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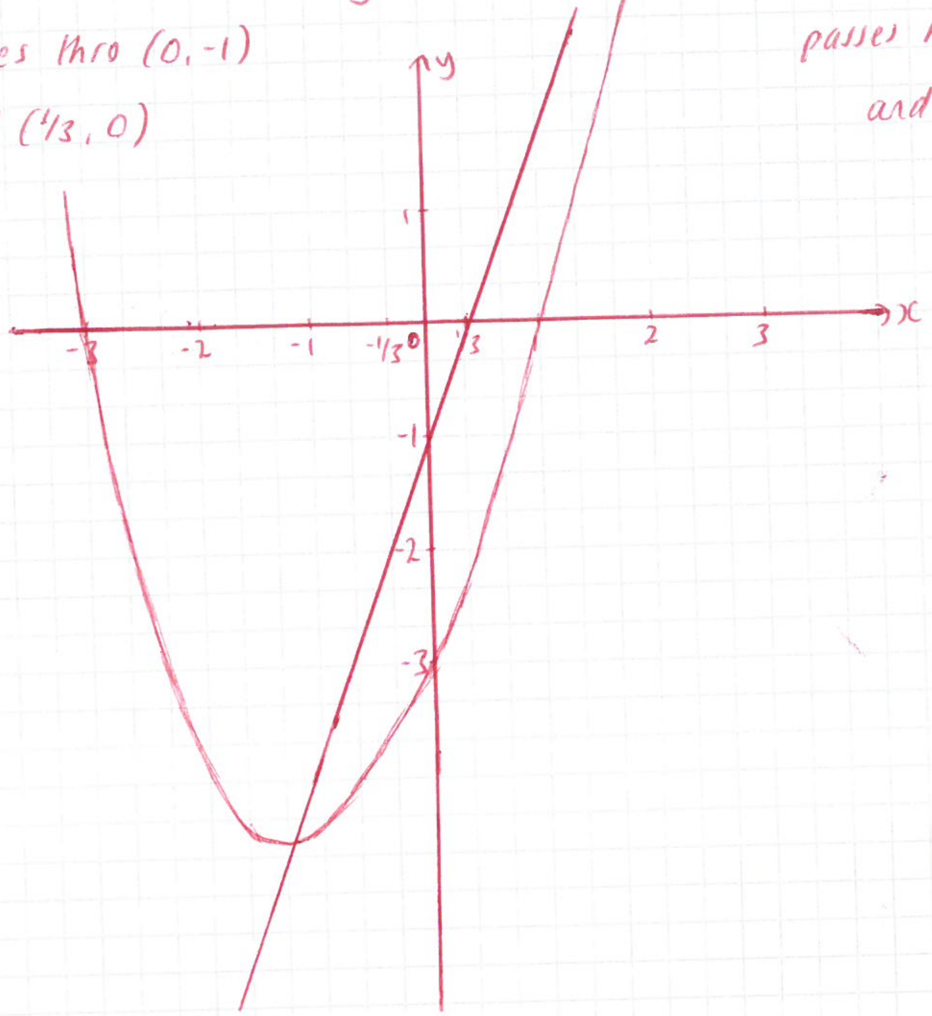
a) $y = 3x - 1$ passes thro $(0, -1)$ and $(1/3, 0)$

$y = (x+3)(x-1) \rightarrow x^2 + 2x - 3$ passes thro $(0, -3)$ and $(-3, 0)$ and $(1, 0)$



b) $3x - 1 = (x+3)(x-1)$

$3x - 1 = x^2 + 2x - 3$

$\underline{x^2 - x - 2 = 0}$ (as required)

c) $(x-2)(x+1) = 0$

$x = 2, \quad x = -1$

$y = 3(2) - 1, \quad y = 3(-1) - 1$

$y = 5 \quad\quad y = -4$

$\underline{(2, 5)} \quad\quad \underline{(-1, -4)}$

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② $x = \sqrt{3} \qquad y = \sqrt{12}$

a) $xy = \sqrt{3} \times \sqrt{12}$

$= \sqrt{36}$

$= \underline{\underline{6}}$

b) $\dfrac{y}{x} = \dfrac{\sqrt{12}}{\sqrt{3}} = \dfrac{\sqrt{4}\cancel{\sqrt{3}}}{\cancel{\sqrt{3}}} = \sqrt{4} = \underline{\underline{2}}$

c) $(x+y)^2 = (\sqrt{3} + \sqrt{12})^2$

$= (\sqrt{3} + \sqrt{12})(\sqrt{3} + \sqrt{12})$

$= \sqrt{9} + \sqrt{36} + \sqrt{36} + \sqrt{144}$

$= 3 + 6 + 6 + 12$

$= \underline{\underline{27}}$

③ $3x + y = 9 \qquad V = xy^2$

a) $y = 9 - 3x$ → sub in $V = xy^2$

$V = x(9-3x)^2$

$V = x(9-3x)(9-3x)$

$V = x(81 - 54x + 9x^2)$

$V = \underline{81x - 54x^2 + 9x^3}$

bi) $V = 81x - 54x^2 + 9x^3$

$\dfrac{dV}{dx} = 81 - 108x + 27x^2$

$= 27x^2 - 108x + 81$

$= \underline{27(x^2 - 4x + 3)} \qquad \underline{\underline{k = 27}}$

ii) $\dfrac{dV}{dx} = 0$

$27(x^2 - 4x + 3) = 0 \quad (\div 27)$

$x^2 - 4x + 3 = 0$

$(x-3)(x-1) = 0$

$\underline{\underline{x = 1}}, \quad \underline{\underline{x = 3}}$

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3. $\frac{dv}{dx} = 27x^2 - 108x + 81$

c) $\frac{d^2v}{dx^2} = \underline{54x - 108}$

di) when $x = 1$, $\frac{d^2v}{dx^2} = 54(1) - 108$

$= \underline{\underline{-54}}$

$x = 3$, $\frac{d^2v}{dx^2} = 54(3) - 108$

$= \underline{\underline{54}}$

ii) maximum when $\frac{d^2v}{dx^2} < 0$, $\therefore$ at $\underline{\underline{x = 1}}$

iii) when $x = 1$, $V = 81(1) - 54(1)^2 + 9(1)^3$

$= 81 - 54 + 9$

$= \underline{\underline{36}}$

4. a) $x^2 - 3x + 4 = (x - \frac{3}{2})^2 - \frac{9}{4} + 4$

$= \underline{(x - \frac{3}{2})^2 + \frac{7}{4}}$

b) minimum value at $\underline{\underline{\frac{7}{4}}}$

c) from $y = x^2 \rightarrow y = (x - \frac{3}{2})^2 + \frac{7}{4}$

Translation $\begin{pmatrix} 3/2 \\ 7/4 \end{pmatrix}$

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⑤

$y = 16 - x^4$

$A(-2,0) \quad B(2,0) \quad C(1,15)$

a) grad $AC = \dfrac{15-0}{1--2} = \dfrac{15}{3} = \underline{5}$ (m) through $A(-2,0)$

$y = 5(x+2)$

$\underline{\underline{y = 5x + 10}}$

bi) $\displaystyle\int_{-2}^{1} (16 - x^4)\,dx = \left[16x - \frac{x^5}{5}\right]_{-2}^{1}$

$= \left(16(1) - \dfrac{(1)^5}{5}\right) - \left(16(-2) - \dfrac{(-2)^5}{5}\right)$

$= \left(16 - \frac{1}{5}\right) - \left(-32 + \frac{32}{5}\right)$

$= 16 + 32 - \frac{1}{5} - \frac{32}{5}$

$= 48 - \frac{33}{5}$

$= 48 - 6\frac{3}{5}$

$= \underline{\underline{41\frac{2}{5}}}$

ii) shaded area = area under curve - area of triangle

area of triangle $= \dfrac{3 \times 15}{2} = \dfrac{45}{2}$

shaded area $= 41\frac{2}{5} - \frac{45}{2}$

$= 41\frac{2}{5} - 22\frac{1}{2}$

$= \underline{\underline{18\frac{9}{10}}}$

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⑥ $p(x) = x^3 + x^2 - 8x - 12$

a) $p(1) = (1)^3 + (1)^2 - 8(1) - 12$

$= 1 + 1 - 8 - 12$

$= \underline{-18}$ Remainder for $(x-1)$ is -18

bi) $p(-2) = (-2)^3 + (-2)^2 - 8(-2) - 12$

$= -8 + 4 + 16 - 12$

$= 0$ $\therefore$ $\underline{(x+2) \text{ is a factor}}$ of $p(x)$

ii)

$$\begin{array}{r|l} & x^2 - x - 6 \\ \hline x+2 & x^3 + x^2 - 8x - 12 \\ - & x^3 + 2x^2 \\ \hline & 0 - x^2 - 8x \\ - & -x^2 - 2x \\ \hline & -6x - 12 \\ - & -6x - 12 \\ \hline & 0 + 0 \end{array}$$

$(x+2)(x^2 - x - 6)$

$(x+2)(x-3)(x+2)$

$\underline{(x+2)^2(x-3)}$

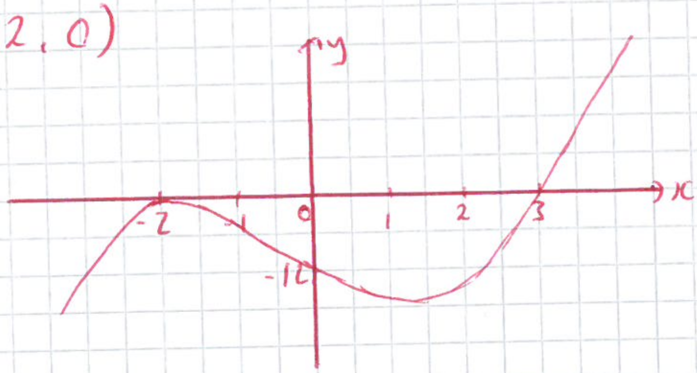
ci) $y = x^3 + x^2 - 8x - 12$

$(0, k)$ $\underline{k = -12}$

ii)

passes thro $(3, 0)$ and $(0, -12)$

touches $(-2, 0)$



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(7) centre $(8,13)$ touches x axis

$\therefore$ radius is 13

a) $(x-8)^2 + (y-13)^2 = 13^2$

b) $P(3,1)$ $C(8,13)$

i) grad $PC = \dfrac{13-1}{8-3} = \dfrac{12}{5}$

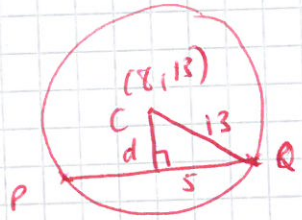
ii) grad tangent $= \dfrac{-5}{12}$ through $P(3,1)$

$$y - 1 = \frac{-5}{12}(x-3)$$

$$12y - 12 = -5x + 15$$

$$5x + 12y = 27$$

iii)



$d^2 = 13^2 - 5^2$

$= 169 - 25$

$d^2 = 144$

$d = 12$

(8) $(k+1)x^2 + 4kx + 9 = 0$

a) $b^2 - 4ac \geqslant 0$ if real roots $a = k+1$, $b = 4k$, $c = 9$

$(4k)^2 - 4(k+1)(9) \geqslant 0$

$16k^2 - 36k - 36 \geqslant 0$ $(\div 4)$

$4k^2 - 9k - 9 \geqslant 0$ (as required)

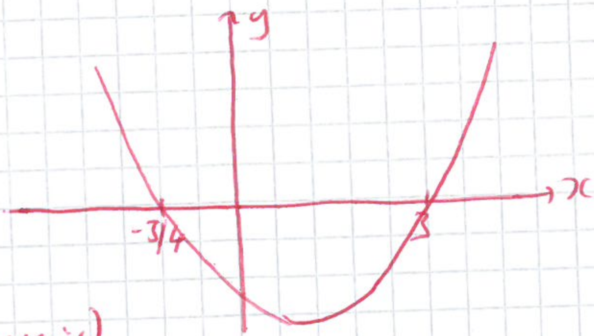
b) $4k^2 - 9k - 9 \geqslant 0$

$(4k+3)(k-3)$

critical values at $k = -3/4$

$k = 3$

graph greater than 0 (above x axis)



$k \leqslant -3/4$, $k \geqslant 3$