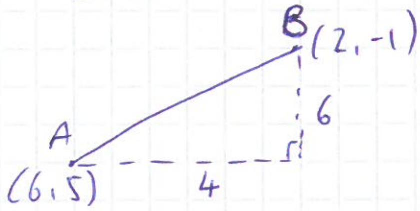
Core 1 June 2005

① A (6, 5) B(2, -1)

a) midpoint AB $\left(\frac{6+2}{2}, \frac{5+-1}{2}\right) = \underline{\underline{(4, 2)}}$



b) length AB $= \sqrt{4^2+6^2}$

$= \sqrt{16+36}$

$= \sqrt{52} = \sqrt{4}\sqrt{13}$

$= \underline{\underline{2\sqrt{13}}}$

c) i) gradient AB $= \frac{-1-5}{2-6} = \frac{-6}{-4} = \underline{\frac{3}{2}}$

ii) $m = \frac{3}{2}$, A (6, 5)

$y - 5 = \frac{3}{2}(x-6)$

$2y - 10 = 3x - 18$

$\underline{3x - 2y = 8}$ (as required)

d) $2x + y = 10$ $(\times 2)$

$3x - 2y = 8$

$+ \quad 4x + 2y = 20$

$7x = 28$

$\underline{x = 4}$ $\rightarrow$ $2(4) + y = 10$

$8 + y = 10$

$\underline{y = 2}$

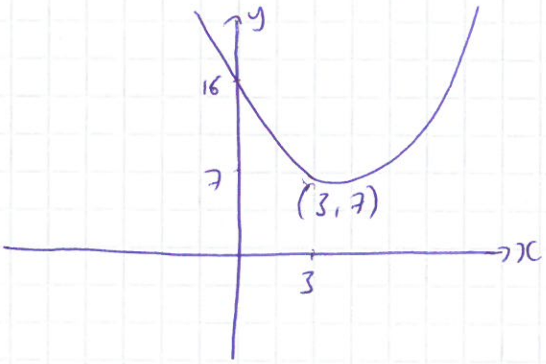
C $\underline{\underline{(4, 2)}}$

Core 1 June 2005

(2) a) $x^2 - 6x + 16 = (x-3)^2 - 9 + 16$
$= (x-3)^2 + 7$

bi) $y = x^2 - 6x + 16$
vertex at $(3, 7)$

ii)



iii) $x = 3$ is line of symmetry

c) from $y = x^2 \rightarrow y = x^2 - 6x + 16$
$y = (x-3)^2 + 7$

Translation $\begin{pmatrix} 3 \\ 7 \end{pmatrix}$

(3) $C(2,-1)$ radius 5 $P(6,2)$

a) $(x-2)^2 + (y+1)^2 = 5^2$

b) sub $(6,2)$ into $(x-2)^2 + (y+1)^2 = 5^2$
$(6-2)^2 + (2+1)^2 = 5^2$
$16 + 9 = 25$ ✓ $\therefore$ P lies on the circle

c) $C(2,-1)$ $P(6,2)$
gradient $CP = \dfrac{2--1}{6-2} = \dfrac{3}{4}$

di) perpendicular gradient is $-\dfrac{4}{3}$

ii) $m = -4/3$, $P(6,2) \rightarrow y - 2 = -\dfrac{4}{3}(x-6)$
$3y - 6 = -4x + 24 \rightarrow 4x + 3y = 30$

Core 1 June 2005

(4) $y = x^3 - 5x^2 + 7x - 3$ $\quad A(1, 0)$

ai) $p(x) = x^3 - 5x^2 + 7x - 3$

$p(3) = 3^3 - 5(3)^2 + 7(3) - 3$

$= 27 - 45 + 21 - 3 = 0$ $\quad \therefore (x-3)$ is a factor of $p(x)$

ii) $B\ \underline{(3, 0)}$

b) $y = x^3 - 5x^2 + 7x - 3$

i) $\frac{dy}{dx} = 3x^2 - 10x + 7$

ii) minimum point at M so $\frac{dy}{dx} = 0$

$3x^2 - 10x + 7 = 0$

$(3x - 7)(x - 1) = 0$

$\underline{x = \frac{7}{3}}$ (M) $\quad x = 1$ (A)

c) $\frac{d^2y}{dx^2} = 6x - 10$

when $x = 1$, $\frac{d^2y}{dx^2} = 6(1) - 10$

$= \underline{-4}$

di) $\int (x^3 - 5x^2 + 7x - 3)\,dx = \underline{\frac{x^4}{4} - \frac{5x^3}{3} + \frac{7x^2}{2} - 3x + C}$

ii) $\left[\frac{x^4}{4} - \frac{5x^3}{3} + \frac{7x^2}{2} - 3x\right]_0^1$

$= \left(\frac{1^4}{4} - \frac{5(1)^3}{3} + \frac{7(1)^2}{2} - 3(1)\right) - 0$

$= \left(\frac{1}{4} - \frac{5}{3} + \frac{7}{2} - 3\right) = \frac{3}{12} - \frac{20}{12} + \frac{42}{12} - \frac{36}{12}$

$= \frac{-11}{12}$

Area of shaded region is $\underline{\frac{11}{12}}$

Core 1 June 2005

(5) a) $(\sqrt{3}+1)^2 = (\sqrt{3}+1)(\sqrt{3}+1)$

$= \sqrt{9} + \sqrt{3} + \sqrt{3} + 1$

$= 3 + 2\sqrt{3} + 1$

$= \underline{4 + 2\sqrt{3}}$

b) $\dfrac{(\sqrt{3}+1)}{(\sqrt{3}-1)} \times \dfrac{(\sqrt{3}+1)}{(\sqrt{3}+1)} = \dfrac{4+2\sqrt{3}}{\sqrt{9}-\sqrt{3}+\sqrt{3}-1}$

$= \dfrac{4+2\sqrt{3}}{3-1}$

$= \dfrac{4+2\sqrt{3}}{2}$

$= \underline{\underline{2+\sqrt{3}}}$

(6) $p(x) = (x-2)(x^2+x+3)$

a) $(x-2)(x^2+x+3)$

$x^3 + x^2 + 3x - 2x^2 - 2x - 6 = \underline{x^3 - x^2 + x - 6}$

b)

$p(-1) = (-1)^3 - (-1)^2 + (-1) - 6$

$= -1 - 1 - 1 - 6$

$= \underline{\underline{-9}}$ Remainder when $p(x)$ divided by $(x+1)$ is $\underline{-9}$

c) $(x-2)(x^2+x+3) = 0$

$x - 2 = 0$ $\qquad$ $x^2 + x + 3 = 0$

$\underline{x = 2}$ $\qquad$ $a = 1,\ b = 1,\ c = 3$

$b^2 - 4ac < 0$ if no real roots

$1^2 - 4(1)(3) = -11$

$\therefore b^2 - 4ac < 0$

$\underline{-11 < 0}$

Core 1 June 2005

7 a) $3(x-1) > 3 - 5(x+6)$

$3x - 3 > 3 - 5x - 30$

$3x - 3 > -27 - 5x \quad (+5x)$

$8x - 3 > -27 \quad (+3)$

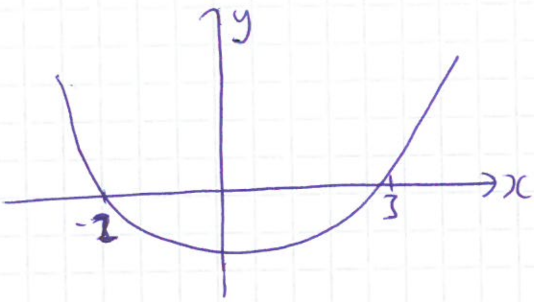
$8x > -24 \quad (\div 8)$

$x > -24/8 \rightarrow x > -3$

b) $x^2 - x - 6 < 0$

$(x-3)(x+2)$

$x = 3, \quad x = -2$ critical values



$x^2 - x - 6 < 0$

$\therefore$ graph below x axis

$-2 < x < 3$

Core 1 June 2005

(8) $y = mx - 1$ $\qquad y = x^2 - 5x + 3$

a) $mx - 1 = x^2 - 5x + 3$

$x^2 - 5x - mx + 4 = 0$

$x^2 - (5+m)x + 4 = 0$ (as required)

b) $b^2 - 4ac = 0$ if equal roots

$a = 1, \quad b = -(5+m), \quad c = 4$

$= -5 - m$

$(-5-m)^2 - 4(1)(4) = 0$

$(-5-m)(-5-m) - 16 = 0$

$25 + 5m + 5m + m^2 - 16 = 0$

$m^2 + 10m + 9 = 0$

$(m+1)(m+9) = 0$

$\underline{\underline{m = -1}}, \quad \underline{\underline{m = -9}}$

c) only one solution at each value of 'm'

$\therefore$ the line is a tangent to the curve