Core 1 - May / June 2006

1a) $A(1,7)$ $B(5,1)$

i) gradient $AB = \frac{1-7}{5-1} = \frac{-6}{4} = \frac{-3}{2}$

ii) $y - 7 = -\frac{3}{2}(x-1)$

$2y - 14 = -3(x-1)$

$2y - 14 = -3x + 3$

$3x + 2y = 17$ (as req)

b) $x - 4y = 8$ ①

$3x + 2y = 17$ ② $(\times 2)$

$$
\begin{array}{rl}
 & 6x + 4y = 34 \\
+ & x - 4y = 8 \\
\hline
 & 7x = 42
\end{array}
$$

$x = 6$, → sub in ②

$3(6) + 2y = 17$

$18 + 2y = 17$

$2y = -1$

$y = -\frac{1}{2}$

$\therefore C(6, -\frac{1}{2})$

c) gradient of perpendicular to AB is $\frac{2}{3}$

Coordinate $A(1,7)$

$y - 7 = \frac{2}{3}(x-1)$

$3y - 21 = 2(x-1)$

$3y - 21 = 2x - 2$

$3y - 2x = 19$

2a) $x^2 + 8x + 19$

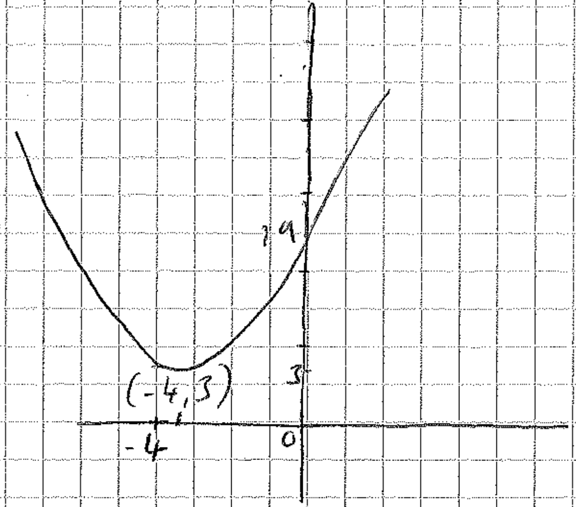
$(x+4)^2 + 3 \quad p = 4, \; q = 3$

b) $(x+4)^2 + 3 = 0$

$(x+4)^2 = -3$

no real solution to $\sqrt{-3}$

c)



d) $y \to x^2$ onto $y = (x+4)^2 + 3$

Translation $\begin{pmatrix} -4 \\ 3 \end{pmatrix}$

3) $y = 7 - 2x^5$

a) $\frac{dy}{dx} = -10x^4$

b) when $x = 1$, $\frac{dy}{dx} = -10(1)^4$

$= -10$ (gradient)

$y = 7 - 2(1)^5$

$y = 5$

$y - 5 = -10(x-1)$

$y - 5 = -10x + 10$

$\underline{y + 10x = 15}$

c) when $x = -2$, $\frac{dy}{dx} = -10(-2)^2 = -160$, $\frac{dy}{dx} < 0$, $\therefore$ y is decreasing

4a) $(4\sqrt{5}-1)(\sqrt{5}+3)$

$4\sqrt{25} + 12\sqrt{5} - \sqrt{5} - 3$

$20 + 11\sqrt{5} - 3 = \underline{17 + 11\sqrt{5}}$

b) $\dfrac{\sqrt{75} - \sqrt{27}}{\sqrt{3}} = \dfrac{\sqrt{25}\sqrt{3} - \sqrt{9}\sqrt{3}}{\sqrt{3}}$

$= \dfrac{5\sqrt{3} - 3\sqrt{3}}{\sqrt{3}}$

$= \dfrac{2\sqrt{3}}{\sqrt{3}} = \underline{2}$

5) $y = x^3 - 10x^2 + 28x$, $A(3, 21)$

a) $\dfrac{dy}{dx} = 3x^2 - 20x + 28$

ii) when $x = 2$, $\dfrac{dy}{dx} = 3(2)^2 - 20(2) + 28$

$= 12 - 40 + 28$

$= 0$

$\therefore$ stationary point as $\dfrac{dy}{dx} = 0$.

$3x^2 - 20x + 28 = 0$

$(3x - 14)(x - 2) = 0$

$\underline{x = 14/3}$, $x = 2$

bi) $\int (x^3 - 10x^2 + 28x)\,dx = \dfrac{x^4}{4} - \dfrac{10x^3}{3} + \dfrac{28x^2}{2} + c$

$= \dfrac{x^4}{4} - \dfrac{10x^3}{3} + 14x^2 + c$

ii) $\left[\dfrac{x^4}{4} - \dfrac{10x^3}{3} + 14x^2\right]_0^3 = \left(\dfrac{3^4}{4} - \dfrac{10(3)^3}{3} + 14(3)^2\right) - 0$

$= \dfrac{81}{4} - 90 + 126 = \underline{56\,1/4}$ (as required)

iii) shaded area = area under curve − area of $\triangle$

area of $\triangle = \dfrac{3 \times 21}{3} = \dfrac{63}{3}$

shaded $= 56\,1/4 - \dfrac{63}{3} = \underline{24\,3/4}$

6) $p(x) = x^3 - 4x^2 + 3x$

a) $p(3) = (3)^3 - 4(3)^2 + 3(3)$

$= 27 - 36 + 9$

$p(3) = 0$, $\therefore$ $(x-3)$ is a factor

b)

$$
\begin{array}{r|l}
 & x^2 - x \\
\hline
x - 3 & x^3 - 4x^2 + 3x \\
 & -\underline{(x^3 - 3x^2)} \\
 & \quad -x^2 + 3x \\
 & \quad -\underline{(-x^2 + 3x)} \\
 & \qquad 0
\end{array}
$$

$(x-3)(x^2 - x)$

$x^2 - x = x(x-1)$

$p(x) = \underline{x(x-1)(x-3)}$

ci) $p(2) = (2)^3 - 4(2)^2 + 3(2)$

$= 8 - 16 + 6$

$= -2$ $\quad\therefore$ remainder is -2

ii)

$$
\begin{array}{r|l}
 & x^2 - 2x - 1 \\
\hline
x - 2 & x^3 - 4x^2 + 3x \\
 & -\underline{(x^3 - 2x^2)} \\
 & \quad -2x^2 + 3x \\
 & \quad -\underline{(-2x^2 + 4x)} \\
 & \quad 0 \; -x + 0 \\
 & \qquad -\underline{(-x + 2)} \\
 & \qquad\quad -2
\end{array}
$$

$(x-2)(x^2 - 2x - 1) - 2$

$a = -2$, $b = -1$, $r = -2$

7) $x^2 + y^2 - 4x - 14 = 0$

a i) $x^2 - 4x + y^2 - 14 = 0$

$(x-2)^2 - 4 + y^2 - 14 = 0$

$(x-2)^2 + y^2 = 18$

centre is $\underline{(2,0)}$

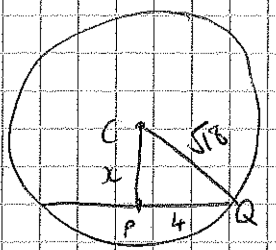
ii) $r^2 = 18$

$r = \sqrt{18}$

$= \sqrt{9}\sqrt{2}$

$\underline{\text{radius} = 3\sqrt{2}}$

b



$PQ = 4$ (perpendicular from centre bisects chord of 8cm)

$x^2 = (\sqrt{18})^2 - 4^2$

$x^2 = 18 - 16$

$\underline{x = \sqrt{2}}$

c) $y = 2k - x$

i) sub $y = 2k - x$ into circle equation

$x^2 + (2k - x)^2 - 4x - 14 = 0$

$x^2 + (2k - x)(2k - x) - 4x - 14 = 0$

$x^2 + 4k^2 - 2kx - 2kx + x^2 - 4x - 14 = 0$

$2x^2 - 4kx - 4x + 4k^2 - 14 = 0 \quad (\div 2)$

$x^2 - 2kx - 2x + 2k^2 - 7 = 0$

$\underline{x^2 - 2(k+1)x + 2k^2 - 7 = 0}$ (as req)

ii) $b^2 - 4ac = 0$ (equal roots) $a = 1$, $b = -2(k+1)$, $c = 2k^2 - 7$

$(-2k-2)^2 - 4(1)(2k^2 - 7) = 0$

$4k^2 + 8k + 4 - 8k^2 + 28 = 0$

$4k^2 - 8k - 32 = 0 \quad (\div 4)$

$k^2 - 2k - 8 = 0$

$(k-4)(k+2) = 0$ $\quad \underline{k = 4}, \underline{k = -2}$

iii) Line is a tangent as only one solution