq k \leq 1}$

**END OF QUESTIONS**