Core 1 Jan 2007

① $p(x) = x^3 - 4x^2 - 7x + k$

ai) $p(-2) = 0$ if $(x+2)$ is a factor of $p(x)$

$(-2)^3 - 4(-2)^2 - 7(-2) + k = 0$

$-8 - 16 + 14 + k = 0$

$-24 + 14 + k = 0$

$-10 + k = 0$

$\underline{k = 10}$ (as req)

ii)

$$
\begin{array}{r l}
 & x^2 - 6x + 5 \\
x + 2 \,\big)\!\! & \overline{x^3 - 4x^2 + 7x + 10} \\
- & \underline{x^3 + 2x^2} \\
 & 0 \; - 6x^2 - 7x \\
 & - \underline{-6x^2 - 12x} \\
 & \quad 0 \; + 5x + 10 \\
 & \quad - \underline{5x + 10} \\
 & \quad\quad 0
\end{array}
$$

$(x+2)(x - 6x + 5)$

$\underline{(x+2)(x-5)(x-1)}$

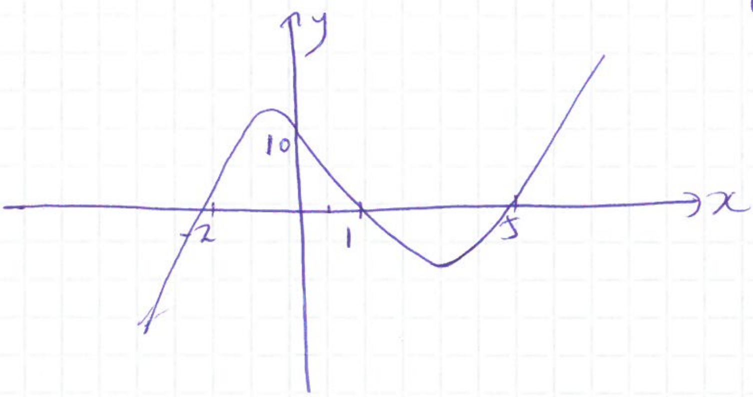
b) $p(3) = (3)^3 - 4(3)^2 - 7(3) + 10$

$= 27 - 36 - 21 + 10$

$= -20$ remainder is $\underline{-20}$

c) $y = x^3 - 4x^2 - 7x + 10$ crosses axes at $(0, 10)$

$(-2, 0), (5, 0), (1, 0)$



Core 1 Jan 2007

② AB $3x + 5y = 8$ $A(6, -2)$

ai) $5y = -3x + 8$

$y = -\frac{3}{5}x + \frac{8}{5}$ gradient is $\underline{\underline{-\frac{3}{5}}}$

ii) perpendicular gradient is $\frac{5}{3}$ through $(6, -2)$

$y + 2 = \frac{5}{3}(x - 6)$

$3y + 6 = 5x - 30$

$\underline{\underline{3y = 5x - 36}}$

b) $3x + 5y = 8$ $(\times 3)$

$2x + 3y = 3$ $(\times 5)$

$$
\begin{array}{r}
9x + 15y = 24 \\
- \quad 10x + 15y = 15 \\
\hline
-x = 9
\end{array}
$$

$\underline{\underline{x = -9}}$ $\rightarrow$ $2(-9) + 3y = 3$

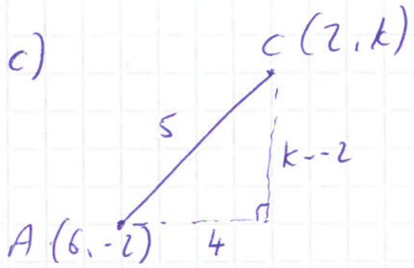
$-18 + 3y = 3$

$3y = 21$

$\underline{\underline{y = 7}}$

B is $\underline{\underline{(-9, 7)}}$

c)



$(6-2)^2 + (k - -2)^2 = 5^2$

$4^2 + (k+2)^2 = 25$

$16 + (k+2)^2 = 25$

$(k+2)^2 = 9$

$(k+2) = \pm\sqrt{9}$

$k + 2 = \pm 3$

$k = -2 + 3$ or $k = -2 - 3$

$\underline{\underline{k = 1}}$ $\underline{\underline{k = -5}}$

Core1 Jan 2007

③ a) $\frac{(\sqrt{5}+3)}{(\sqrt{5}-2)} \times \frac{(\sqrt{5}+2)}{(\sqrt{5}+2)} = \frac{\sqrt{25}+2\sqrt{5}+3\sqrt{5}+6}{\sqrt{25}+2\sqrt{5}-2\sqrt{5}-4}$

$= \frac{5+5\sqrt{5}+6}{5-4}$

$= \frac{11+5\sqrt{5}}{1} = 11+5\sqrt{5}$

$= \underline{5\sqrt{5}+11}$

b) i) $\sqrt{45} = \sqrt{9}\times\sqrt{5}$

$= \underline{3\sqrt{5}}$

ii) $x\sqrt{20} = 7\sqrt{5} - \sqrt{45}$

$x\sqrt{20} = 7\sqrt{5} - 3\sqrt{5}$

$x\sqrt{20} = 4\sqrt{5}$

$x\sqrt{4}\sqrt{5} = 4\sqrt{5}$

$2x\sqrt{5} = 4\sqrt{5}$

$x = \frac{4\cancel{\sqrt{5}}}{2\cancel{\sqrt{5}}} = \underline{2}$

④ $x^2+y^2+2x-12y+12=0$

a) $x^2+2x+y^2-12y+12=0$

$(x+1)^2 - 1 + (y-6)^2 - 36 + 12 = 0$

$(x+1)^2 + (y-6)^2 = 25$

$\underline{(x+1)^2 + (y-6)^2 = 5^2}$

b) i) $C = \underline{(-1, 6)}$

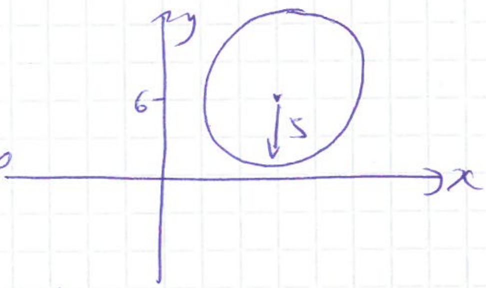
ii) $r^2 = 25$

$r = 5$ radius is $\underline{5}$

Core 1 Jan 2007

4c) centre is $(-1, 6)$

radius is 5



distance from centre to x axis is 6

$6 > 5 \therefore$ does not intersect the x axis.

di) $x + y = 4$ intersects $x^2 + y^2 + 2x - 12y + 12 = 0$

$\boxed{\cancel{x = 4 - y}}$ sub in

$\boxed{y = (4 - x)}$

$$x^2 + (4-x)^2 + 2x - 12(4 - x) + 12 = 0$$

$$x^2 + (4-x)(4-x) + 2x - 48 + 12x + 12 = 0$$

$$x^2 + 16 - 8x + x^2 + 2x - 48 + 12x + 12 = 0$$

$$2x^2 + 6x - 20 = 0 \quad (\div 2)$$

$$\underline{x^2 + 3x - 10 = 0} \quad \text{(as req)}$$

ii) $x^2 + 3x - 10 = 0$

$(x + 5)(x - 2) = 0$

$x = -5, \quad x = 2 \ (P)$

$x + y = 4$

$-5 + y = 4$ $\qquad \underline{(-5, 9)}$ is point Q

$y = 9$

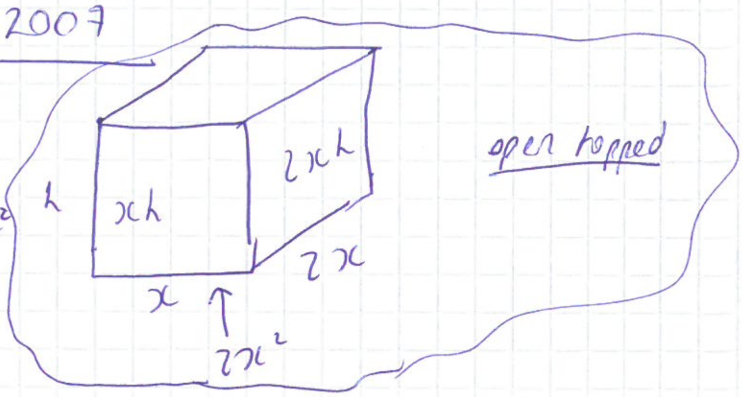
iii) $P(2, 2) \qquad Q(-5, 9)$

$$\text{midpoint} = \left(\frac{2 + -5}{2}, \frac{2 + 9}{2} \right)$$

$$= \left(\frac{-3}{2}, \frac{11}{2} \right) \text{ or } \underline{\left(-1\tfrac{1}{2}, 5\tfrac{1}{2}\right)}$$

Core 1 Jan 2007

(5) SA of 5 faces is $54m^2$



ai) $SA = xh + 2xh + xh + 2xh + 2x^2$

$6xh + 2x^2 = 54 \quad (\div 2)$

$\underline{x^2 + 3xh = 27}$ (as req)

ii) $x^2 + 3xh = 27 \quad (-x^2)$

$3xh = 27 - x^2 \quad (\div 3x)$

$\underline{h = \dfrac{27 - x^2}{3x}}$

iii) $Vol = 2x^2 h$

$h = \boxed{\dfrac{27 - x^2}{3x}}$ → sub into $V = 2x^2 h$

$V = 2\cancel{x^2}^{x}\left(\dfrac{27 - x^2}{3\cancel{x}}\right)$

$V = 2x\left(\dfrac{27 - x^2}{3}\right)$

$V = \dfrac{54x}{3} - \dfrac{2x^3}{3}$

$\underline{V = 18x - \dfrac{2x^3}{3}}$ (as req)

bi) $\dfrac{dV}{dx} = 18 - \dfrac{6x^2}{3} \rightarrow \underline{18 - 2x^2}$

ii) when $x = 3$, $\dfrac{dV}{dx} = 18 - 2(3)^2$

$= 18 - 18 = 0$ ∴ stationary point when $x = 3$

c) $\dfrac{d^2V}{dx^2} = \dfrac{-12x}{3} = \underline{-4x}$

when $x = 3$, $\dfrac{d^2V}{dx^2} = -4(3)$

$= -12$

$\dfrac{d^2V}{dx^2} < 0$ ∴ maximum value when $x = 3$

Core 1 Jan 2007

⑥ $y = 3x^5 + 2x + 5 \qquad A(-1, 0)$

ai) $x = 0$ at point B $\qquad$ Area of $\Delta = \frac{1 \times 5}{2} = \underline{\frac{5}{2}}$ or $2\frac{1}{2}$

$y = 5 \qquad \underline{(0, 5)}$

ii) $\int (3x^5 + 2x + 5)\,dx = \frac{3x^6}{6} + \frac{2x^2}{2} + 5x + C$

$= \frac{x^6}{2} + \frac{\cancel{2}x^2}{\cancel{2}} + 5x + C$

iii) shaded region = area under curve − area of Δ

area under curve $= \left[\frac{x^6}{2} + x^2 + 5x\right]_{-1}^{0}$

$= 0 - \left(\frac{(-1)^6}{2} + (-1)^2 + 5(-1)\right)$

$= 0 - \left(\frac{1}{2} + 1 - 5\right)$

$= 0 - \left(-3\frac{1}{2}\right)$

$= 3\frac{1}{2}$

shaded area $= 3\frac{1}{2} - 2\frac{1}{2} = \underline{1}$

bi) $y = 3x^5 + 2x + 5 \qquad A(-1, 0)$

$\frac{dy}{dx} = 15x^4 + 2$ $\qquad$ when $x = -1$, $\frac{dy}{dx} = 15(-1)^4 + 2$

$= \underline{17}$

gradient is $\underline{17}$ when $x = -1$

ii) $m = 17$, $(-1, 0)$

$y = 17(x + 1)$

$y = \underline{17x + 17}$

Core 1 Jan 2007

7. $(k+1)x^2 + 12x + (k-4) = 0$

a) Real roots $\rightarrow$ $b^2 - 4ac \geqslant 0$

$a = k+1 \qquad b = 12 \qquad c = k-4$

$12^2 - 4(k+1)(k-4) \geqslant 0$

$144 - 4(k^2 - 3k - 4) \geqslant 0$

$144 - 4k^2 + 12k + 16 \geqslant 0$

$160 - 4k^2 + 12k \geqslant 0$

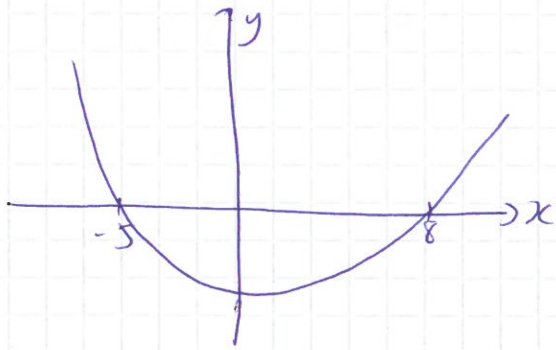
$4k^2 - 12k - 160 \leq 0$ $(\div 4)$

$k^2 - 3k - 40 \leq 0$ (as req)

b) $k^2 - 3k - 40 = 0$

$(k+5)(k-8) = 0$

$k = -5 \qquad k = 8$ (critical values)



$k^2 - 3k - 40 \leq 0$

$\therefore$ graph below x axis

$-5 \leq k \leq 8$