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① $A(1, 6)$ $B(5, -2)$

a) $M = \left(\frac{1+5}{2}, \frac{6+-2}{2}\right) = \underline{\underline{(3, 2)}}$

b) grad $AB = \frac{-2-6}{5-1} = \frac{-8}{4} = \underline{\underline{-2}}$

ci) perpendicular gradient is $\frac{1}{2}$ through $M(3, 2)$

$y - 2 = \frac{1}{2}(x-3)$

$2y - 4 = x - 3$

$2y = x + 1$

$\underline{x - 2y + 1 = 0}$ (as required)

ii) $x = k$, $y = k+5$

$k - 2(k+5) + 1 = 0$

$k - 2k - 10 + 1 = 0$

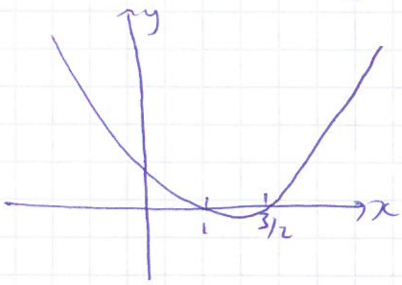
$-k - 9 = 0$

$\underline{k = -9}$

② a) $2x^2 - 5x + 3 = \underline{(2x-3)(x-1)}$

b) $2x^2 - 5x + 3 < 0$

critical values at $x = \frac{3}{2}$, $x = 1$



$2x^2 - 5x + 3 < 0$

$\therefore$ graph below x axis

$\underline{1 < x < \frac{3}{2}}$

③ a) $\frac{(7+\sqrt{5})}{(3+\sqrt{5})} \times \frac{(3-\sqrt{5})}{(3-\sqrt{5})} = \frac{21 - 7\sqrt{5} + 3\sqrt{5} - \sqrt{25}}{9 - 3\sqrt{5} + 3\sqrt{5} - \sqrt{25}}$

$= \frac{21 - 4\sqrt{5} - 5}{9 - 5} = \frac{16 - 4\sqrt{5}}{4}$

$= \underline{\underline{4 - \sqrt{5}}}$

b) $\sqrt{45} = \sqrt{9}\sqrt{5}$
$= 3\sqrt{5}$

$\frac{20}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{20\sqrt{5}}{5}$

$= 4\sqrt{5}$

$\sqrt{45} + \frac{20}{\sqrt{5}} = 3\sqrt{5} + 4\sqrt{5}$

$= \underline{\underline{7\sqrt{5}}}$

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4 ai) $x^2 + 2x + 5 = (x+1)^2 - 1 + 5$

$= \underline{(x+1)^2 + 4}$

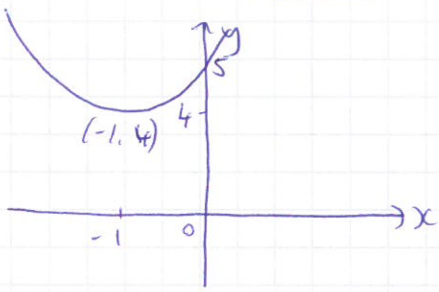
ii) $(x+1)^2 \geq 0$

$\therefore (x+1)^2 + 4 > 0$

b) $y = x^2 + 2x + 5$

i) minimum at $\underline{(-1, 4)}$

ii)



c) $y = x^2 \rightarrow y = x^2 + 2x + 5$

Translation $\begin{pmatrix} -1 \\ 4 \end{pmatrix}$ (1 left, 4 up)

5 $x = \frac{1}{2}t^4 - 20t^2 + 66t \qquad 0 \leq t \leq 4$

ai) $\frac{dx}{dt} = 2t^3 - 40t + 66$

ii) $\frac{d^2x}{dt^2} = 6t^2 - 40$

b) when $t = 3$, $\frac{dx}{dt} = 2(3)^3 - 40(3) + 66$

$= 54 - 120 + 66 = 0 \quad \therefore$ stationary value

$\frac{d^2x}{dt^2} = 6(3)^2 - 40$

$= 54 - 40 = 14 \quad \frac{d^2x}{dt^2} > 0 \quad \therefore$ minimum value

c) when $t = 1$, $\frac{dx}{dt} = 2(1)^3 - 40(1) + 66$

$= 2 - 40 + 66$

$= \underline{28}$

d) when $t = 2$, $\frac{dx}{dt} = 2(2)^3 - 40(2) + 66$

$= 16 - 80 + 66 = 2$

$\frac{dx}{dt} > 0 \quad \therefore$ increasing when $t = 2$

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⑥ $p(x) = x^3 + x - 10$

ai) $p(2) = (2)^3 + 2 - 10 = 0$ $(x-2)$ is a factor of $p(x)$

ii)
$$\begin{array}{r} x^2 + 2x + 5 \\ x-2 \enclose{longdiv}{x^3 + 0x^2 + x - 10} \\ -\underline{x^3 - 2x^2} \\ 0 + 2x^2 + x \\ -\underline{2x^2 - 4x} \\ 0 + 5x - 10 \\ -\underline{5x - 10} \\ 0 \quad 0 \end{array}$$

$\underline{(x-2)(x^2+2x+5)}$

b) $y = x^3 + x - 10$

i) $\frac{dy}{dx} = 3x^2 + 1$ when $x = 2$, $\frac{dy}{dx} = 3(2)^2 + 1$

$= \underline{\underline{13}} \rightarrow$ gradient

ii) $m = 13$, $(2, 0)$

$y = 13(x-2)$

$y = \underline{13x - 26}$

iii) $\int (x^3 + x - 10)\,dx = \underline{\frac{x^4}{4} + \frac{x^2}{2} - 10x + C}$

iv) $\int_0^2 (x^3 + x - 10)\,dx = \left[\frac{x^4}{4} + \frac{x^2}{2} - 10x\right]_0^2$

$= \left(\frac{(2)^4}{4} + \frac{(2)^2}{2} - 10(2)\right) - 0$

$= 4 + 2 - 20$

$= -14$

$\therefore$ area of shaded region is $\underline{14}$

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7) $x^2 + y^2 - 6x + 10y + 9 = 0$

a) $x^2 - 6x + y^2 + 10y + 9 = 0$

$(x-3)^2 - 9 + (y+5)^2 - 25 + 9 = 0$

$\underline{(x-3)^2 + (y+5)^2 = 25}$

bi) $C = \underline{(3, -5)}$

ii) $r^2 = 25$, radius $= \underline{5}$

c) $D(7, -2)$

i) $x = 7, \quad y = -2$

$(7-3)^2 + (-2+5)^2 = 25$

$4^2 + 3^2 = 25$

$16 + 9 = 25$ ✓ $\therefore$ point D lies on the circle

ii) grad $CD = \dfrac{-2 - -5}{7-3} = \dfrac{3}{4}$ $\qquad D(7, -2)$

$y + 2 = \frac{3}{4}(x - 7)$

$4y + 8 = 3x - 21$

$\underline{3x - 4y = 29}$ (as req)

di) $y = kx \rightarrow$ sub in $\quad x^2 + (kx)^2 - 6x + 10(kx) + 9 = 0$

$x^2 + k^2x^2 - 6x + 10kx + 9 = 0$

$\underline{(k^2+1)x^2 + 2(5k-3)x + 9 = 0}$ (as req)

ii) $b^2 - 4ac = 0$ (equal roots) $\quad a = k^2 + 1,\ b = 2(5k-3) = 10k - 6,\ c = 9$

$(10k-6)^2 - 4(k^2+1)(9) = 0$

$100k^2 - 120k + 36 - 36(k^2+1) = 0$

$100k^2 - 120k + 36 - 36k^2 - 36 = 0$

$64k^2 - 120k = 0 \quad (\div 8)$

$8k^2 - 15k = 0, \quad k(8k - 15) = 0$

$\underline{k = 0}, \quad \underline{k = 15/8}$

iii) The line is a tangent to the circle as only one solutio