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① $A(-2,3)$ $B(4,1)$ $C(2,-5)$

a) midpoint $BC = \left(\frac{4+2}{2}, \frac{1+-5}{2}\right)$

$= \underline{(3,-2)}$

bi) grad $AB = \frac{3-1}{-2-4} = \frac{2}{-6} = \underline{\underline{-\frac{1}{3}}}$

ii) $m = -\frac{1}{3}$, $B(4,1)$

$y - 1 = -\frac{1}{3}(x-4)$

$3y - 3 = -x + 4$

$\underline{\underline{x + 3y = 7}}$

iii) grad $= -\frac{1}{3}$ (parallel to AB)

thro' $C(2,-5)$

$y + 5 = -\frac{1}{3}(x-2)$

$3y + 15 = -x + 2$

$\underline{x + 3y + 13 = 0}$

c) grad $AB = -\frac{1}{3}$

grad $BC = \frac{-5-1}{2-4} = \frac{-6}{-2} = 3$

$-\frac{1}{3} \times 3 = -1$ $\therefore$ gradients are perpendicular and angle ABC is a right angle

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② $y = x^4 - 32x + 5$

a) $\frac{dy}{dx} = \underline{4x^3 - 32}$

b) stationary point at M → $\frac{dy}{dx} = 0$

$4x^3 - 32 = 0$

$4x^3 = 32$

$x^3 = 8$

$\underline{\underline{x = 2}}$

c) i) $\frac{d^2y}{dx^2} = \underline{\underline{12x^2}}$

ii) when $x = 2$, $\frac{d^2y}{dx^2} = 12(2)^2$

$= 48$

$\frac{d^2y}{dx^2} > 0$ ∴ minimum point at $x = 2$ (M)

d) when $x = 0$, $\frac{dy}{dx} = 4(0)^3 - 32$

$= -32$

∴ gradient is negative and the curve is decreasing at $x = 0$.

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③ a) $5\sqrt{8} + \frac{6}{\sqrt{2}}$

$5\sqrt{4}\sqrt{2} + \frac{6}{\sqrt{2}}$

$10\sqrt{2} + 3\sqrt{2}$

$\underline{\underline{13\sqrt{2}}}$

$$\boxed{\begin{aligned} 5\sqrt{8} &= 5\sqrt{4}\sqrt{2} \\ &= \underline{10\sqrt{2}} \\ \frac{6}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} &= \frac{6\sqrt{2}}{2} = 3\sqrt{2} \end{aligned}}$$

b) $\frac{(\sqrt{2}+2)}{(3\sqrt{2}-4)} \times \frac{(3\sqrt{2}+4)}{(3\sqrt{2}+4)} = \frac{3\sqrt{4}+4\sqrt{2}+6\sqrt{2}+8}{9\sqrt{4}+12\sqrt{2}-12\sqrt{2}-16}$

$= \frac{3(2)+10\sqrt{2}+8}{9(2)-16}$

$= \frac{14+10\sqrt{2}}{2} = \underline{7+5\sqrt{2}}$

$= \underline{\underline{5\sqrt{2}+7}}$

④ $x^2+y^2-10y+20=0$

a) $x^2+(y-5)^2-25+20=0$

$\underline{x^2+(y-5)^2=5}$

bi) $C = \underline{\underline{(0,5)}}$

ii) $r^2 = 5$, radius $= \underline{\underline{\sqrt{5}}}$

c) $y=2x$, $x^2+y^2-10y+20=0$

a) sub $y=2x$ into circle :-

$x^2+(2x)^2-10(2x)+20=0$

$x^2+4x^2-20x+20=0$

$5x^2-20x+20=0$ $(\div 5)$

$\underline{x^2-4x+4}=0$ (as required)

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(4)

c ii) $x^2 - 4x + 4 = 0$

$(x-2)(x-2) = 0$ (repeated root)

$\underline{x = 2}$ only one solution, $\therefore$ line is a tangent to the circle

when $x = 2$, $y = 2x$

$y = 2(2)$

$y = 4$

$P\ \underline{\underline{(2, 4)}}$

d) $Q(-1, 4)$ $C(0, 5)$

length $CQ = \sqrt{1^2 + 1^2}$

$= \sqrt{2}$ radius is $\sqrt{5}$

$\sqrt{2} < \sqrt{5}$ $\therefore$ Q lies inside the circle

(5) a) $9 - 8x - x^2$

$\underline{(9 + x)(1 - x)}$

b) $25 - (x+4)^2$

$25 - (x+4)(x+4)$

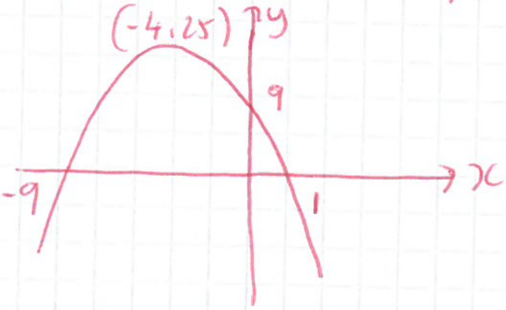
$25 - (x^2 + 8x + 16) = 9 - x^2 - 8x$

$= \underline{9 - 8x - x^2}$ (as req)

c i) line of symmetry at $\underline{\underline{x = -4}}$

ii) vertex $(\underline{\underline{-4, 25}})$

iii) max at $(-4, 25)$ passes thro $(0, 9)$ and $(-9, 0)$, $(1, 0)$



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6. $p(x) = x^3 - 7x + 6$

i) $p(-1) = (-1)^3 - 7(-1) - 6$

$= -1 + 7 - 6 = 0 \quad \therefore (x+1)$ is a factor of $p(x)$

ii)

$$\begin{array}{r|l} & x^2 - x - 6 \\ \hline x+1 & x^3 + 0x^2 - 7x - 6 \\ & -\ x^3 + x^2 \\ \hline & 0 \ - x^2 - 7x \\ & -\ -x^2 - x \\ \hline & 0 \ -6x - 6 \\ & -\ -6x - 6 \\ \hline & 0 \end{array}$$

$(x+1)(x^2 - x - 6)$

$\underline{(x+1)(x-3)(x+2)}$

b i) $A = \underline{(-2, 0)}$

ii) $\int_{-1}^{3} (x^3 - 7x - 6)\,dx = \left[\frac{x^4}{4} - \frac{7x^2}{2} - 6x\right]_{-1}^{3}$

$= \left(\frac{3^4}{4} - \frac{7(3)^2}{2} - 6(3)\right) - \left(\frac{(-1)^4}{4} - \frac{7(-1)^2}{2} - 6(-1)\right)$

$= \left(\frac{81}{4} - \frac{63}{2} - 18\right) - \left(\frac{1}{4} - \frac{7}{2} + 6\right)$

$= \underline{\underline{-32}}$

iii) shaded area $= \underline{\underline{32}}$

iv) $y = x^3 - 7x - 6$

$\frac{dy}{dx} = 3x^2 - 7$, when $x = -1$ (B)

$\frac{dy}{dx} = 3(-1)^2 - 7$

$= \underline{-4}$

v) grad of normal is $\frac{1}{4}$ through $(-1, 0)$

$y = \frac{1}{4}(x+1)$

$\underline{4y = x + 1}$

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(7) $y = x^2 + 7$ $\quad y = k(3x+1)$

a) $x^2 + 7 = k(3x+1)$ → point of intersection

$x^2 + 7 = 3kx + k$

$x^2 - 3kx + 7 - k = 0$ (as req)

b) $b^2 - 4ac > 0$ if two distinct roots

$a = 1, \quad b = -3k, \quad c = 7-k$

$(-3k)^2 - 4(1)(7-k) > 0$

~~$9k^2 + 4k$~~

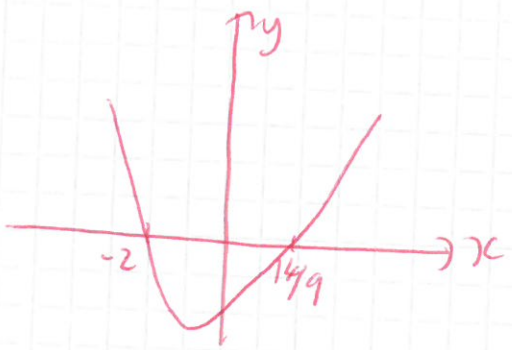
$9k^2 - 4(7-k) > 0$

$9k^2 - 28 + 4k > 0$

$9k^2 + 4k - 28 > 0$ (as req)

c) $(9k - 14)(k + 2)$

$k = \frac{14}{9} \quad k = -2$ (critical values)



$9k^2 + 4k - 28 > 0$

∴ graph above x axis

$k < -2, \quad k > 14/9$