Core 1 Jan 2005

① A $(11, 2)$ $\quad$ B$(-1, -1)$

ai) gradient AB $= \dfrac{-1-2}{-1-11} = \dfrac{-3}{-12} = \dfrac{1}{4}$

ii) $m = \dfrac{1}{4}$ $\quad$ A $(11, 2)$

$$y - 2 = \frac{1}{4}(x - 11)$$

$$4y - 8 = x - 11$$

$x - 4y = 3$ (as required)

b) $3x + 5y = 26$ $(\times 4)$

$x - 4y = 3$ $(\times 5)$

$$\begin{array}{r} 12x + 20y = 104 \\ + \quad 5x - 20y = 15 \\ \hline 17x = 119 \end{array} \quad (\div 17)$$

$x = 7$, $\quad 3(7) + 5y = 26$

$21 + 5y = 26$ (-21)

$5y = 5$

$y = 1$

C $(7, 1)$

Core 1 Jan 2005

② $y = x^5 - 6x^3 - 3x + 25$

a) $\dfrac{dy}{dx} = \underline{5x^4 - 18x^2 - 3}\cancel{x}$

b) $P(2, 3)$

i) when $x = 2$, $\dfrac{dy}{dx} = 5(2)^4 - 18(2)^2 - 3$

$= 80 - 72 - 3$

$= \underline{5}$ (gradient at P)

ii) grad normal is $-\frac{1}{5}$ (m) $(2, 3)$

$y - 3 = -\frac{1}{5}(x - 2)$

$5y - 15 = -x + 2$

$\underline{x + 5y = 17}$

c) when $x = 1$, $\dfrac{dy}{dx} = 5(1)^4 - 18(1)^2 - 3$

$= 5 - 18 - 3$

$= -16$

$\therefore$ y is decreasing as $\dfrac{dy}{dx} < 0$

③ $x^2 + y^2 - 12x - 6y + 20 = 0$

a) $x^2 - 12x + y^2 - 6y + 20 = 0$

$(x-6)^2 - 36 + (y-3)^2 - 9 + 20 = 0$

$\underline{(x-6)^2 + (y-3)^2 = 25}$ OR $\underline{(x-6)^2 + (y-3)^2 = 5^2}$

bi) centre $\underline{(6, 3)}$

ii) $r^2 = 5^2$ radius $\underline{5}$

ci) $y = x + 4$ → sub into circle

$x^2 + (x+4)^2 - 12x - 6(x+4) + 20 = 0$

$x^2 + (x+4)(x+4) - 12x - 6x - 24 + 2$

$x^2 + x^2 + 4x + 4x + 16 - 12x - 6x - 24 + 20 =$

$2x^2 + 8x - 12x - 6x + 12 = 0$

$2x^2 - 10x + 12 = 0$ $(\div 2)$

$\underline{x^2 - 5x + 6 = 0}$ (as req)

ii) $x^2 - 5x + 6 = 0$

$(x-3)(x-2) = 0$

$x = 3$ $x = 2$

$y = 3 + 4$ $y = 2 + 4$

$= 7$ $= 6$

$\underline{(3, 7)}$ and $\underline{(2, 6)}$

Core 1 Jan 2005

④ $f(x) = x^3 - 3x^2 - 6x + 8$

a i) $f(-1) = (-1)^3 - 3(-1)^2 - 6(-1) + 8$

$= -1 - 3 + 6 + 8 = \underline{\underline{10}}$ remainder is 10 when divided by $(x+1)$

ii) $\underline{\underline{(x-1)(x+2)}}$ ~~kx~~ are factors of $f(x)$

iii) $x^3 - 3x^2 - 6x + 8 = \underline{(x-1)(x+2)}(x-4)$ (by inspection)

b) i) $y = x^3 - 3x^2 - 6x + 8$

y intersect at A, $\underline{\underline{y = 8}}$

ii) x intersects, when $y = 0$,

$(x-1)(x+2)(x-4) = 0$

$x = 1, \quad x = -2, \quad \underline{\underline{x = 4}}$ (B)

c i) $\int (x^3 - 3x^2 - 6x + 8)\,dx = \frac{x^4}{4} - \frac{3x^3}{3} - \frac{6x^2}{2} + 8x + C$

$= \underline{\frac{x^4}{4} - x^3 - 3x^2 + 8x + C}$

ii) area under curve (shaded) :-

$\left[\frac{x^4}{4} - x^3 - 3x^2 + 8x\right]_{-2}^{1}$

$\left(\frac{1}{4} - 1 - 3 + 8\right) - \left(\frac{(-2)^4}{4} - (-2)^3 - 3(-2)^2 + 8(-2)\right)$

$4\frac{1}{4} - (4 + 8 - 12 - 16)$

$4\frac{1}{4} - (-16) = \underline{\underline{20\frac{1}{4}}}$

Core 1 Jan 2005

5 a) $(\sqrt{12}+2)(\sqrt{12}-2)$

$\sqrt{144} - 2\sqrt{12} + 2\sqrt{12} - 4$

$12 - 4 = \underline{8}$

b) $\sqrt{12} = \sqrt{4}\sqrt{3} = \underline{\underline{2\sqrt{3}}}$

c) $\dfrac{(\sqrt{12}+2)}{(\sqrt{12}-2)} \times \dfrac{(\sqrt{12}+2)}{(\sqrt{12}+2)} = \dfrac{\sqrt{144} + 2\sqrt{12} + 2\sqrt{12} + 4}{8}$

$= \dfrac{12 + 4\sqrt{12} + 4}{8}$

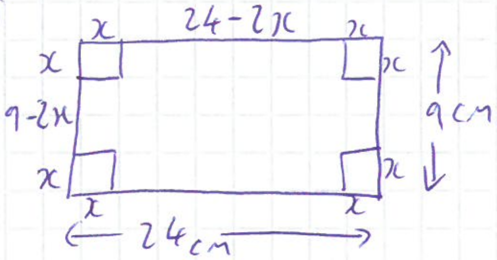
$= \dfrac{16 + 4\sqrt{4}\sqrt{3}}{8}$

$= \dfrac{16 + 8\sqrt{3}}{8}$

$= \underline{\underline{2 + \sqrt{3}}}$

Core 1 Jan 2005

6



a) $Vol = (24-2x)(9-2x)x$

$= (216 - 48x - 18x + 4x^2)x$

$= 216x - 66x^2 + 4x^3$

$= \underline{4x^3 - 66x^2 + 216x}$ (as req)

bi) $\frac{dy}{dx} = \underline{12x^2 - 132x + 216}$

ii) $\frac{dy}{dx} = 0$ at stationary values of V

$12x^2 - 132x + 216 = 0$ $(\div 12)$

$\underline{x^2 - 11x + 18 = 0}$ (as req)

iii) $x^2 - 11x + 18 = 0$

$(x - 9)(x - 2) = 0$

$\underline{x = 9}$ or $\underline{x = 2}$

iv)

width is $9 - 2x$ if $x = 9$, $9 - 2(9) = -9$

$9 - 2x < 0$ which is not possible for a width

$\therefore$ reject $x = 9$

Only value is $\underline{x = 2}$

ci) $\frac{dy}{dx} = 12x^2 - 132x + 216$

$\frac{d^2y}{dx^2} = \underline{24x - 132}$

ii) when $x = 2$, $\frac{d^2y}{dx^2} = 24(2) - 132$

$= -84$

$\frac{d^2y}{dx^2} < 0$ $\therefore$ $\underline{\text{maximum value}}$ at $x = 2$

Core 1 Jan 2005

7 a) $(k+5)^2 - 12k(k+2)$

$(k+5)(k+5) - 12k^2 - 24k$

$k^2 + 5k + 5k + 25 - 12k^2 - 24k$

$-11k^2 - 14k \cancel{+24k} + 25$

$\cancel{-11k^2 - 14k + 25}$

$\underline{-11k^2 - 14k + 25}$

b) i) $3(k+2)x^2 + (k+5)x + k = 0$

$b^2 - 4ac \geqslant 0$ if real roots

$a = 3(k+2)$, $b = k+5$, $c = k$

$(k+5)^2 - 4(3k+6)(k) \geqslant 0$

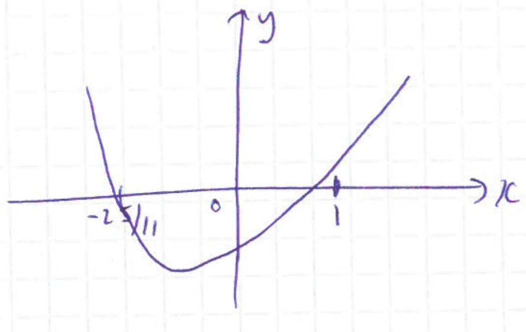
$(k+5)^2 - 12k(k+2) \geqslant 0$

$-11k^2 - 14k + 25 \geqslant 0$ (from part a))

$11k^2 + 14k - 25 \leqslant 0$

$\underline{(11k+25)(k-1) \leqslant 0}$ (as req)

ii) $k = -\frac{25}{11}$ $k = 1$



$y = (11k+25)(k-1)$

$y \leqslant 0$

graph is below x axis

$\underline{-\frac{25}{11} \leqslant k \leqslant 1}$