Core1 - Jan 2010

① $p(x) = x^3 - 13x - 12$

a) $p(-3) = (-3)^3 - 13(-3) - 12$

$= -27 + 39 - 12$

$= 0 \quad \therefore (x+3)$ is a factor of $p(x)$

b)

$$\begin{array}{r|l} & x^2 - 3x - 4 \\ x+3 & x^3 + 0x^2 - 13x - 12 \\ - & x^3 + 3x^2 \\ & 0 - 3x^2 - 13x \\ - & -3x^2 - 9x \\ & 0 - 4x - 12 \\ - & -4x - 12 \\ & 0 + 0 \end{array}$$

$(x+3)(x^2 - 3x - 4)$

$(x+3)(x-4)(x+1)$

② $A(1,3) \quad B(3,7) \quad C(-1,9)$

ai) grad $AB = \frac{7-3}{3-1} = \frac{4}{2} = \underline{\underline{2}}$

ii) grad $BC = \frac{9-7}{-1-3} = \frac{2}{-4} = \underline{\underline{-\frac{1}{2}}}$

$2 \times -\frac{1}{2} = -1 \quad \therefore$ right angle at B as perpendicular

$A(1,3)\ C(-1,9)$

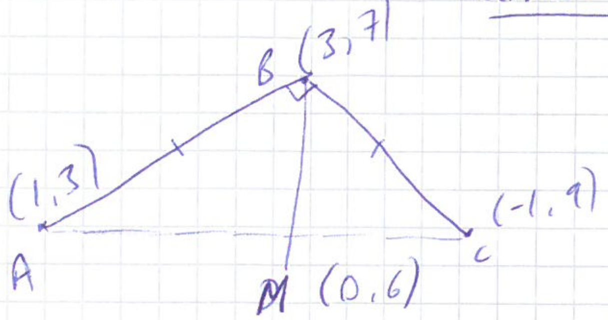
bi) Midpoint $M = \left(\frac{1+-1}{2}, \frac{3+9}{2}\right) = \underline{\underline{(0,6)}}$

ii) length $AB = \sqrt{2^2 + 4^2} = \sqrt{20}$ $\quad \therefore$ lengths are equal

$BC = \sqrt{4^2 + 2^2} = \sqrt{20}$

Core 1 - Jan 2010

2biii)



line of symmetry at BM

grad BM is perp to AC

$$\text{grad } BM = \frac{7-6}{3-0} = \frac{1}{3}$$

B (3,7) OR M (0,6)

$$y - 7 = \frac{1}{3}(x-3)$$

$$3y - 21 = x - 3$$

$$\underline{\underline{3y = x + 18}}$$

③ $y = \frac{1}{8}t^4 - 2t^2 + 4t$

ai) $\frac{dy}{dt} = \frac{4}{8}t^3 - 4t + 4$

$= \frac{1}{2}t^3 - 4t + 4$

ii) $\frac{d^2y}{dt^2} = \frac{3}{2}t^2 - 4$

b) when $t = 2$

$\frac{dy}{dt} = \frac{1}{2}(2)^3 - 4(2) + 4$

$= 4 - 8 + 4$

$= 0$ ∴ stationary value when $t = 2$

ci) when $t = 1$

$\frac{dy}{dt} = \frac{1}{2}(1)^3 - 4(1) + 4$

$= \frac{1}{2} - 4 + 4 = \frac{1}{2}$

ii) $\frac{dy}{dt} > 0$ ∴ increasing when $t = 1$

Core 1 - Jan 2010

4 a) $\dfrac{\sqrt{50}+\sqrt{18}}{\sqrt{8}} = \dfrac{5\sqrt{2}+3\sqrt{2}}{2\sqrt{2}}$

$= \dfrac{8\cancel{\sqrt{2}}}{2\cancel{\sqrt{2}}} = \underline{\underline{4}}$

b) $\dfrac{(2\sqrt{7}-1)}{(2\sqrt{7}+5)} \times \dfrac{(2\sqrt{7}-5)}{(2\sqrt{7}-5)} = \dfrac{4\sqrt{49}-10\sqrt{7}-2\sqrt{7}+5}{4\sqrt{49}-10\sqrt{7}+10\sqrt{7}-25}$

$= \dfrac{28-12\sqrt{7}+5}{28-25} = \dfrac{33-12\sqrt{7}}{3}$

$= \underline{\underline{11-4\sqrt{7}}}$

5 a) $(x-5)(x-3)+2$

$x^2-3x-5x+15+2$

$x^2-8x+17$

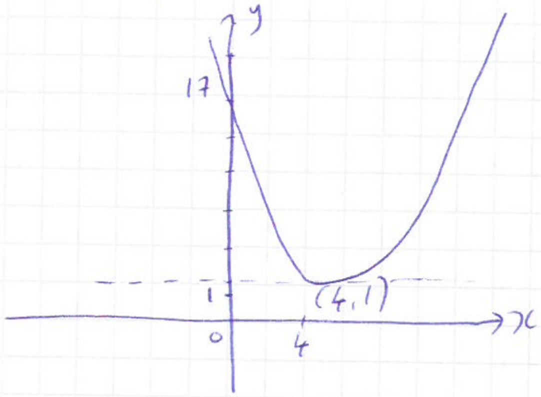
$(x-4)^2-16+17$

$\underline{(x-4)^2+1}$

b i) $y=(x-5)(x-3)+2$

$y = x^2-8x+17$ crosses at $(0,17)$

Min point at $(4,1)$



ii) equation of tangent is $\underline{\underline{y=1}}$

c) $y=x^2 \rightarrow (x-4)^2+1$

Translation $\begin{pmatrix}4\\1\end{pmatrix}$

Core1 - Jan 2010

⑥ $y = 12x^2 - 19x - 2x^3$ $\qquad$ $A(2, -6)$

ai) $\frac{dy}{dx} = 24x - 19 - 6x^2$

when $x = 2$, $\frac{dy}{dx} = 24(2) - 19 - 6(2)^2$

$= 48 - 19 - 24$

grad $= \underline{-5}$ at A

ii) grad normal is $\frac{1}{5}$ $\qquad$ $A(2, -6)$

$y + 6 = \frac{1}{5}(x - 2)$

$5y + 30 = x - 2$

$\underline{x - 5y - 32 = 0}$

bi) $\int_0^2 (12x^2 - 19x - 2x^3)\,dx$

$\left[\frac{12x^3}{3} - \frac{19x^2}{2} - \frac{2x^4}{4}\right]_0^2 = \left[4x^3 - \frac{19x^2}{2} - \frac{x^4}{2}\right]_0^2$

$= \left(4(2)^3 - \frac{19(2)^2}{2} - \frac{(2)^4}{2}\right) - (0)$

$= 32 - 38 - 8$

$= \underline{\underline{-14}}$

ii) shaded region = area under curve - area of triangle

area under curve = 14

area of triangle $= \frac{2 \times 6}{2} = 6$

shaded area = 14 - 6

$= \underline{\underline{8}}$

Core1 Jan 2010

(7) $x^2 + y^2 - 4x + 12y + 15 = 0$

ai) $x^2 - 4x + y^2 + 12y + 15 = 0$

$(x-2)^2 - 4 + (y+6)^2 - 36 + 15 = 0$

$(x-2)^2 + (y+6)^2 = 25$

$C = \underline{(2, -6)}$

ii) $r^2 = 25$

radius $= \underline{5}$

b) distance from x axis to centre $(2, -6)$ is 6

radius is 5

$5 < 6$ $\therefore$ entirely below ~~x~~ axis

c) $P(5, k)$ $\quad C(2, -6)$

i) $PC^2 = 3^2 + (k - -6)^2$

$= 9 + (k+6)(k+6)$

$= 9 + k^2 + 12k + 36$

$= \underline{k^2 + 12k + 45}$ (as required)

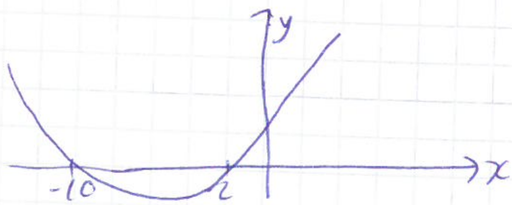
ii) $PC^2 > 5^2$

$k^2 + 12k + 45 > 25$

$\underline{k^2 + 12k + 20 > 0}$ (as required)

iii) $(k+10)(k+2)$

critical values at $k = -10$ and $k = -2$



$k^2 + 12k + 20 > 0$

$\therefore \underline{k < -10}, \underline{k > 2}$