Core1 - January 2006 - SOLUTIONS

1a) $(\sqrt{5}+2)(\sqrt{5}-2)$

$\sqrt{25} - 2\sqrt{5} + 2\sqrt{5} - 4$

$5 - 4 = \underline{1}$

b) $\sqrt{8} + \sqrt{18}$

$\sqrt{4}\sqrt{2} + \sqrt{9}\sqrt{2} = 2\sqrt{2} + 3\sqrt{2}$

$= \underline{5\sqrt{2}}$

2a) $A(1,1)$ $B(5,k)$

$3x + 4y = 7$

i) when $x = 5$, $y = k$

$3(5) + 4k = 7$

$15 + 4k = 7 \quad (-15)$

$4k = -8 \quad (\div 4)$

$\underline{k = -2}$ (as req)

ii) $(1,1)$ $(5,-2)$

Midpoint $= \left(\frac{1+5}{2}, \frac{1-2}{2}\right)$

$= \underline{\left(3, -\frac{1}{2}\right)}$

b) gradient $= \frac{-2-1}{5-1} = \underline{-\frac{3}{4}}$

c) i) gradient AC $= \underline{\frac{4}{3}}$ $\rightarrow M_1 \times M_2 = -1$ (perpendicular)

ii) $A(1,1)$ $m = 4/3$

$y - 1 = \frac{4}{3}(x-1)$

$3y - 3 = 4x - 4$

$\underline{3y = 4x - 1}$

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2c(iii) $y = 0$ ∴ lies on x axis

$3y = 4x - 1$

$3(0) = 4x - 1$

$4x - 1 = 0$

$4x = 1$

$x = 1/4$

3a(i) $x^2 - 4x + 9$

$(x-2)^2 - 4 + 9$

$(x-2)^2 + 5$

ii) minimum point at $(2, 5)$

b)i) $y = -2x + 12$ intersects with $y = x^2 - 4x + 9$

$x^2 - 4x + 9 = -2x + 12$

$x^2 - 2x - 3 = 0$ (as req)

ii) $x^2 - 2x - 3 = 0$

$(x+1)(x-3) = 0$

$x = -1, \quad x = 3$

$y = -2x + 12$

$x = -1, \quad y = -2(-1) + 12$, $x = 3, \quad y = -2(3) + 12$

$y = 14$ $\qquad y = 6$

$(-1, 14)$ $\qquad (3, 6)$

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4a) $x^2 + (m+4)x + (4m+1) = 0$ if equal roots, $b^2 - 4ac = 0$

$a = 1,\ b = m+4,\ c = 4m+1$

$(m+4)^2 - 4(1)(4m+1) = 0$

$m^2 + 8m + 16 - 4(4m+1) = 0$

$m^2 + 8m + 16 - 16m - 4 = 0$

$\underline{m^2 - 8m + 12 = 0}$ (as req)

b) $(m-2)(m-6) = 0$

$\underline{m = 2}$, $\underline{m = 6}$

5) $x^2 + y^2 - 8x + 6y = 11$

a) $x^2 - 8x + y^2 + 6y = 11$

$(x-4)^2 - 16 + (y+3)^2 - 9 = 11$

$(x-4)^2 + (y+3)^2 - 25 = 11$

$(x-4)^2 + (y+3)^2 = 36$

bi) $C(4, -3)$

ii) $r^2 = 36$

$\underline{r = 6}$

ci) $C(4, 3)$ $O(0, 0)$

$CO = \sqrt{4^2 + 3^2}$

$CO = \sqrt{25} = \underline{\underline{5}}$

ii) CO length is less than radius length

$5 < 6$

so, O lies inside the circle

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6) $p(x) = x^3 + x^2 - 10x + 8$

ai) $p(2) = (2)^3 + (2)^2 - 10(2) + 8$

$= 8 + 4 - 20 + 8$

$= 0$ $\therefore$ $(x-2)$ is a factor of $p(x)$

ii)

$$\begin{array}{r|l} & \quad x^2 + 3x - 4 \\ x-2 & x^3 + x^2 - 10x + 8 \\ - & x^3 - 2x^2 \\ & \quad 3x^2 - 10x \\ - & \quad 3x^2 - 6x \\ & \qquad -4x + 8 \\ - & \qquad -4x + 8 \\ & \qquad\quad 0 \end{array}$$

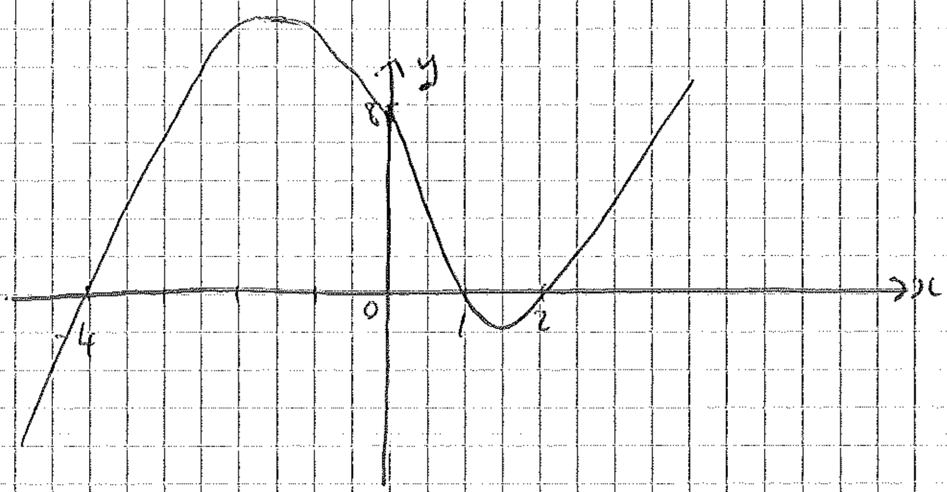
$(x-2)(x^2+3x-4)$

$p(x) = (x-2)(x+4)(x-1)$

b) $(x-2)(x+4)(x-1) = 0$

$x = 2,\ x = -4,\ x = 1$

$(2,0)\ (-4,0)\ (1,0)$ - x axis $(0,8)$ - y axis



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7) $V = \frac{1}{3}t^6 - 2t^4 + 3t^2$

ai) $\frac{dV}{dt} = \frac{6}{3}t^5 - 8t^3 + 6t$

$= 2t^5 - 8t^3 + 6t$

ii) $\frac{d^2V}{dt^2} = 10t^4 - 24t^2 + 6$

b) when $t = 2$,

$\frac{dV}{dt} = 2(2)^5 - 8(2)^3 + 6(2)$

$= 64 - 64 + 12$

$= 12$ rate of change is $\underline{12 m^3 s^{-1}}$

ci) when $t = 1$,

$\frac{dV}{dt} = 2(1)^5 - 8(1)^3 + 6(1)$

$= 2 - 8 + 6 = 0$ $\therefore$ stationary value when $t = 1$

ii) when $t = 1$, $\frac{d^2y}{dx^2} = 10(1)^4 - 24(1)^2 + 6$

$= 10 - 24 + 6 = -8$

$\frac{d^2y}{dx^2} < 0$ $\therefore$ maximum value when $t = 1$

8a) when $x = 2$, $y = 3(2)^2 - (2)^3$
$= 4$ (height of rectangle is 4)

Area of rectangle $= 3 \times 4$
$= \underline{12}$

bi) $\int (3x^2 - x^3)\,dx = \frac{3x^3}{3} - \frac{x^4}{4} + c = \underline{x^3 - \frac{x^4}{4} + c}$

ii) Shaded area = rectangle area − area under curve

area under curve $= \left[x^3 - \frac{x^4}{4}\right]_{-1}^{2}$

$= \left(2^3 - \frac{2^4}{4}\right) - \left((-1)^3 - \frac{(-1)^4}{4}\right) = (8 - 4) - \left(-1 - \frac{1}{4}\right)$

$= 4 + 1\frac{1}{4} = \underline{5\frac{1}{4}}$

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8 b iii) shaded $= 12 - 5\frac{1}{4}$

$= \underline{6\frac{3}{4}}$

c i) $y = 3x^2 - x^3$

$\frac{dy}{dx} = 6x - 3x^2$

ii) when $x = 1$,

$\frac{dy}{dx} = 6(1) - 3(1)^2$, $y = 3(1)^2 - (1)^3$

$= \underline{3}$ $\quad$ $y \underline{= 2}$

$y - 2 = 3(x - 1)$

$y - 2 = 3x - 3$

$y = \underline{3x - 1}$

iii) $\frac{dy}{dx} < 0$ if y is decreasing

$6x - 3x^2 < 0 \quad (\div 3)$

$2x - x^2 < 0$

$\underline{x^2 - 2x > 0}$ (as req)

d) $x^2 - 2x = 0$

$x(x - 2) = 0$

$x = 0, \quad x = 2$

$\underline{x < 0, \quad x > 2}$