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① $3x + 5y = 11$

ai) $5y = -3x + 11$

$y = -\frac{3}{5}x + \frac{11}{5}$ grad is $\underline{-\frac{3}{5}}$

ii) $A\ (2, 1)$ perp grad is $\frac{5}{3}$

$y - 1 = \frac{5}{3}(x - 2)$

$3y - 3 = 5x - 10$

$3y = \underline{5x - 7}$

b) $2x + 3y = 8 \quad (\times 5)$

$3x + 5y = 11 \quad (\times 3)$

$$\begin{array}{r} 10x + 15y = 40 \\ - \quad 9x + 15y = 33 \\ \hline x = 7 \end{array}$$

$\underline{\underline{x = 7}} \rightarrow 2(7) + 3y = 8$

$14 + 3y = 8$

$3y = -6$

$\underline{\underline{y = -2}}$

$C\ (\underline{\underline{7, -2}})$

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② a) $\frac{(5+\sqrt{7})}{(3-\sqrt{7})} \times \frac{(3+\sqrt{7})}{(3+\sqrt{7})} = \frac{15+5\sqrt{7}+3\sqrt{7}+\sqrt{49}}{9+3\sqrt{7}-3\sqrt{7}-\sqrt{49}}$

$= \frac{15+8\sqrt{7}+7}{9-7}$

$= \frac{22+8\sqrt{7}}{2}$

$= \underline{\underline{11+4\sqrt{7}}}$

b) $x^2 = (2\sqrt{5})^2 - (3\sqrt{2})^2$

$x^2 = 4\sqrt{25} - 9\sqrt{4}$

$x^2 = 4(5) - 9(2)$

$x^2 = 20 - 18$

$x^2 = 2$

$\underline{\underline{x = \sqrt{2}}}$ (cannot be $-\sqrt{2}$ as it is a length)

③ $y = x^5 + 20x^2 - 8$ $\quad P,\ x = -2$

a) $\frac{dy}{dx} = \underline{\underline{5x^4 + 40x}}$

b) when $x = -2$, $\frac{dy}{dx} = 5(-2)^4 + 40(-2)$

$= 80 - 80 = 0$

$\therefore$ a stationary point when $x = -2$

ci) $\frac{d^2y}{dx^2} = 20x^3 + 40$

when $x = -2$, $\frac{d^2y}{dx^2} = 20(-2)^3 + 40$

$= -160 + 40 = \underline{-120}$

ii) $\frac{d^2y}{dx^2} < 0$, $\therefore$ a $\underline{\text{maximum}}$ at $x = -2$

d) when $x = 1$, $\frac{dy}{dx} = 5(1)^4 + 40(1) = \underline{45}$ (m)

$y = (1)^5 + 20(1)^2 - 8 = 13$ (y_1)

$y - 13 = 45(x-1)$

$\underline{\underline{y = 45x - 32}}$

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④ $p(x) = x^3 - x + 6$

i) $p(3) = (3)^3 - 3 + 6 = 30$

$\therefore$ remainder is $\underline{\underline{30}}$ when $p(x)$ is divided by $x-3$

ii) $p(-2) = (-2)^3 - (-2) + 6$

$= -8 + 2 + 6 = 0$ $\quad\therefore (x+2)$ is a factor of $p(x)$

iii)

$$\begin{array}{r l}
 & x^2 - 2x + 3 \\
x+2 & \overline{\big)\, x^3 + 0x^2 - x + 6} \\
 & \underline{-\,x^3 + 2x^2} \\
 & 0 - 2x^2 - x \\
 & -\ \underline{-2x^2 - 4x} \\
 & 0 + 3x + 6 \\
 & -\ \underline{3x + 6} \\
 & 0 \quad 0
\end{array}$$

$\underline{(x+2)(x^2-2x+3)}$

iv) $(x+2)(x^2-2x+3) = 0$

$x + 2 = 0$ OR $x^2 - 2x + 3 = 0$

$x = -2$ $\quad a = 1, \quad b = -2, \quad c = 3$

$b^2 - 4ac < 0$ if no real roots

$(-2)^2 - 4(1)(3) < 0$

$4 - 12 < 0$ ✓ $\therefore$ no roots

bi) $y = \underline{6}$

ii) $\int_{-2}^{0} (x^3 - x + 6)\,dx = \left[\frac{x^4}{4} - \frac{x^2}{2} + 6x\right]_{-2}^{0}$

$= (0) - \left(\frac{(-2)^4}{4} - \frac{(-2)^2}{2} + 6(-2)\right)$

$= 0 - (4 - 2 - 12) = \underline{\underline{10}}$

iii) shaded region = curve − triangle

triangle $= \frac{2 \times 6}{2} = \underline{\underline{6}}$ $\qquad$ shaded $= 10 - 6$

$= \underline{\underline{4}}$

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⑤ $(x-5)^2 + (y+12)^2 = 169$

a i) $C = (5, -12)$

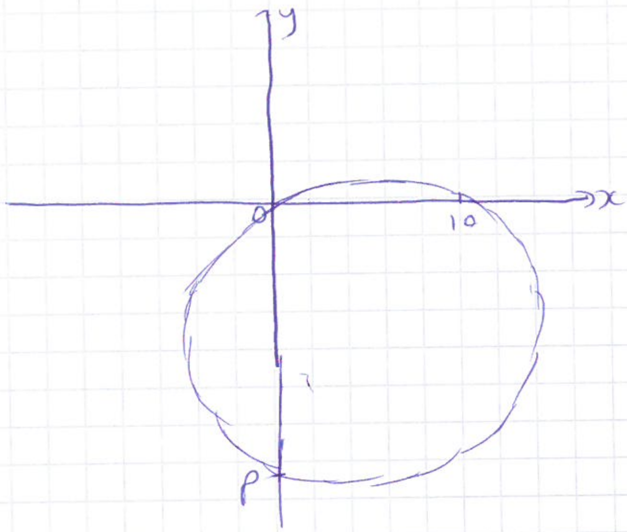
ii) $r^2 = 169$, radius $= 13$

b i) sub in $(0,0)$:-

$(-5)^2 + (+12)^2 = 169$

$25 + 144 = 169$ ✓ ∴ point $(0,0)$ lies on circle

ii)



$x = 0$:-

$(-5)^2 + (y+12)^2 = 169$

$25 + (y+12)^2 = 169$

$(y+12)^2 = 144$

$y + 12 = \pm\sqrt{144}$

$y + 12 = \pm 12$

$y = 0$, or $y = -24$

$p = -24$

c) $A(-7, -7)$ $C(5, -12)$

i) grad $AC = \dfrac{-12 - -7}{5 - -7} = \dfrac{-5}{12}$

ii) gradient tangent $= \dfrac{12}{5}$ $A(-7, -7)$

$y + 7 = \dfrac{12}{5}(x + 7)$

$5y + 35 = 12x + 84$

$12x - 5y + 49 = 0$

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⑥ ai) $x^2 - 8x + 17 = (x-4)^2 - 16 + 17$

$= \underline{(x-4)^2 + 1}$

ii) minimum value is $\underline{1}$

iii) minimum when $\underline{x = 4}$

b) $A(5, 4)$ $B(x, 7-x)$

i) $(x-5)^2 = \underline{x^2 - 10x + 25}$

ii) $AB^2 = (x-5)^2 + (7-x-4)^2$

$= x^2 - 10x + 25 + (3-x)^2$

$= x^2 - 10x + 25 + 9 - 6x + x^2$

$= 2x^2 - 16x + 34$

$= \underline{2(x^2 - 8x + 17)}$ (as req)

iii) minimum value of $x^2 - 8x + 17$ is 1

$AB^2 = 2(1)$

$\underline{AB = \sqrt{2}}$

⑦ $y = k(x^2 + 3)$ $y = 2x + 2$

a) $k(x^2 + 3) = 2x + 2$

$kx^2 + 3k = 2x + 2$

$kx^2 - 2x + 3k - 2 = 0$ (as req)

b) $b^2 - 4ac > 0$ if two distinct points

$a = k$ $b = -2$ $c = 3k - 2$

$(-2)^2 - 4(k)(3k-2) > 0$

$4 - 4k(3k-2) > 0$

$4 - 12k^2 + 8k > 0$

$12k^2 - 8k - 4 < 0$ $(\div 4)$ $\underline{3k^2 - 2k - 1 < 0}$ (as req)

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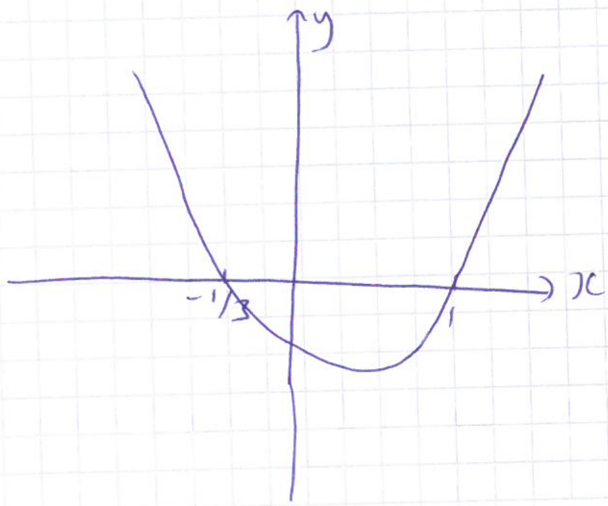
ii)

$3k^2 - 2k - 1 < 0$

$(3k+1)(k-1)$

$k = -1/3 \quad k = 1$ (critical values)

SKETCH!



graph < 0

$\therefore$ below x axis

$-1/3 < k < 1$