GREAT YARMOUTH CHARTER ACADEMY

Y11

Changing the Subject and Rearranging Formulae

One step rearranging

Worked Examples

We will learn how to rearrange a formula so it starts with a particular unknown, this is known as changing the subject of a formula. Here are some examples where a is the subject of the formula

$$a = b + x$$

$$a = b - x$$

$$b - x = a$$

$$a = bx$$

$$\frac{b}{x} = a$$

$$a = \sqrt{b^2 - x^2}$$

Here a is the subject of the formula because it is isolated on one side of the equation. It doesn't matter what side of the equation the subject is, just that it is isolated. Here are some examples where a is NOT the subject of the formula:

$$a + x = b$$

'a' is not the subject of the equation because it is not isolated, it has x on the same side of the equation.

$$\frac{a}{x} = b$$

'a' is not the subject of the equation because it is not isolated, it is being divided by 'x' on the same side of the equation.

$$a^2 = b + x$$

'a' is not the subject of the equation, 'a^2' is the subject of the equation.

We are going to learn how to change the subject of the equation where we only have to apply one step, only. Before we look at this we need to understand the inverse of certain operations, here they are below:

$$+ \leftrightarrow -$$

$$\times \leftrightarrow \div$$

$$x^2 \leftrightarrow \sqrt[2]{x}$$

$$x^3 \leftrightarrow \sqrt[3]{x}$$

$$x^y \leftrightarrow \sqrt[y]{x}$$

Now we know the inverse of each operation let’s look at a few examples where we want to change the subject of an equation. We want to make ‘y’ the subject of the equation so I want my equation to look like this ‘y = …’. We can write an equation where y is the subject like this too ‘ … = y’ but conventionally we write an equation where the subject of the equation is on the left hand side of the equation. When we have ‘y’ as the subject of an equation we can also say that we have written an equation ‘in terms of y’.

Here are the steps:

1. Circle the subject that we want to isolate
2. Identify what needs to be eliminated
3. Inverse the operation, and apply to both sides

Example 1

Make ‘y’ the subject:

$$a + \boldsymbol{y} = x$$

2. ‘a’ needs to be eliminated, ‘a’ has been added to ‘y’.

3. Subtract ‘a’ on both sides of the equation

$$a + \boldsymbol{y} = x$$

$$-a \qquad\quad -a$$

$$\boldsymbol{y} = x - a$$

‘y’ is the subject of the equation because it is isolated to one side of the equation.

Example 2

Make ‘y’ the subject:

$$x = \boldsymbol{y} - a$$

2. ‘a’ needs to be eliminated, ‘a’ has been subtracted from ‘y’.

3. Add ‘a’ on both sides of the equation

$$x = \boldsymbol{y} - a$$

$$+a \qquad\quad +a$$

$$x + a = y$$

$$\boldsymbol{y} = x + a$$

I have written the equation in terms of ‘y’ where it is present on the left hand side of the equation, by convention.

Example 3

Make ‘y’ the subject:

$$x = \frac{\boldsymbol{y}}{a}$$

2. ‘a’ needs to be eliminated, ‘a’ is dividing ‘y’.

3. Multiply ‘a’ on both sides of the equation

$$x = \frac{\boldsymbol{y}}{a}$$

$$\times a \quad \times a$$

$$ax = y$$

$$\boldsymbol{y} = ax$$

I have written the equation in terms of ‘y’ where it is present on the left hand side of the equation, by convention.

Example 4

Make ‘y’ the subject:

$$x = \boldsymbol{y}a$$

2. ‘a’ needs to be eliminated, ‘a’ has been multiplied with‘y’.

3. Divide ‘a’ on both sides of the equation

$$x = \boldsymbol{y}a$$

$$\div a \quad \div a$$

$$\frac{x}{a} = y$$

$$\boldsymbol{y} = \frac{x}{a}$$

I have written the equation in terms of ‘y’ where it is present on the left hand side of the equation, by convention.

Example 5

Make ‘y’ the subject:

$$x - a = 3\boldsymbol{y}$$

2. ‘3’ needs to be eliminated, ‘3’ has been multiplied with‘y’.

3. Divide ‘3’ on both sides of the equation

$$x - a = 3\boldsymbol{y}$$

$$\div 3 \quad \div 3$$

$$\frac{x-a}{3} = y$$

$$\boldsymbol{y} = \frac{x-a}{3}$$

I have written the equation in terms of ‘y’ where it is present on the left hand side of the equation, by convention.

Example 6

Make ‘y’ the subject:

$$y + m = 3 + \boldsymbol{x^2}$$

2. ‘m’ needs to be eliminated, ‘m’ has been added to ‘y’.

3. Subtract ‘m’ on both sides of the equation

$$y + m = 3 + \boldsymbol{x^2}$$

$$-m \quad -m$$

$$y = 3 + \boldsymbol{x^2} - \boldsymbol{m}$$

Mini-quiz

Make ‘b’ the subject of the equation					
(a)	(b)	(c)	(d)	(e)	(f)
$a + \boldsymbol{b} = x$	$x = \boldsymbol{b} - a$	$x = \frac{\boldsymbol{b}}{a}$	$x = a\boldsymbol{b}$	$x - a = 7\boldsymbol{b}$	$b - m = \boldsymbol{y^3} - \boldsymbol{6}$

Example 7

Make ‘y’ the subject:

$$x = \frac{2}{3}\boldsymbol{y}$$

1. ‘$\frac{2}{3}$’ needs to be eliminated, ‘$\frac{2}{3}$’ has been multiplied with‘y’.

2. Multiply both sides by the reciprocal of $\frac{2}{3}$, which is $\frac{3}{2}$ to make ‘y’ the subject

$$x = \frac{2}{3}\boldsymbol{y}$$

$$\times\frac{3}{2} \quad \times\frac{3}{2}$$

$$x \times \frac{3}{2} = y$$

$$\frac{3}{2}x = y$$

$$y = \frac{3}{2}x$$

I have written the equation in terms of ‘y’ where it is present on the left hand side of the equation, by convention.

Practice Exercises

Question 1: Make ‘x’ the subject:

(a)	(b)	(c)	(d)
d + x = m	x – d = m	d + a + x = m	a^2 + x = m
(e)	(f)	(g)	(h)
a^2 + m = x	a^2 + m = x – y	a^2 + m = -y + x	$3a^2$ + x + m = -y
(i)	(j)	(k)	(l)
$a = \frac{x}{m}$	$m = \frac{x}{a^2}$	$ab = \frac{x}{a}$	$abc = \frac{x}{4}$
(m)	(n)	(o)	(p)
mx = q	8x = p	8x = P – 4	8 + x = P – 4

Question 2: Make ‘y’ the subject

(a)	(b)	(c)	(d)
a^2 + m = x – y	$m = \frac{y}{a^2}$	8x = py	$abc = \frac{y}{4}$
(e)	(f)	(g)	(h)
a^2 + y = x	my = P – 4	$a = \frac{y}{m}$	$3a^2$ + x + m = y
(i)	(j)	(k)	(l)
my = q	a^2 + m = -y + x	$ab = \frac{x}{a}$	9 + y = P – 4

Question 3: Rearrange the formula to find the circumference of a circle, C = $2\pi r$ to express r in terms of C.

Question 4: Rearrange the formula, v = u + at to express u in terms of v, a and t

Question 5: Make ‘b’ the subject

(a)	(b)	(c)	(d)	(e)	(f)
$a = \frac{4}{3}b$	$a = \frac{5}{6}b$	$\frac{9}{10}b = a$	$\frac{9}{10}a = b$	$\frac{9}{10}b = m$	$\frac{1}{10}b = a$
(g)	(h)	(i)	(j)	(k)	(l)
$\frac{1b}{9} = a$	$m = \frac{1b}{9}$	$m = \frac{6b}{9}$	$m = \frac{11b}{9}$	$m = \frac{b}{9}$	$m = \frac{b}{10}$

Question 6: The volume of a cylinder is given by V = $\pi r^2 h$. Express h in terms of V, π, and r.

Question 7: The volume of a cone is given by V = $\frac{1}{3}\pi r^2 h$. Rearrange the volume formula to make h the subject.

Question 8. The surface area of a cone is given by A = V = $\pi r l$. Rearrange the area formula to make r the subject.

Question 9. The surface area of a cone is given by A = V = $\pi r l$. Rearrange the area formula to make l the subject.

One step rearranging – Powers and Roots

Recap

Make ‘a’ the subject of the equation					
(a)	(b)	(c)	(d)	(e)	(f)
$a + c = x$	$x = c - a$	$x = \frac{c}{a}$	$x = ac$	$x - a = 7c$	$c - m = m^{\mathbf{3}} - 11$

We have learnt how to change the subject of a formula using the four operations. We had to perform one step. We will now learn how to change the subject of a formula but where we have to apply powers or roots to an equation to write an equation in terms of a certain subject. This will also perform one step only here.

Let’s recap the inverse relationship between powers and roots:

$$x^2 \leftrightarrow \sqrt[2]{x}$$

$$x^3 \leftrightarrow \sqrt[3]{x}$$

$$x^y \leftrightarrow \sqrt[y]{x}$$

Let’s look at a few worked examples:

<u>Example 1:</u>

Make ‘b’ the subject of the equation

$$b^2 = m$$

2. ‘b’ has a power of 2. We need to elimate ‘b^2’.

3. Square root both sides of the equation

$$b^2 = m$$

$$\sqrt[2]{} \quad \sqrt[2]{}$$

$$b = \pm\sqrt{m}$$

When we root an unknown where it is an even root then our unknown must have a $\pm$ to the left of the root. This is not required when we apply an odd root to an unknown.

<u>Example 2:</u>

Make ‘b’ the subject of the equation

$$b^2 = m - a$$

2. ‘b’ has a power of 2. We need to elimate ‘b^2’.

3. Square root both sides of the equation

$$b^2 = m - a$$

$$\sqrt[2]{} \quad \sqrt[2]{}$$

$$b = \pm\sqrt{m - a}$$

When we root an unknown where it is an even root then our unknown must have a $\pm$ to the left of the root. This is not required when we apply an odd root to an unknown.

Example 3:

Make 'b' the subject of the equation

$$b^3 = m$$

2. 'b' has a power of 3. We need to elimate 'b^3'.

3. Cube root both sides of the equation

$$b^3 = m$$

$$\sqrt[3]{} \quad \sqrt[3]{}$$

$$b = \sqrt[3]{m}$$

When we apply an odd root to a positive unknown, then our result is still positive.

When we apply an odd root to a negative unknown, then our result is still negative.

Example 4:

Make 'b' the subject of the equation

$$b^3 = -a$$

2. 'b' has a power of 3. We need to elimate 'b^3'.

3. Cube root both sides of the equation

$$b^3 = -a$$

$$\sqrt[3]{} \quad \sqrt[3]{}$$

$$b = \sqrt[3]{-a}$$

When we apply an odd root to a negative unknown, the result will remain negative. We do not place $\pm$ to the left of the root.

Example 5:

Make ‘b’ the subject of the equation

$$\sqrt{b} = m$$

2. ‘b’ is square rooted. We need to elimate ‘$\sqrt{b}$’.

3. Square both sides of the equation

$$\sqrt{b} = m$$

$$x^2 \quad x^2$$

Example 6:

Make ‘b’ the subject of the equation

$$\sqrt[3]{b} = m$$

2. ‘b’ is cube rooted. We need to elimate ‘$\sqrt[3]{b}$’.

3. Cube both sides of the equation

$$\sqrt[3]{b} = m$$

$$x^3 \quad x^3$$

$$b = m^3$$

Example 7:

Make ‘b’ the subject of the equation

$$\sqrt[5]{b} = -m$$

2. ‘b’ is cube rooted. We need to elimate $\sqrt[5]{b}$’.

3. Apply the fifth root to both sides of the equation

$$\sqrt[5]{b} = -m$$

$$x^5 \quad x^5$$

$$b = (-m)^5$$

When you apply an even power to a negative unknown, the result will always be positive

When you apply an odd power to a negative unknown, the result will always be negative.

When you apply an odd power to a positive unknown, the result will always be positive.

Mini Quiz

Make 'a' the subject of the equation						
(a)	(b)	(c)	(d)	(e)	(f)	
$a^2 = b$	$a^2 = b - m$	$m = a^3$	$-m = a^3$	$\sqrt{a} = m$	$\sqrt[3]{a} = m$	$\sqrt[7]{a} = -m$

Practice Exercises

Question 1: Make 'd' the subject:

(a)	(b)	(c)	(d)
$d^2 = b$	$d^2 = m - b$	$d^2 = M + b$	$d^2 = \frac{M}{B}$
(e)	(f)	(g)	(h)
$d^2 = \frac{M}{B-3}$	$d^3 = \frac{M}{B}$	$d^3 = M$	$d^3 = -M$
(i)	(j)	(k)	(l)
$d^3 = M + b$	$d^5 = M + b$	$M + A = d^3$	$Ma = d^3$
(m)	(n)	(o)	(p)
$-M = d^5$	$d^7 = M + b$	$d^2 = m^2 + b^2$	$d^2 = m^2 + b^2 - a$

Question 2: Make 'g' the subject

(a)	(b)	(c)	(d)
$\sqrt{g} = m$	$\sqrt{g} = m + 2$	$\sqrt{g} = m - 2$	$\sqrt{g} = a + b$
(e)	(f)	(g)	(h)
$\sqrt[3]{g} = m$	$\sqrt[3]{g} = m + 2$	$\sqrt[3]{g} = m - 2$	$\sqrt[3]{g} = m^4$
(i)	(j)	(k)	(l)
$\sqrt[3]{g} = -m$	$\sqrt{g} = -m$	$\sqrt{g} = 5m$	$\sqrt[3]{g} = -5m$

Question 3: Make 'h' the subject

(a)	(b)	(c)	(d)
$h^2 = b$	$\sqrt{h} = m$	$\sqrt{h} = 12m$	$-m = h^3$
(e)	(f)	(g)	(h)
$h^2 = \frac{a}{B-3}$	$h^2 = b - m$	$M + A = h^3$	$h^3 = -M$
(i)	(j)	(k)	(l)
$\sqrt[3]{h} = m$	$h^5 = M + b$	$\sqrt[7]{h} = -m$	$Ma = d^3$
(m)	(n)	(o)	(p)
$-M = h^5$	$\sqrt{h} = a - 1$	$\sqrt{h} = 2a - 1$	$h^2 = a^2 + b^2 - c$

Question 4: Rearrange the formula, $v^2 = u^2 + 2as$ to express v in terms of u, a and s.

Question 5: Rearrange the formula $a^2 + b^2 = c^2$ to express c in terms of a and b.

Question 6: Rearrange the formula $a^2 = b^2 + c^2 – 2bcCosA$ to express a in terms of b, c, and CosA

Question 7: The formula for the area of a circle in A = πr^2. Express r in terms of A and π.

Question 8: The volume of a cylinder is given by V = $\pi r^2 h$. Express r in terms of V, π, and r.

Recap

Example 7

Make ‘y’ the subject:

$$x = \frac{2}{3}\boldsymbol{y^2}$$

1. ‘$\frac{2}{3}$’ needs to be eliminated, ‘$\frac{2}{3}$’ has been multiplied with‘y’.

2. Multiply both sides by the reciprocal of $\frac{2}{3}$, which is $\frac{3}{2}$ to make ‘y’ the subject

$$x = \frac{2}{3}\boldsymbol{y^2}$$

$$\times\frac{3}{2} \quad \times\frac{3}{2}$$

$$x \times \frac{3}{2} = \boldsymbol{y^2}$$

3. ‘y’ is square rooted. We need to eliminate’$\sqrt{y}$’.

3. Apply the square root to both sides of the equation

$$\frac{3}{2}x = \boldsymbol{y^2}$$

$$\sqrt[2]{\quad} \quad \sqrt[2]{\quad}$$

$$y = \pm\sqrt{\frac{3}{2}x}$$

Practice Exercises

Question 1: Make 'b' the subject

(a)	(b)	(c)	(d)	(e)	(f)
$a = \frac{4}{3}b^2$	$a = \frac{5}{6}b^2$	$\frac{9}{10}b^2 = a$	$\frac{9}{10}a^2 = b$	$\frac{9}{10}b^2 = m$	$\frac{1}{10}b^2 = a$
(g)	(h)	(i)	(j)	(k)	(l)
$\frac{1b^2}{9} = a$	$m = \frac{1b^2}{9}$	$m = \frac{6b^2}{9}$	$m = \frac{11b^2}{9}$	$m = \frac{b^2}{9}$	$m = \frac{b^2}{10}$

Question 2. The surface area, A, and volume, V, of a sphere are given b y the formulae A = $4\pi r^2$ and V = $\frac{4}{3}\pi r^3$. Make r the subject of each formula.

Question 3: The volume of a cone is given by V = $\frac{1}{3}\pi r^2 h$. Rearrange the volume formula to make r the subject.

One step rearranging - Groups

Recap

Make ‘c’ the subject					
(a)	(b)	(c)	(d)	(e)	(f)
$c^2 = a^2 + b^2$	$c^3 = -b$	$m = c + a^3$	$\frac{c}{b} = a^3$	$\sqrt{c} = m - 1$	$c^2 = \frac{a}{B-3}$

Worked Examples

We have learnt how to change the subject where we apply one step only. We simply perform the inverse operation to both sides to ensure that the subject is isolated on one side of the equation. We will now learn how to change the subject of an equation, again with only one step, but where we have to apply the one step to a group rather than to one term.

Let’s look at a few examples:

<u>Example 1:</u>

Make ‘c’ the subject

$$\frac{c}{(m+a)} = z$$

2. ‘c’ is divided by a group ‘m+a’. We need to eliminate this group.

3. Multiply both sides by this group

$$\frac{\boldsymbol{c}}{(m+a)} = z$$

$$\times (m+a) \qquad \times (m+a)$$

$$\boldsymbol{c} = z \times (m+a)$$

We can multiply the group by ‘z’. This is known as ‘expand the brackets’.

$$\boldsymbol{c} = z(m+a) = mz + az$$

Either option are correct.

<u>Example 2:</u>

Make ‘c’ the subject

$$c(m+a) = z$$

2. We are multiplying ‘c’ with the group ‘(m+a)’. We need to eliminate the group.

3. Divide both sides by this group

$$c(m + a) = z$$

$$\div (m + a) \qquad \div (m + a)$$

$$c = \frac{z}{m + a}$$

Example 3:

Make ‘c’ the subject

$$cm + ca = z$$

Here we have an additional step where we want to make ‘c’ the subject but it presents itself twice. When this happens we can factorise ‘c’ from the expression so we have ‘c’ and a group.

$$c(m + a) = z$$

‘c’ is now present once, and it is easy to isolate ‘c’ by dividing by the group.

2. We are multiplying ‘c’ with the group ‘(m+a)’. We need to eliminate the group.

3. Divide both sides by this group

$$c(m + a) = z$$

$$\div (m + a) \qquad \div (m + a)$$

$$c = \frac{z}{m + a}$$

Mini-quiz

Make ‘a’ the subject				
(a)	(b)	(c)	(d)	(e)
$\frac{a}{b-3} = 5$	$\frac{a}{b^3} = 5$	$a(b-3) = 5$	$a(2+m-3) = 5$	$ab + ac = 9$

Practice Exercises

Question 1: Make ‘p’ the subject:

(a)	(b)	(c)	(d)
$\frac{p}{l+w} = 2$	$\frac{p}{b+3} = 2$	$\frac{p}{b+3} = -2$	$\frac{p}{b-3} = 2$
(e)	(f)	(g)	(h)
$\frac{p}{m^2} = 2$	$\frac{p}{a+b+c} = 2$	$2 = \frac{p}{5+b-3}$	$2 = \frac{p-3}{5+b}$
(i)	(j)	(k)	(l)

$\frac{p^2}{l+w} = 2$	$\frac{\sqrt{p}}{l+w} = 2$	$\frac{p^3}{b+3} = 2$	$\frac{p^3}{b+3} = -2$

Question 2: Make 'a' the subject

(a)	(b)	(c)	(d)
$a(3+m) = 5$	$5 = a(3+m)$	$5 = a(m+2)$	$5 = a(6+m+2)$
(e)	(f)	(g)	(h)
$5 = a(6+m-4)$	$5 = a(6m+2-4m)$	$5 = a(2m+2-4m)$	$a(m^2+3+m) = 5$
(i)	(j)	(k)	(l)
$a^2(3+m) = 5$	$a^3(3+m) = 5$	$\sqrt{a}(3+m) = 5$	$a(3+m) = m+3$

Question 3: Make 'b' the subject

(a)	(b)	(c)	(d)
$bh + ab = 9$	$5b + ab = 9$	$bh - ab = 9$	$6b - ab = 9$
(e)	(f)	(g)	(h)
$bh + ab + b = 11$	$bh + ab + b^2 = 11$	$ab + b^2 = 11$	$M = ab^2 - b^2$
(i)	(j)	(k)	(l)
$bh - ab = 9m$	$bh - ab = 9 + m$	$M = a\sqrt{b} - \sqrt{b}$	$bh - ab = h - a$

Question 4: Make 'm' the subject

(a)	(b)	(c)	(d)
$\frac{m}{l+a} = 2$	$5 = a(3+m)$	$2 = \frac{m-3}{5+b}$	$\frac{m^2}{l+w} = 2$
(e)	(f)	(g)	(h)
$5m + am = 9$	$\frac{m}{b+3} = 2$	$5 = a(m+2)$	$\frac{m}{p^2} = 2$
(i)	(j)	(k)	(l)
$6m - am = 9$	$\frac{\sqrt{m}}{a+b} = 2$	$mh + am + m = 11$	$c = a\sqrt{m} + \sqrt{m}$

Worked Examples

We will learn how to rearrange a quadratic equation to make the repeated unknown the subject. This is similar to what we have learnt previously but the first step is to factorise the common unknown.

Example 1:

Make 'x' the subject

$$x^2 - 7x - 9 = 0$$

1. 'x is a repeated unknown which needs to be factorised from the first two terms.

$$x(x-7)-9=0$$

2. To isolate x, the unknown, we need to eliminate ‘-9’ and ‘(x-7)’. Follow the order of operations, and reverse it

$$x \ \rightarrow \times\,(x-7) \rightarrow \ -9$$

$$x \ \leftarrow \div\,(x-7) \leftarrow \ +9$$

$$x(x-7)-9=0$$

$$+9 \qquad +9$$

$$x(x-7)=9$$

$$\div\,(x-7) \qquad \div\,(x-7)$$

$$\boldsymbol{x}=\frac{9}{x-7}$$

__Example 2:__

Make ‘x’ the subject

$$x^3-7-9=0$$

1. ‘x is a repeated unknown which needs to be factorised from the first two terms.

$$x^2(x-7)-9=0$$

2. To isolate x^2, the unknown, we need to eliminate ‘-9’ and ‘(x-7)’. Follow the order of operations, and reverse it

$$x \ \rightarrow \ x^2 \rightarrow \times\,(x-7) \rightarrow \ -9$$

$$x \ \leftarrow \sqrt{x} \ \leftarrow \div\,(x-7) \leftarrow \ +9$$

$$x^2(x-7)-9=0$$

$$+9 \qquad +9$$

$$x^2(x-7)=9$$

$$\div\,(x-7) \qquad \div\,(x-7)$$

$$x^2=\frac{9}{x-7}$$

$$x = \pm\sqrt{\frac{9}{x-7}}$$

Mini – quiz

(a)	(b)	(c)
$x^2 - 5x - 11 = 0$	$6x^2 - 5x = 0$	$x^3 + 5x^2 - 11 = 0$

Practice Exercise

Question 1: Make 'x' the subject

(a)	(b)	(c)
$x^2 - 5x - 11 = 0$	$x^2 - 11x - 11 = 0$	$x^2 - 5x + 11 = 0$
(d)	(e)	(f)
$x^2 - 9x - 11 = 11$	$x^2 - 13x - 11 = -13x$	$2x^2 - 5x - 11 = 4$
(g)	(h)	(i)
$6x^2 - 5x - 11 = 0$	$6x^2 - 5x = 0$	$6x^2 - 11 = 0$

Question 2: Make 'x' the subject

(a)	(b)	(c)
$x^3 + 5x^2 - 11 = 0$	$x^3 + 11x^2 - 11 = 0$	$x^3 + 5x^2 - 5 = 0$
(d)	(e)	(f)
$x^3 + 5x^2 - 11 = 13$	$x^4 + 5x^3 - 11 = 13$	$x^4 + 5x^2 - 11 = 0$
(g)	(h)	(i)
$6x^4 + x^3 - 11 = 13$	$6x^4 + x^3 - 11 = -11$	$6x^4 + x^2 = 13$

One step rearranging – Groups and Roots

Recap

Make ‘q’ the subject				
(a)	(b)	(c)	(d)	(e)
$\frac{q}{b-3} = 5$	$\frac{q}{b^3} = 5$	$q(b-3) = 5$	$q(2+m-3) = 5$	$qb + qc = 9$

We have learnt how to change the subject of an equation where we have a group involved. We will learn how to change the equation of a subject given that we have a group held to a power, or under a root.

Let’s look at a few examples:

<u>Example 1:</u>

Make ‘b’ the subject of the equation

$$b^2 = m - a$$

2. ‘b’ has a power of 2. We need to elimate ‘b^2’.

3. Square root both sides of the equation

$$b^2 = m - a$$

$$\sqrt[2]{\quad} \quad \sqrt[2]{\quad}$$

$$b = \pm\sqrt{m-a}$$

When we root an unknown where it is an even root then our unknown must have a $\pm$ to the left of the root. This is not required when we apply an odd root to an unknown.

<u>Example 2:</u>

Make ‘b’ the subject of the equation

$$b^3 = -a + m$$

2. ‘b’ has a power of 3. We need to elimate ‘b^3’.

3. Cube root both sides of the equation

$$b^3 = -a + m$$

$$\sqrt[3]{\quad} \quad \sqrt[3]{\quad}$$

$$b = \sqrt[3]{-a+m}$$

When we apply an odd root to a negative unknown, the result will remain negative. We do not place $\pm$ to the left of the root.

Example 3:

Make 'b' the subject of the equation

$$\sqrt{b} = m + a$$

2. 'b' is square rooted. We need to elimate '$\sqrt{b}$'.

3. Square both sides of the equation

$$\sqrt{b} = m + a$$

$$x^2 \quad x^2$$

$$b = (m + a)^2$$

We are squaring a group, therefore we can expand the brackets:

$$b = (m + a)^2 = (m + a)(m + a) = m^2 + 2am + a^2$$

Either answer is correct

Example 4:

Make 'b' the subject of the equation

$$\sqrt[3]{b} = m - a$$

2. 'b' is cube rooted. We need to elimate '$\sqrt[3]{b}$'.

3. Cube both sides of the equation

$$\sqrt[3]{b} = m + a$$

$$x^3 \quad x^3$$

$$b = (m + a)^3$$

We are squaring a group, therefore we can expand the brackets:

$$b = (m + a)^3 = (m + a)(m + a)(m + a) = m^3 + 3am^2 + 3a^2m + a^3$$

Either answer is correct

Example 5:

Make 'b' the subject of the equation

$$\sqrt[3]{b} = -m$$

2. 'b' is cube rooted. We need to elimate '$\sqrt[3]{b}$'.

3. Cube both sides of the equation

$$\sqrt[3]{b} = -m$$

$$x^3 \quad x^3$$

$$b = (-m)^3$$

When you apply an even power to a negative unknown, the result will always be positive

When you apply an odd power to a negative unknown, the result will always be negative.

When you apply an odd power to a positive unknown, the result will always be positive.

Mini-quiz

Make 'a' the subject				
(a)	(b)	(c)	(d)	(e)
$a^2 = n - b$	$a^3 = n + b$	$\sqrt{a} = M + b$	$\sqrt[3]{a} = m - q + b$	$\sqrt[3]{a} = -m$

Practice Exercises

Question 1: Make 'd' the subject:

(a)	(b)	(c)	(d)
$d^2 = b + a$	$d^2 = m - b$	$d^2 = M + b$	$d^2 = \frac{M}{B}$
(e)	(f)	(g)	(h)
$d^2 = \frac{M}{B-3}$	$d^3 = \frac{M}{B}$	$d^3 = M$	$d^3 = -M$
(i)	(j)	(k)	(l)
$d^3 = M + b$	$d^5 = M + b$	$M + A = d^3$	$Ma = d^3$
(m)	(n)	(o)	(p)
$-M = d^5$	$d^7 = M + b$	$d^2 = m^2 + b^2$	$d^2 = m^2 + b^2 - a$

Question 2: Make 'g' the subject

(a)	(b)	(c)	(d)
$\sqrt{g} = m$	$\sqrt{g} = m + 2$	$\sqrt{g} = m - 2$	$\sqrt{g} = a + b$
(e)	(f)	(g)	(h)
$\sqrt[3]{g} = m$	$\sqrt[3]{g} = m + 2$	$\sqrt[3]{g} = m - 2$	$\sqrt[3]{g} = m^4$
(i)	(j)	(k)	(l)
$\sqrt[3]{g} = -m$	$\sqrt{g} = -m$	$\sqrt{g} = 5m$	$\sqrt[3]{g} = -5m$

Question 3: Make 'h' the subject

(a)	(b)	(c)	(d)
$h^2 = b$	$\sqrt{h} = m$	$\sqrt{h} = 12m$	$-m = h^3$
(e)	(f)	(g)	(h)
$h^2 = \dfrac{a}{B-3}$	$h^2 = b - m$	$M + A = h^3$	$h^3 = -M$
(i)	(j)	(k)	(l)
$\sqrt[3]{h} = m$	$h^5 = M + b$	$\sqrt[7]{h} = -m$	$Ma = d^3$
(m)	(n)	(o)	(p)
$-M = h^5$	$\sqrt{h} = a - 1$	$\sqrt{h} = 2a - 1$	$h^2 = a^2 + b^2 - c$

Two step rearranging

Recap

Make 'd' the subject				
(a)	(b)	(c)	(d)	(e)
$d^3 = M$	$d^3 = n + b$	$d^2 = \dfrac{a}{B-3}$	$\sqrt[3]{d} = m - q + b$	$\sqrt[3]{d} = -m$

Worked Examples

We have learnt how to change the subject of an equation where we only apply one step. We will now learn how to change the subject of an equation but where we have two steps to take to isolate the unknown, that becomes the subject of the equation.

Let's look at the steps that we apply to all problem types where it takes two steps to change the subject of an equation:

1. List the order of operations for the unknown
2. Inverse the order of operations in the right order
    a. Eliminate a term
    b. Isolate the unknown
3. Check that you have the correct subject

Let's look at a few examples:

<u>*Example 1:*</u>

Make 'y' the subject of the equation

$$5y + a = m$$

1. List the order of operations for the unknown.
   y → × 5 → +a → m
2. Inverse the order of operations in the right order
   m → -a → ÷5 →

$$5y + a = m$$

$$-a \quad\quad -a$$

$$5y = m - a$$

$$\div 5 \quad\quad\quad \div 5$$

$$y = \frac{m-a}{5}$$

Example 2:

Make ‘y’ the subject of the equation

$$\frac{y}{5} - a = m$$

1. List the order of operations for the unknown.
   y → ÷ 5 → -a → m
2. Inverse the order of operations in the right order
   m → +a → ×5 →

$$\frac{y}{5} - a = m$$

$$+a \qquad + a$$

$$\frac{y}{5} = m + a$$

$$\times 5 \qquad\qquad \times 5$$

$$y = 5(m + a)$$

It is important to note that we are multiplying the entire expression, ‘m+a’ by 5, we can expand the brackets to get an expression.

$$y = 5(m + a) = 5m + 5a$$

Either answer is correct.

Example 3:

Make ‘y’ the subject of the equation

$$-5 + \frac{y}{a} = m$$

1. List the order of operations for the unknown.
   y → ÷ a → -5 → m
2. Inverse the order of operations in the right order
   m → +5 → ×a →

$$-5 + \frac{y}{a} = m$$

$$+5 \qquad + 5$$

$$\frac{y}{a} = m + 5$$

$$\times a \qquad\qquad \times a$$

$$y = a(m + 5)$$

It is important to note that we are multiplying the entire expression, ‘m+5’ by ‘a’, we can expand the brackets to get an expression.

$$y = a(m+5) = am + 5a$$

Either answer is correct.

Mini –quiz

Make 'd' the subject				
(a)	(b)	(c)	(d)	(e)
$5d + a = b$	$5a + d = b$	$\frac{d}{5} + a = b$	$\frac{d-a}{5} = b$	$-a + \frac{\sqrt{d}}{5} = b$

Practice Exercises

Question 1: Make 'a' the subject

(a)	(b)	(c)	(d)
$7a + b = 9$	$7a - b = 9$	$-b + 7a = 9$	$7a + c - b = 9$
(e)	(f)	(g)	(h)
$7a + \ m = 11$	$a^2 + m = 11$	$7a^2 + m = 11$	$7\sqrt{a} + m = 11$

Question 2: Make x the subject

(a)	(b)	(c)	(d)
$\frac{x}{3} + 9 = r$	$\frac{x}{3} + a = r$	$\frac{x}{3} - 5 = r$	$-5 + \frac{x}{3} = m$
(e)	(f)	(g)	(h)
$p = \frac{x}{3} - m$	$\frac{x}{5} - p = r$	$\frac{x}{5} - q = 10 + m$	$\frac{x}{m} - 2 = m$
(i)	(j)	(k)	(l)
$\frac{x^2}{3} + 9 = r$	$\frac{\sqrt{x}}{3} + 9 = r$	$\frac{x}{3} - a^2 = b^2$	$\frac{x}{3} - a^2 = b^2 + 2ab$

Question 3: Make 'x' the subject

(a)	(b)	(c)	(d)
$\frac{x+4}{3} = m$	$\frac{x+4}{m} = m$	$\frac{x-4}{m} = m$	$\frac{-4+x}{m} = m$
(e)	(f)	(g)	(h)
$11 = \frac{x-n}{3}$	$\frac{x-n}{5} = n$	$\frac{x+m^2}{m} = 5m$	$\frac{x-5abc}{a} = b$
(i)	(j)	(k)	(l)
$\frac{7x+b}{3} = 9$	$9 = \frac{7a-b}{3}$	$\frac{-b+7a}{3} = 3$	$\frac{7x+c-b}{3} = 3$
(m)	(n)	(o)	(p)
$1 = \frac{7x+m}{11}$	$\frac{7x^2+m}{11} = 1$	$1 = \frac{7\sqrt{x}+m}{11}$	$-1 = \frac{x^2+m}{11}$

Two step rearranging - Powers and Roots

Recap

Make ‘e’ the subject				
(a)	(b)	(c)	(d)	(e)
$5e + a = b$	$5e + d = b$	$\frac{e}{5} + a = b$	$\frac{e - a}{5} = b$	$\frac{e + m^2}{m} = 5m$

Worked Examples

We have learnt how to change the subject of an equation using two steps, we will again learn how to do this but when we have an unknown with a power, or an unknown with a root.

Here are some examples of unknowns (in this case a) with roots or powers :

$$5 + \sqrt{a} = m$$

$$a^3 - 5 = m$$

Similarly, here are some examples with groups (in this case a) with roots and powers:

$$(a - 5)^3 = m$$

$$m = \sqrt{5 + a}$$

Let’s look at a few examples of changing the subject of an equation, before we do this here are the steps that apply in all cases:

1. List the order of operations for the unknown
2. Inverse the order of operations in the right order
    a. Eliminate a term
    b. Isolate the unknown
3. Check that you have the correct subject

__Example 1:__

Make ‘f’ the subject

$$2 + f^2 = m$$

1. List the order of operations for the unknown.
   f → x^2 → +2 → m
2. Inverse the order of operations in the right order
   m → -2 → $\sqrt{x}$ →

$$2 + f^2 = m$$

$$-2 \qquad -2$$

$$f^2 = m - 2$$

$$\sqrt{x} \qquad\qquad \sqrt{x}$$

$$f = \pm\sqrt{m - 2}$$

It is important to note that when you square root both sides of the equation, you are square rooting the group.

We also must include the $\pm$ whenever we have an even root. This is placed to the left of the root.

Example 2:

Make 'f' the subject

$$2f^2 = m$$

1. List the order of operations for the unknown.
   f → x^2 → ×2 → m
2. Inverse the order of operations in the right order
   m → ÷2 → $\sqrt{x}$ →

$$2f^2 = m$$

$$\div 2 \qquad\qquad \div 2$$

$$f^2 = \frac{m}{2}$$

$$\sqrt{x} \qquad\qquad \sqrt{x}$$

$$f = \sqrt{\frac{m}{2}}$$

It is important to note that when you square root both sides of the equation, you are square rooting the group.

We also must include the $\pm$ whenever we have an even root. This is placed to the left of the root.

Example 3:

Make 'f' the subject

$$\frac{f^2}{2} = m$$

1. List the order of operations for the unknown.
   f → x^2 → ÷2 → m
2. Inverse the order of operations in the right order
   m → ×2 → $\sqrt{x}$ →

$$\frac{f^2}{2} = m$$

$$\times 2 \qquad \times 2$$

$$f^2 = 2m$$

$$\sqrt{x} \qquad \sqrt{x}$$

$$f = \pm\sqrt{2m}$$

It is important to note that when you square root both sides of the equation, you are square rooting the whole term, 2m.

We also must include the $\pm$ whenever we have an even root. This is placed to the left of the root.

Example 4:

Make 'f' the subject

$$\sqrt{f} - 90 = m$$

1. List the order of operations for the unknown.
   f → $\sqrt{x}$ → -91 → m
2. Inverse the order of operations in the right order
   m → $+91$ → x^2 →

$$\sqrt{f} - 90 = m$$

$$+90 \qquad +90$$

$$\sqrt{f} = m + 90$$

$$x^2 \qquad x^2$$

$$f = (m + 90)^2$$

The above answer is correct. We can write the answer in a different way by expanding the brackets.

$$f = (m + 90)^2 = (m + 90)(m + 90) = m^2 + 180m + 8100$$

Example 5:

Make 'f' the subject

$$\sqrt{f - 90} = m$$

1. List the order of operations for the unknown.
   f → -90 → $\sqrt{f}$ → m
2. Inverse the order of operations in the right order
   m → x^2 →$+90$ →

$$\sqrt{f - 90} = m$$

$$x^2 \qquad x^2$$

$$f - 90 = m^2$$

$$+90 \qquad\qquad + 90$$

$$f = m^2 + 90$$

<u>Example 6:</u>

Make ‘f’ the subject

$$\sqrt{\frac{f}{4}} = m$$

1. List the order of operations for the unknown.
   f → $\div\, 4$ → $\sqrt{x}$ → m
2. Inverse the order of operations in the right order
   m → x^2 → $\times\, 4$ →

$$\sqrt{\frac{f}{4}} = m$$

$$x^2 \qquad\qquad x^2$$

$$\frac{f}{4} = m^2$$

$$\times\, 4 \qquad\qquad \times\, 4$$

$$f = 4m^2$$

<u>Example 7:</u>

Make ‘f’ the subject

$$f^2 - a^2 = b^2$$

1. List the order of operations for the unknown.
   f → x^2 → $-a^2$ → m
2. Inverse the order of operations in the right order
   m →+ a^2 →$\sqrt{x}$ →

$$f^2 - a^2 = b^2$$

$$+a^2 \qquad\qquad + a^2$$

$$f^2 = b^2 + a^2$$

$$\sqrt{x} \qquad\qquad \sqrt{x}$$

$$f = \pm\sqrt{b^2 + a^2}$$

It is important to note that this is not possible:

$$\sqrt{b^2 + a^2} \neq \sqrt{b^2} + \sqrt{a^2} \neq b + a$$

The entire group is $b^2 + a^2$ is being square rooted and it is left as that.

Mini-quiz

Make 'd' the subject				
(a)	(b)	(c)	(d)	(e)
$\sqrt{d} - 6 = m$	$3d^2 = g$	$\frac{d^2}{3} = m$	$\sqrt{d} - 10 = g$	$d^2 + a^2 + c^2 = b^2$

Practice Exercises

Question 1: Make 'f' the subject

(a)	(b)	(c)	(d)
$6 + \sqrt{f} = m$	$\sqrt{f} - 6 = m$	$\sqrt{f} - n = m$	$\sqrt{2f} - n = m$
(e)	(f)	(g)	(h)
$\sqrt{f} - ab = m$	$6 + \sqrt{f} + 11 = m$	$6 + \sqrt{f} + p = m$	$6 + \sqrt{f} = 11$
(i)	(j)	(k)	(l)
$6 + \sqrt{2f} = 11$	$6 + \sqrt{2f} = 3$	$6 + \sqrt{2f} = m$	$6 + \sqrt{f^3} = 11$

Question 2: Make 'b' the subject

(a)	(b)	(c)	(d)
$3b^2 = g$	$3\pi b^2 = g$	$3ab^2 = g$	$(m-3)b^2 = g$
(e)	(f)	(g)	(h)
$(m-n)b^2 = g$	$\frac{b^2}{2} = m$	$\frac{b^2}{n} = m$	$\frac{b^2}{n} = 25$
(i)	(j)	(k)	(l)
$\frac{5b^2}{n} = m$	$\frac{4b^2}{n} = m$	$\frac{b^3}{n} = m$	$\frac{b^3}{n} = -m$
(m)	(n)	(o)	(p)
$\frac{\sqrt{b}}{3} = m$	$\frac{\sqrt{b}}{n} = m$	$\frac{6\sqrt{b}}{n} = m$	$\frac{6 + \sqrt{b}}{2} = m$

Question 3: Make 'e' the subject

(a)	(b)	(c)	(d)
$\sqrt{e} - 10 = g$	$\sqrt{e} + 10 = g$	$\sqrt{e} - a = b$	$\sqrt{e} - 10 = g$
(e)	(f)	(g)	(h)
$4 + \sqrt{e} - 10 = g$	$2\sqrt{e} - 10 = g$	$\frac{\sqrt{e}}{2} - 10 = g$	$\frac{\sqrt{e}}{a} - 10 = a$

(i)	(j)	(k)	(l)
$\sqrt{e-10}=g$	$\sqrt{e+10}=g$	$\sqrt{e+16}=g$	$\sqrt{e-16}=2g$
(m)	(n)	(o)	(p)
$\sqrt{4e}=2g$	$\sqrt{ef}=2g$	$\sqrt{\frac{e}{4}}=2g$	$\sqrt{\frac{e}{8}}=2g$

Question 4: Make ‘d’ the subject

(a)	(b)	(c)	(d)
$d^2-a^2=b^2$	$d^2+a^2=b^2$	$-a^2+d^2=b^2$	$d^2+a^2+c^2=b^2$
(e)	(f)	(g)	(h)
$d^2+a^2=9$	$d^2+9=a^2$	$d^2+9+b=a^2$	$d^2+ab=a^2$
(i)	(j)	(k)	(l)
$5d^2+ab=a^2$	$\frac{d^2}{5}+ab=a^2$	$\frac{d^2}{10}+b=a^2$	$\frac{d^2}{5}-a=b$

Rearranging with the subject in the denominator

Recap

Make 'a' the subject				
(a)	(b)	(c)	(d)	(e)
$\sqrt{a} - 6 = m$	$3a^2 = g$	$\frac{a^2 - 1}{3} = m$	$\sqrt{a - 10} = g$	$d^2 + a^2 + c^2 = b^2$

Worked Examples

We will learn how to change the subject of an equation where the subject is the denominator of a fraction. This is a two step process.

Make 'g' the subject	
Subject is the numerator	Subject is the denominator
$\frac{g}{3} = m$	$\frac{3}{g} = m$
Here the subject is the numerator. If we multiply both sides by '3' then we have made 'g' the subject of the equation.	Here the subject is the denominator. If we multiply both sides by the denominator, 'g' is still not the subject of the equation.
$\frac{g}{3} = m$ $\times 3 \qquad \times 3$ $g = 3m$	$\frac{3}{g} = m$ $\times g \qquad \times g$ $3 = gm$ We must isolate 'g' by dividing both sides by 'm'. $3 = gm$ $\div m \qquad \div m$ $\frac{3}{m} = g$

1 Let’s look at a few examples:

2 *Example 1:*

3 Make ‘g’ the subject

$$\frac{a^2}{g} = m$$

$$\times g \qquad \times g$$

$$a^2 = gm$$

4 We must isolate ‘g’ by dividing both sides by ‘m’.

$$a^2 = gm$$

$$\div m \qquad \div m$$

$$\frac{a^2}{m} = g$$

5 Conventionally, we write an equation where the subject is on the left hand side of the equal sign.

$$g = \frac{a^2}{m}$$

6 *Example 2:*

7 Make ‘g’ the subject

$$\frac{a^2}{g} = m - 9$$

$$\times g \qquad \times g$$

$$a^2 = g(m - 9)$$

8 We must isolate ‘g’ by dividing both sides by ‘(m – 9)’.

$$a^2 = g(m - 9)$$

$$\div (m - 9) \qquad \div (m - 9)$$

$$\frac{a^2}{m - 9} = g$$

9 Conventionally, we write an equation where the subject is on the left hand side of the equal sign.

$$g = \frac{a^2}{m - 9}$$

10

1 *Example 3:*

2 Make ‘g’ the subject

$$\frac{a}{\sqrt{g}} = m$$

$$\times\sqrt{g} \qquad \times\sqrt{g}$$

$$a = m\sqrt{g}$$

3 We must isolate ‘$\sqrt{g}$’ by dividing both sides by ‘m’.

$$a = m\sqrt{g}$$

$$\div m \qquad \div m$$

$$\frac{a}{m} = \sqrt{g}$$

4 We haven’t isolated ‘g’ so we must square both sides of the equation.

$$\frac{a}{m} = \sqrt{g}$$

$$x^2 \qquad x^2$$

$$\left(\frac{a}{m}\right)^2 = g$$

5 We can expand the bracket to get the following:

$$\frac{a^2}{m^2} = g$$

6 Conventionally, we write an equation where the subject is on the left hand side of the equal sign.

$$g = \frac{a^2}{m^2}$$

7 Mini-quiz

Make ‘d’ the subject				
(a)	(b)	(c)	(d)	(e)
$\frac{p^2}{d} = m$	$\frac{p^3}{d} = m$	$\frac{3+p}{d} = m^2$	$\frac{g}{\sqrt{d}} = 4$	$\frac{mg}{\sqrt{d-5}} = 5$

8

9

10

Practice Exercises

Question 1: Make ‘w’ the subject:

(a)	(b)	(c)	(d)
$\frac{g^2}{w} = m$	$\frac{g^3}{w} = m$	$\frac{3g^3}{w} = m$	$\frac{3\pi}{w} = m$
(e)	(f)	(g)	(h)
$\frac{3}{w} = m$	$\frac{3\pi}{w} = am$	$\frac{3\pi}{w} = m^2$	$\frac{3+\pi}{w} = m^2$
(i)	(j)	(k)	(l)
$\frac{3-\pi}{w} = m^2$	$\frac{m^2}{w} = 3-\pi$	$\frac{m^2}{w} = 3-a$	$\frac{m^2}{w} = a-3$

Question 2: Make ‘a’ the subject:

(a)	(b)	(c)	(d)
$\frac{g}{w} = m$	$\frac{g}{\sqrt{w}} = m$	$\frac{g}{\sqrt{w}} = 4$	$\frac{3g}{\sqrt{w}} = 4$
(e)	(f)	(g)	(h)
$\frac{g}{\sqrt{w}} = 5$	$\frac{g}{\sqrt{w+1}} = 5$	$\frac{g}{\sqrt{w+n}} = 5$	$\frac{mg}{\sqrt{w-5}} = 5$
(i)	(j)		
$\frac{g}{\sqrt{2w}} = 5$	$\frac{g}{\sqrt{25w}} = 5$		

One step rearranging – Fractions

Recap

Make 'r' the subject				
(a)	(b)	(c)	(d)	(e)
$\frac{p^2}{r} = m$	$\frac{p^3}{r} = m$	$\frac{3+p}{r} = m^2$	$\frac{g}{\sqrt{r}} = 4$	$\frac{m^2}{r} = a - 3$

Worked Examples

We will learn how to change the subject of an equation, with one step, but this time one side of the equation will have a fraction. We will learn to change the subject of the following equations.

For example, make 'd' the subject

$$\frac{d}{a} = \frac{3}{b}$$

$$da = \frac{3}{b}$$

$$a + d = \frac{3}{b}$$

Before we do this let's recap how to multiply fractions:

To multiply fractions:

1) Top x Top
2) Base x Base

Example 1	Example 2	Example 3	Example 4
$\frac{2}{4} \times \frac{3}{5} = \frac{6}{20}$	$\frac{2}{4} \times \frac{3}{5} \times \frac{1}{2} = \frac{6}{40}$	$\frac{3}{b} \times \frac{a}{4} = \frac{3a}{4b}$	$\frac{2}{b} \times \frac{5}{m} \times \frac{a}{12} = \frac{10a}{12bm}$

To multiply a fraction with an integer or term

1) Write the integer or term with a denominator of 1
2) Top x Top
3) Base x Base

Example 1	Example 2	Example 3	Example 4
$\frac{2}{4} \times 3 = \frac{6}{4}$	$7 \times \frac{1}{5} \times 4 = \frac{28}{5}$	$\frac{3}{b} \times 4 = \frac{12}{b}$	$\frac{2}{b} \times m \times \frac{a}{12} = \frac{2am}{12b}$

$\frac{2}{4} \times \frac{3}{1} = \frac{6}{4}$	$\frac{7}{1} \times \frac{1}{5} \times \frac{4}{1} = \frac{28}{5}$	$\frac{3}{b} \times \frac{4}{1} = \frac{12}{b}$	$\frac{2}{b} \times \frac{m}{1} \times \frac{a}{12} = \frac{2am}{12b}$

Let’s look at a few examples of changing the subject where the equation is equal to a fraction or an algebraic fraction, here are the steps:

1. List the order of operations for the unknown
2. Inverse the order of operations in the right order
    a. Eliminate a term
    b. Isolate the unknown
3. Check that you have the correct subject

Example 1:

Make ‘p’ the subject

$$\frac{p}{4} = \frac{t}{g}$$

1. p → ÷4
2. $\times 4$ → p
3. ‘P’ is divided by 4, we need to eliminate the denominator
4. Multiply both sides of the equation by the denominator to isolate ‘p’.

$$\frac{p}{4} = \frac{t}{g}$$

$$\times 4 \qquad \times 4$$

$$p = \frac{t}{g} \times 4 = \frac{t}{g} \times \frac{4}{1} = \frac{4t}{g}$$

$$p = \frac{4t}{g}$$

Example 2

Make ‘p’ the subject

$$\frac{p}{4} = \frac{t}{g-3}$$

1. p → ÷4
2. $\times 4$ → p
3. ‘P’ is divided by 4, we need to eliminate the denominator
4. Multiply both sides of the equation by the denominator to isolate ‘p’.

1

$$\frac{p}{4} = \frac{t}{g-3}$$

$$\times 4 \qquad \times 4$$

$$p = \frac{t}{g-3} \times 4 = \frac{t}{g-3} \times \frac{4}{1} = \frac{4t}{g-3}$$

$$p = \frac{4t}{g-3}$$

2 *Example 3:*

3 Make ‘p’ the subject

$$\frac{p}{4} = \frac{t-m}{3}$$

4 1. p → ÷4
5 2. × 4 → p
6 3. ‘P’ is divided by 4, we need to eliminate the denominator
7 4. Multiply both sides of the equation by the denominator to isolate ‘p’.

8

$$\frac{p}{4} = \frac{t-m}{3}$$

$$\times 4 \qquad \times 4$$

$$p = \frac{t-m}{3} \times 4 = \frac{t-m}{3} \times \frac{4}{1} = \frac{4(t-m)}{3}$$

$$p = \frac{4(t-m)}{3}$$

9 We can simplify the numerator by expanding the bracket, either answer is correct.

$$p = \frac{4t-4m}{3}$$

10 Mini-quiz

Make ‘p’ the subject				
(a)	(b)	(c)	(d)	(e)
$\frac{p}{4} = \frac{t}{g}$	$\frac{p^2}{4} = \frac{t}{g}$	$\frac{p}{4} = \frac{g-3}{t}$	$\frac{g-3}{t} = \frac{p}{g^2}$	$\frac{g-3}{t} = \frac{p^2}{g^2}$

11

12

Practice Exercises

Question 1: Make ‘t’ the subject

(a)	(b)	(c)	(d)
$\frac{t}{4}=\frac{p}{g}$	$\frac{t}{4}=\frac{3}{g}$	$\frac{t}{4}=\frac{g}{3}$	$\frac{t}{4}=g$
(e)	(f)	(g)	(h)
$\frac{p}{4}=\frac{t}{g}$	$\frac{4}{p}=\frac{t}{g}$	$\frac{4}{p}=\frac{t}{5}$	$\frac{4}{p}=\frac{\sqrt{t}}{5}$
(i)	(j)	(k)	(l)
$\frac{4}{p}=\frac{\sqrt[3]{t}}{5}$	$\frac{2}{p}=\frac{t^2}{8}$	$\frac{16}{p}=\frac{t^3}{4}$	$-\frac{16}{p}=\frac{t^3}{4}$
(m)	(n)	(o)	(p)
$\frac{4}{p}=\frac{t+1}{5}$	$\frac{4}{p}=\frac{t-1}{5}$	$\frac{4}{p}=\frac{t-p}{5}$	$\frac{4}{p}=\frac{-p+t}{5}$

Question 2: Make ‘k’ the subject

(a)	(b)	(c)	(d)
$\frac{k}{4}=\frac{3}{g+3}$	$\frac{k}{4}=\frac{5}{g+3}$	$\frac{k}{4}=\frac{g}{g+3}$	$\frac{k}{5}=\frac{g}{g+3}$
(e)	(f)	(g)	(h)
$\frac{k}{g}=\frac{g}{g+3}$	$\frac{g}{g+3}=\frac{k}{3}$	$\frac{g}{g+3}=\frac{k}{3h}$	$\frac{g}{g+3}=\frac{k}{mp}$
(i)	(j)	(k)	(l)
$\frac{1}{g+3}=\frac{k}{\sqrt{g}}$	$\frac{5}{g+3}=\frac{k}{\sqrt{g}}$	$\frac{5}{g+3}=\frac{k}{g}$	$\frac{g+3}{5}=\frac{k}{4}$
(m)	(n)	(o)	(p)
$\frac{g+3}{5}=\frac{k}{g}$	$\frac{2g+3}{5}=\frac{k}{2}$	$\frac{10g+5}{5}=\frac{k}{2}$	$\frac{2g+3}{5}=\frac{k}{-2}$

Question 3: Make ‘g’ the subject

(a)	(b)	(c)	(d)
$\frac{g}{4}=\frac{p}{5}$	$\frac{g}{4}=\frac{p}{m}$	$\frac{4}{p}=\frac{\sqrt{g}}{5}$	$\frac{2}{p}=\frac{g^2}{8}$
(e)	(f)	(g)	(h)

$\frac{4}{p} = \frac{g-p}{5}$	$\frac{5}{k} = \frac{g}{k+3}$	$\frac{g}{k+3} = \frac{k}{mp}$	$\frac{5}{k+3} = \frac{g}{\sqrt{k}}$

Now let's look at example where when we change the subject of an equation we must divide a fraction by an integer or a term. We will learn how to change the subject of an equation for the following problem types:

$$da = \frac{3}{b}$$

$$da = \frac{3}{m+a}$$

$$da = \frac{m+a}{3}$$

Before we learn how make 'd' the subject of the following equations above let's recap how to divide fractions, here are the steps:

1. Keep the first fraction
2. Flip the second
3. Change (division) to times

What if we are dividing a fraction by an integer or a term, there is one additional step:

ADDITIONAL STEP: Write the integer or term as a fraction with a denominator of 1

1. Keep the first fraction
2. Flip the second
3. Change (division) to times

Here are some examples to show how we divide fractions:

$\frac{4}{5} \div \frac{2}{3} = \frac{4}{5} \times \frac{3}{2} = \frac{12}{10}$	1. Keep the first fraction 2. Flip the second 3. Change (division) to times
$\frac{1}{5} \div \frac{2}{3} = \frac{1}{5} \times \frac{3}{2} = \frac{3}{10}$	
$\frac{2}{a} \div \frac{4}{b} = \frac{2}{a} \times \frac{b}{4} = \frac{2b}{4a}$	
$\frac{2}{a} \div \frac{b}{4} = \frac{2}{a} \times \frac{4}{b} = \frac{8}{ab}$	

$\frac{2}{m^2} \div \frac{m^4}{n} = \frac{2}{m^2} \times \frac{n}{m^4} = \frac{2n}{m^6}$	

Here are some examples of when we divide a fraction by an integer:

$\frac{4}{5} \div 2 = \frac{4}{5} \div \frac{2}{1} = \frac{4}{5} \times \frac{1}{2} = \frac{4}{10}$	ADDITIONAL STEP: Write the integer or term as a fraction with a denominator of 1 1. Keep the first fraction 2. Flip the second 3. Change (division) to times
$\frac{1}{5} \div 2 = \frac{1}{5} \div \frac{2}{1} = \frac{1}{5} \times \frac{1}{2} = \frac{1}{10}$	
$\frac{2}{a} \div b = \frac{2}{a} \div \frac{b}{1} = \frac{2}{a} \times \frac{1}{b} = \frac{2}{ab}$	
$\frac{2}{am} \div b = \frac{2}{am} \div \frac{b}{1} = \frac{2}{am} \times \frac{1}{b} = \frac{2}{abm}$	
$\frac{2}{m^2} \div m^4 = \frac{2}{m^2} \div \frac{m^4}{1} = \frac{2}{m^2} \times \frac{1}{m^4} = \frac{2}{m^6}$	

Now let's look at a few examples

Example 1:

Make 'd' the subject

$$da = \frac{3}{b}$$

1. d → $\times a$
2. $\div a$ → d
3. 'd' is multiplied by 'a', we need to eliminate 'a'.
4. Divide both sides of the equation by 'a' to isolate 'd'.

$$da = \frac{3}{b}$$

$$\div a \qquad \div a$$

$$d = \frac{3}{b} \div a = \frac{3}{b} \div \frac{a}{1} = \frac{3}{b} \times \frac{1}{a} = \frac{3}{ab}$$

$$d = \frac{3}{ab}$$

1 *Example 2:*

2 Make ‘d’ the subject

$$da = \frac{3}{m+a}$$

3 1. d → $\times a$
4 2. $\div a$ → d
5 3. ‘d’ is multiplied by ‘a’, we need to eliminate ‘a’.
6 4. Divide both sides of the equation by ‘a’ to isolate ‘d’.

7

$$da = \frac{3}{m+a}$$

$$\div a \qquad \div a$$

$$d = \frac{3}{m+a} \div a = \frac{3}{m+a} \div \frac{a}{1} = \frac{3}{m+a} \times \frac{1}{a} = \frac{3}{a(m+a)}$$

8 We can expand the bracket in the denominator. Either answer would be correct:

$$d = \frac{3}{a(m+a)} = \frac{3}{am+a^2}$$

9 *Example 3:*

10 Make ‘d’ the subject

$$da = \frac{m+a}{3}$$

11 1. d → $\times a$
12 2. $\div a$ → d
13 3. ‘d’ is multiplied by ‘a’, we need to eliminate ‘a’.
14 4. Divide both sides of the equation by ‘a’ to isolate ‘d’.

15

$$da = \frac{m+a}{3}$$

$$\div a \qquad \div a$$

$$d = \frac{m+a}{3} \div a = \frac{m+a}{3} \div \frac{a}{1} = \frac{m+a}{3} \times \frac{1}{a} = \frac{m+a}{3a}$$

$$d = \frac{m+a}{3a}$$

Mini-quiz

Make 'p' the subject				
(a)	(b)	(c)	(d)	(e)
$mp = \frac{5}{m}$	$mp = \frac{5}{m+5}$	$mp = \frac{m+5}{5}$	$\sqrt{p} = \frac{m+5}{5}$	$p\sqrt{m} = \frac{m+5}{5}$

Practice Exercises

Question 1: Make 'b' the subject

(a)	(b)	(c)	(d)
$3b = \frac{g}{5}$	$5b = \frac{g}{5}$	$\frac{g}{5} = 6b$	$\frac{7g}{5} = 6b$
(e)	(f)	(g)	(h)
$\frac{10g}{5} = 6b$	$\frac{10gh}{5} = 6b$	$\frac{gh}{5} = 6b$	$\frac{gh}{5} = ab$
(i)	(j)	(k)	(l)
$\frac{gh}{5} = 5ab$	$\frac{gh}{5} = \sqrt{b}$	$\frac{gh}{5} = b^2$	$\frac{gh}{5} = b^3$

Question 2: Make 'k' the subject

(a)	(b)	(c)	(d)
$4k = \frac{3}{g+3}$	$6k = \frac{5}{g+3}$	$3k = \frac{g}{g+3}$	$gk = \frac{g}{g+3}$
(e)	(f)	(g)	(h)
$\frac{g+3}{5} = 4k$	$\frac{g+3}{6} = 6k$	$\frac{g+3}{3} = k^2$	$\frac{g+3}{3} = \sqrt{k}$

One step rearranging – Groups in the denominator

Recap

Make ‘p’ the subject				
(a)	(b)	(c)	(d)	(e)
$\frac{p}{4}=\frac{t}{g}$	$mp=\frac{5}{m}$	$\frac{p}{4}=\frac{g-3}{t}$	$mp=\frac{5}{m+5}$	$mp=\frac{m+5}{5}$

Worked Examples

We will learn how to change the subject of an equation, with one step, but this time the subject will have a group in the denominator.

We will learn to change the subject of the following equations.

For example, make ‘d’ the subject

$$\frac{d}{a+b}=30$$

$$\frac{d}{a+b}=\frac{30}{b}$$

Before we look at any examples, here are the steps:

1. To isolate the subject we must eliminate the denominator

Here are a few examples:

Example 1:

Make ‘d’ the subject

$$\frac{d}{a+b}=30$$

1. To isolate the subject we must eliminate the denominator

$$\frac{d}{a+b}=30$$

$$\times(a+b) \qquad \times(a+b)$$

$$d=\ 30(a+b)$$

At this point we can expand the single bracket. Either answer is correct.

$$d=\ 30a+30b$$

1 *Example 2:*

2 Make ‘d’ the subject

$$\frac{d}{a+b} = \frac{30}{g-3}$$

3 1. To isolate the subject we must eliminate the denominator

$$\frac{d}{a+b} = \frac{30}{g-3}$$

$$\times (a+b) \qquad \times (a+b)$$

$$d = \frac{30}{g-3} \times (a+b) = \frac{30}{g-3} \times \frac{(a+b)}{1} = \frac{30(a+b)}{g-3}$$

$$d = \frac{30(a+b)}{g-3}$$

4 We can expand the bracket on the numerator. Either answer is correct.

$$d = \frac{30a+30b}{g-3}$$

5 *Example 3:*

6 Make ‘d’ the subject

$$\frac{d}{a+b} = \frac{a-b}{3}$$

7 1. To isolate the subject we must eliminate the denominator

8

$$\frac{d}{a+b} = \frac{a-b}{3}$$

$$\times (a+b) \qquad \times (a+b)$$

$$p = \frac{a-b}{3} \times (a+b) = \frac{a-b}{3} \times \frac{(a+b)}{1} = \frac{(a-b)(a+b)}{3}$$

$$p = \frac{(a-b)(a+b)}{3}$$

9 We can simplify the numerator by expanding the bracket, either answer is correct.

$$p = \frac{a^2-b^2}{3}$$

10

Mini quiz

Make 'd' the subject				
(a)	(b)	(c)	(d)	(e)
$\frac{d}{m-p}=30$	$\frac{d}{m-p}=\frac{30}{m+p}$	$\frac{d}{m-p}=1$	$\frac{d}{m-p}=\frac{1}{5}$	$\frac{d}{m-p}=\frac{m+p}{t}$

Practice Exercises

Question 1: Make 't' the subject

(a)	(b)	(c)	(d)
$\frac{t}{a+4}=a$	$\frac{t}{a+4}=b$	$\frac{t}{a+4}=1$	$\frac{t}{a+4}=3$
(e)	(f)	(g)	(h)
$\frac{t}{a+4}=\frac{3}{4}$	$\frac{t}{a+4}=\frac{1}{4}$	$\frac{t}{a+4}=\frac{1}{a+4}$	$\frac{t}{a+4}=\frac{a+4}{a+4}$
(i)	(j)	(k)	(l)
$\frac{t}{a+4}=\frac{a-4}{a+4}$	$\frac{t}{a+4}=\frac{1}{a+4}$	$\frac{t}{a+4}=a+4$	$\frac{t}{a+4}=\frac{a+4}{b}$
(m)	(n)	(o)	(p)
$\frac{t}{a+4}=\frac{a-4}{b}$	$\frac{t}{a+4}=\frac{a-4}{a}$	$\frac{t^2}{a+4}=\frac{b}{a+4}$	$\frac{\sqrt{t}}{a+4}=\frac{b}{a+4}$

Question 2: Make 'k' the subject

(a)	(b)	(c)	(d)
$\frac{k}{4}=\frac{g+3}{3}$	$\frac{k}{4}=\frac{g+3}{5}$	$\frac{k}{4}=\frac{g+3}{2}$	$\frac{k}{4}=\frac{g+3}{4}$
(e)	(f)	(g)	(h)
$\frac{k}{g}=\frac{g+3}{4}$	$\frac{k}{g-3}=\frac{g+3}{4}$	$\frac{k}{g+3}=\frac{g+3}{4}$	$\frac{k}{g+3}=\frac{1}{g+3}$

Question 3: Make 'g' the subject

(a)	(b)	(c)	(d)
$\frac{g}{a+4}=\frac{1}{5}$	$\frac{g}{a+4}=\frac{4}{5}$	$\frac{g}{a+4}=\frac{10}{5}$	$\frac{g}{a+4}=\frac{a+4}{5}$

Two step – Subject in the denominator

Recap

Make ‘f’ the subject				
(a)	(b)	(c)	(d)	(e)
$\frac{f}{m-p} = 30$	$\frac{f}{m-p} = 1$	$\frac{f}{m-p} = \frac{1}{5}$	$\frac{1}{m-p} = \frac{f}{m+p}$	$\frac{f}{m-1} = \frac{m+1}{t}$

Worked Examples

We will learn how to change the subject of an equation where both sides of the equation have fractions (potentially algebraic fractions) where the subject is the denominator of one of those fractions. This will be a two step process. We will also learn two methods. The first method will show you the two step process. The second step is a faster technique where you can change the subject of an equation in one step.

Let’s look at a few examples to remind ourselves of what an equation will look like when the subject is in the denominator.

Make ‘g’ the subject:			
$\frac{a^2}{g} = \frac{3}{b}$	$\frac{a^2}{g} = \frac{m-9}{4}$	$\frac{\sqrt{a}}{g} = \frac{3}{b}$	

We will learn how to change the subject of an equation where the subject is the denominator of a fraction. This is a two step process.

Before we look at a few examples, here are the steps:

1. Multiply both sides by one fraction’s denominator
2. Multiply both sides by the second fraction’s denominator
3. Eliminate any terms to isolate the unknown
4. Isolate the unknown to make it the subject

Let’s look at a few examples:

Example 1:

Make ‘g’ the subject

$$\frac{a^2}{g} = \frac{3}{b}$$

1. Multiply both sides by one fraction’s denominator

1

$$\times g \qquad \times g$$

$$a^2 = \frac{3}{b} \times g = \frac{3}{b} \times \frac{g}{1} = \frac{3g}{b}$$

$$a^2 = \frac{3g}{b}$$

2 2. Multiply both sides by the second fraction's denominator

3 We must isolate 'g' by dividing both sides by the denominator 'b'.

$$a^2 = \frac{3g}{b}$$

$$\times b \qquad \times b$$

$$a^2 b = 3g$$

4 3. Eliminate any terms to isolate the unknown

5 To isolate 'g' we must eliminate '3' by dividing both sides of the equation by '3':

$$a^2 b = 3g$$

$$\div 3 \qquad \div 3$$

$$\frac{a^2 b}{3} = g$$

6 Conventionally, we write an equation where the subject is on the left hand side of the equal sign.

$$g = \frac{a^2 b}{3}$$

7 *Example 2:*

8 Make 'g' the subject

$$\frac{a^2}{g} = \frac{m-9}{4}$$

9 1. Multiply both sides by one fraction's denominator

$$\times g \qquad \times g$$

$$a^2 = g \times \frac{(m-9)}{4} = \frac{g}{1} \times \frac{(m-9)}{4} = \frac{g(m-9)}{4}$$

$$a^2 = \frac{g(m-9)}{4}$$

10 2. Multiply both sides by the second fraction's denominator

1 We must isolate ‘g’ by dividing both sides by the denominator ‘4’:

$$a^2 = \frac{g(m-9)}{4}$$

$$\times 4 \qquad \times 4$$

$$4a^2 = \; g(m-9)$$

2 3. Eliminate any terms to isolate the unknown

$$4a^2 = \; g(m-9)$$

$$\div (m-9) \qquad \div (m-9)$$

$$\frac{4a^2}{(m-9)} = \; g$$

3 Conventionally, we write an equation where the subject is on the left hand side of the equal sign.

$$g = \frac{4a^2}{m-9}$$

4 *Example 3:*

5 Make ‘g’ the subject

$$\frac{\sqrt{a}}{g} = \frac{3}{b}$$

6 1. Multiply both sides by one fraction’s denominator

$$\frac{\sqrt{a}}{g} = \frac{3}{b}$$

$$\times g \qquad \times g$$

$$\sqrt{a} = \frac{3}{b} \times g = \; \frac{3}{b} \times \frac{g}{1} = \frac{3g}{b}$$

$$\sqrt{a} = \frac{3g}{b}$$

7 2. Multiply both sides by the second fraction’s denominator

8 We must isolate ‘g’ by dividing both sides by the denominator ‘b’:

$$\sqrt{a} = \frac{3g}{b}$$

$$\times b \qquad \times b$$

$$b\sqrt{a} = 3g$$

9 3. Eliminate any terms to isolate the unknown

1 To isolate 'g' we must eliminate '3' by dividing both sides of the equation by '3'

$$b\sqrt{a} = 3g$$

$$\div 3 \qquad \div 3$$

$$\frac{b\sqrt{a}}{3} = g$$

2 Conventionally, we write an equation where the subject is on the left hand side of the equal sign.

$$g = \frac{b\sqrt{a}}{3}$$

3 Mini-quiz

Make 'm' the subject				
(a)	(b)	(c)	(d)	(e)
$\frac{a}{m} = \frac{1}{5}$	$\frac{a}{m} = \frac{3}{5}$	$\frac{a^2}{m} = \frac{3}{5}$	$\frac{a^2}{m} = \frac{m-9}{5}$	$\frac{\sqrt{a}}{m} = \frac{m-9}{5}$

4

5 Practice Exercises

6 Question 1: Make 'g' the subject

(a)	(b)	(c)	(d)
$\frac{t}{4} = \frac{p}{g}$	$\frac{t}{4} = \frac{3}{g}$	$\frac{t}{4} = \frac{g}{3}$	$\frac{t}{4} = g$
(e)	(f)	(g)	(h)
$\frac{p}{4} = \frac{t}{g}$	$\frac{4}{p} = \frac{t}{g}$	$\frac{4}{g} = \frac{t}{5}$	$\frac{4}{g} = \frac{\sqrt{t}}{5}$
(i)	(j)	(k)	(l)
$\frac{4}{g} = \frac{\sqrt[3]{t}}{5}$	$\frac{2}{g} = \frac{t^2}{8}$	$\frac{16}{g} = \frac{t^3}{4}$	$-\frac{16}{g} = \frac{t^3}{4}$
(m)	(n)	(o)	(p)
$\frac{4}{g} = \frac{t+1}{5}$	$\frac{4}{g} = \frac{t-1}{5}$	$\frac{4}{g} = \frac{t-p}{5}$	$\frac{4}{g} = \frac{-p+t}{5}$

7

8 Question 2: Make 'k' the subject

(a)	(b)	(c)	(d)
$\frac{4}{k} = \frac{3}{g+3}$	$\frac{4}{k} = \frac{5}{g+3}$	$\frac{4}{k} = \frac{g}{g+3}$	$\frac{4}{k} = \frac{g}{g+3}$

(e)	(f)	(g)	(h)
$\frac{g}{k}=\frac{g}{g+3}$	$\frac{g}{g+3}=\frac{3}{k}$	$\frac{g}{g+3}=\frac{3h}{k}$	$\frac{g}{g+3}=\frac{mp}{k}$
(i)	(j)	(k)	(l)
$\frac{1}{g+3}=\frac{\sqrt{g}}{k}$	$\frac{5}{g+3}=\frac{\sqrt{g}}{k}$	$\frac{5}{g+3}=\frac{g}{k}$	$\frac{g+3}{5}=\frac{4}{k}$
(m)	(n)	(o)	(p)
$\frac{g+3}{5}=\frac{g}{k}$	$\frac{2g+3}{5}=\frac{g}{k}$	$\frac{10g+5}{5}=\frac{2}{k}$	$\frac{2g+3}{5}=\frac{-2}{k}$

Question 3: Make ‘g’ the subject

(a)	(b)	(c)	(d)
$\frac{4}{g}=\frac{p}{5}$	$\frac{4}{g}=\frac{p}{m}$	$\frac{4}{p}=\frac{5}{\sqrt{g}}$	$\frac{2}{p}=\frac{8}{g^2}$
(e)	(f)	(g)	(h)
$\frac{4}{p}=\frac{5}{g-p}$	$\frac{5}{k}=\frac{k+3}{g}$	$\frac{k+5}{g}=\frac{k}{mp}$	$\frac{5}{k+3}=\frac{\sqrt{k}}{g}$

Rearranging with Negative unknowns

Recap

Make ‘q’ the subject				
(a)	(b)	(c)	(d)	(e)
$\frac{a}{q}=\frac{1}{5}$	$\frac{a}{q}=\frac{3}{5}$	$\frac{a^2}{q}=\frac{3}{5}$	$\frac{a^2}{q}=\frac{m-9}{5}$	$\frac{\sqrt{a}}{q}=\frac{m-9}{5}$

Worked Examples

We have learnt how to change the subject of an equation when the equation is unknown. If the subject is negative then we must multiply both sides of the equation by -1. We can also divide both sides by -1 too. The result is the same if we divide by -1 or multiply by -1.

Here are the steps:

1. Circle the subject that we want to isolate
2. Identify what needs to be eliminated
3. Inverse the operation, and apply to both sides
4. If the subject is negative then multiply both sides by -1

Example 1:

Make ‘y’ the subject:

$$a-\boldsymbol{y}=x$$

2. ‘a’ needs to be eliminated, ‘a’ has been added to ‘y’.

3. Subtract ‘a’ on both sides of the equation

$$a-\boldsymbol{y}=x$$

$$-a \qquad -a$$

$$-\boldsymbol{y}=x-a$$

$$\times -1 \quad \times -1$$

$$\boldsymbol{y}=-x+a$$

‘y’ is the subject of the equation because it is isolated to one side of the equation.

Example 2:

Make ‘y’ the subject:

$$x=-\boldsymbol{y}-a$$

2. ‘a’ needs to be eliminated, ‘a’ has been subtracted from ‘y’.

3. Add ‘a’ on both sides of the equation

$$x = -\boldsymbol{y} - a$$

$$+a \qquad + a$$

$$x + a = -\boldsymbol{y}$$

$$\times -1 \quad \times -1$$

$$-x - a = y$$

I have written the equation in terms of ‘y’ where it is present on the left hand side of the equation, *by convention.*

<u>Example 3:</u>

Make ‘y’ the subject:

$$x = \frac{-\boldsymbol{y}}{a}$$

2. ‘a’ needs to be eliminated, ‘a’ is dividing ‘y’.

3. Multiply ‘a’ on both sides of the equation

$$x = \frac{-\boldsymbol{y}}{a}$$

$$\times a \quad \times a$$

$$ax = -y$$

$$\times -1 \quad \times -1$$

$$\boldsymbol{y} = -ax$$

I have written the equation in terms of ‘y’ where it is present on the left hand side of the equation, by convention.

<u>Example 4:</u>

Make ‘y’ the subject:

$$x = -\boldsymbol{y}a$$

2. ‘a’ needs to be eliminated, ‘a’ has been multiplied with‘y’.

3. Divide ‘a’ on both sides of the equation

$$x = -\boldsymbol{y}a$$

$$\div a \quad \div a$$

$$\frac{x}{a} = -y$$

$$\times -1 \quad \times -1$$

$$\boldsymbol{y} = -\frac{x}{a}$$

I have written the equation in terms of ‘y’ where it is present on the left hand side of the equation, by convention.

Example 5:

Make ‘y’ the subject:

$$x - a = -3\boldsymbol{y}$$

2. ‘3’ needs to be eliminated, ‘3’ has been multiplied with‘y’.

3. Divide ‘3’ on both sides of the equation

$$x - a = -3\boldsymbol{y}$$

$$\div 3 \quad \div 3$$

$$\frac{x-a}{3} = -y$$

$$\times -1 \quad \times -1$$

$$\boldsymbol{y} = -\frac{x-a}{3}$$

I have written the equation in terms of ‘y’ where it is present on the left hand side of the equation, by convention.

Example 6:

Make ‘y’ the subject:

$$-y + m = 3 + \boldsymbol{x^2}$$

2. ‘m’ needs to be eliminated, ‘m’ has been added to ‘y’.

3. Subtract ‘m’ on both sides of the equation

$$-y + m = 3 + \boldsymbol{x^2}$$

$$-m \quad -m$$

$$-y = 3 + \boldsymbol{x^2} - \boldsymbol{m}$$

$$\times -1 \quad \times -1$$

$$y = -3 - \boldsymbol{x^2} + \boldsymbol{m}$$

Mini-quiz

Make 'b' the subject of the equation					
(a)	(b)	(c)	(d)	(e)	(f)
$a - \boldsymbol{b} = x$	$x = -\boldsymbol{b} - a$	$x = \frac{-\boldsymbol{b}}{a}$	$x = -a\boldsymbol{b}$	$x - a = -7\boldsymbol{b}$	$-b - m = \boldsymbol{y^3 - 6}$

Practice Exercises

Question 1: Make 'x' the subject:

(a)	(b)	(c)	(d)
d - x = m	-x + d = m	d + a - x = m	a^2 - x = m
(e)	(f)	(g)	(h)
a^2 + m = -x	a^2 + m = -x + y	a^2 + m = -y - x	$3a^2$ - x + m = -y
(i)	(j)	(k)	(l)
$a = \frac{-x}{m}$	$m = \frac{-x}{a^2}$	$ab = \frac{-x}{a}$	$abc = \frac{-x}{4}$
(m)	(n)	(o)	(p)
-mx = q	-8x = p	-8x = P – 4	8 - x = P – 4

Question 2: Make 'y' the subject

(a)	(b)	(c)	(d)
a^2 + m = x – y	$m = \frac{-y}{a^2}$	8x = -py	$abc = \frac{-y}{4}$
(e)	(f)	(g)	(h)
a^2 - y = x	-my = P – 4	$a = \frac{-y}{m}$	$3a^2$ + x + m = -y
(i)	(j)	(k)	(l)
-my = q	a^2 + m = -y + x	$ab = \frac{-y}{a}$	9 - y = P – 4

Two step rearranging with negative unknowns

Recap

Make 'm' the subject of the equation					
(a)	(b)	(c)	(d)	(e)	(f)
$a - \boldsymbol{m} = x$	$x = -\boldsymbol{m} - a$	$x = \frac{-\boldsymbol{m}}{a}$	$x = -a\boldsymbol{m}$	$x - a = -7\boldsymbol{m}$	$-b - m = \boldsymbol{y^3 - 6}$

Worked Examples

We will now learn how to change the subject of an equation but where we have two steps to take to isolate the unknown, that becomes the subject of the equation.

We have learnt how to change the subject of an equation when the equation is unknown. If the subject is negative then we must multiply both sides of the equation by -1. We can also divide both sides by -1 too. The result is the same if we divide by -1 or multiply by -1.

Let's look at the steps that we apply to all problem types where it takes two steps to change the subject of an equation:

4. List the order of operations for the unknown
5. Inverse the order of operations in the right order
    a. Eliminate a term
    b. Isolate the unknown
6. Check that you have the correct subject
7. If the subject is negative then multiply both sides of the equation by -1.

Let's look at a few examples:

<u>Example 1:</u>

Make 'y' the subject of the equation

$$-5y + a = m$$

3. List the order of operations for the unknown.
   y → × 5 → +a → m
4. Inverse the order of operations in the right order
   m → -a → ÷5 →

$$-5y + a = m$$

$$-a \quad -a$$

$$-5y = m - a$$

$$\div 5 \qquad \div 5$$

$$-y = \frac{m-a}{5}$$

5. If the subject is negative then multiply both sides of the equation by -1

$$-y = \frac{m-a}{5}$$

$$\times -1 \qquad\qquad \times -1$$

$$y = -\frac{m-a}{5}$$

Example 2:

Make ‘y’ the subject of the equation

$$\frac{-y}{5} - a = m$$

3. List the order of operations for the unknown.
   y → ÷ 5 → -a → m
4. Inverse the order of operations in the right order
   m → +a → ×5 →

$$-\frac{y}{5} - a = m$$

$$+a \qquad +a$$

$$-\frac{y}{5} = m + a$$

$$\times 5 \qquad\qquad \times 5$$

$$-y = 5(m+a)$$

5. If the subject is negative then we multiply both sides of the equation by -1.

$$-y = 5(m+a)$$

$$\times -1 \qquad\qquad \times -1$$

$$y = -5(m+a)$$

It is important to note that we are multiplying the entire expression, ‘m+a’ by -5, we can expand the brackets to get an expression.

$$y = -5(m+a) = -5m - 5a$$

Either answer is correct.

Example 3:

Make ‘y’ the subject of the equation

$$-5 - \frac{y}{a} = m$$

3. List the order of operations for the unknown.
   -y → ÷ a → -5 → m
4. Inverse the order of operations in the right order
   m → +5 → ×a →

$$-5-\frac{y}{a}=m$$

$$+5 \qquad +5$$

$$-\frac{y}{a}=m+5$$

$$\times a \qquad \times a$$

$$-y=a(m+5)$$

5. If the subject is negative then we multiply both sides by -1.

$$-y=a(m+5)$$

$$\times -1 \qquad \times -1$$

$$y=-a(m+5)$$

It is important to note that we are multiplying the entire expression, 'm+5' by 'a', we can expand the brackets to get an expression.

$$y=-a(m+5)=\ -am-5a$$

Either answer is correct.

Mini-quiz

Make 'e' the subject				
(a)	(b)	(c)	(d)	(e)
$-5e+a=b$	$-5e+d=b$	$\frac{-e}{5}+a=b$	$\frac{-e-a}{5}=b$	$\frac{-e+m^2}{m}=5m$

Practice Exercises

Question 1: Make 'a' the subject

(a)	(b)	(c)	(d)
$-7a+b=9$	$-7a-b=9$	$-b-7a=9$	$-7a+c-b=9$
(e)	(f)	(g)	(h)
$-7a+\ m=11$	$-a^2+m=11$	$-7a^2+m=11$	$-7\sqrt{a}+m=11$

Question 2: Make x the subject

(a)	(b)	(c)	(d)
$\frac{-x}{3}+9=r$	$\frac{-x}{3}+a=r$	$\frac{-x}{3}-5=r$	$-5-\frac{x}{3}=m$

(e)	(f)	(g)	(h)
$p = -\frac{x}{3} - m$	$\frac{-x}{5} - p = r$	$\frac{-x}{5} - q = 10 + m$	$\frac{-x}{m} - 2 = m$
(i)	(j)	(k)	(l)
$\frac{-x^2}{3} + 9 = r$	$\frac{-\sqrt{x}}{3} + 9 = r$	$\frac{-x}{3} - a^2 = b^2$	$\frac{-x}{3} - a^2 = b^2 + 2ab$

Question 3: Make ‘x’ the subject

(a)	(b)	(c)	(d)
$-\frac{x+4}{3} = m$	$-\frac{x+4}{m} = m$	$-\frac{x-4}{m} = m$	$\frac{-4-x}{m} = m$
(e)	(f)	(g)	(h)
$11 = -\frac{x-n}{3}$	$-\frac{x-n}{5} = n$	$-\frac{x+m^2}{m} = 5m$	$-\frac{x-5abc}{a} = b$
(i)	(j)	(k)	(l)
$-\frac{7x+b}{3} = 9$	$9 = -\frac{7a-b}{3}$	$-\frac{-b+7a}{3} = 3$	$-\frac{7x+c-b}{3} = 3$
(m)	(n)	(o)	(p)
$1 = -\frac{7x+m}{11}$	$-\frac{7x^2+m}{11} = 1$	$1 = -\frac{7\sqrt{x}+m}{11}$	$-1 = -\frac{x^2+m}{11}$

Formulae with the unknown on both sides

Recap

Make 'd' the subject				
(a)	(b)	(c)	(d)	(e)
$-5d + a = b$	$-5d + p = b$	$\frac{-d}{5} + a = b$	$\frac{-e - d}{5} = b$	$\frac{-d + m^2}{m} = 5m$

Worked Examples

We will now learn how to change the subject of an equation but where we have two steps to take to isolate the unknown, that becomes the subject of the equation.

We have learnt how to change the subject of an equation when the equation is unknown. If the subject is negative then we must multiply both sides of the equation by -1. We can also divide both sides by -1 too. The result is the same if we divide by -1 or multiply by -1.

Let's look at the steps that we apply to all problem types where it takes two steps to change the subject of an equation:

1. List the order of operations for the unknown
2. Inverse the order of operations in the right order
   a. Eliminate a term
   b. Isolate the unknown
3. Check that you have the correct subject

Let's look at a few examples:

Example 1:

Make 'y' the subject of the equation

$$9(y - p) = 8(y + n)$$

1. Expand the brackets

$$9y - 9p = 8y + 8n$$

2. Rearrange the equation so all terms with 'y' are on the same side

$$9y - 9p = 8y + 8n$$

$$-8y \qquad -8y$$

$$\mathbf{9y - 8y} - 9p = 8n$$

$$y - 9p = 8n$$

3. List the order of operations for the unknown.

y → -9p → 8n

4. Inverse the order of operations in the right order

8n → +9p → y

$$y - 9p = 8n$$

$$+9p \quad + 9p$$

$$y = 8n + 9p$$

Example 2:

Make 'e' the subject of the equation

$$6ae + 9 = 7 + ke + ae$$

1. Rearrange the equation so all terms with 'e' are on the same side

$$6ae + 9 = 7 + ke + ae$$

$$-ae \quad - ke \qquad\qquad - ae \quad - ke$$

$$6ae - ke - ae + 9 = 7$$

2. Factorise 'e' from all the terms

$$e(6a - k - a) + 9 = 7$$

$$e(5a - k) + 9 = 7$$

6. List the order of operations for the unknown.

e → $\times\,(5a - k)$ → +9 → 7

7. Inverse the order of operations in the right order

7 → -9 → $\div\,(5a - k)$ → e

$$e(5a - k) + 9 = 7$$

$$-9 \quad - 9$$

$$e(5a - k) = -2$$

$$\div\,(5a - k) \qquad\qquad \div\,(5a - k)$$

$$e = \frac{-2}{(5a - k)}$$

Example 3:

Make 'e' the subject of the equation

$$y = \frac{7e + 1}{9e - 5}$$

1. When we have a fraction, we must eliminate the denominator by multiplying both sides by the denominator

$$y = \frac{7e+1}{9e-5}$$

$$\times (9e-5) \qquad \qquad \times (9e-5)$$

$$y(9e-5) = 7e+1$$

2. Expand the brackets

$$y(9e-5) = 7e+1$$

$$9ey - 5y = 7e+1$$

3. Rearrange the equation so all terms with ‘e’ are on the same side

$$9ey - 5y = 7e + 1$$
$$-7e \qquad \qquad -7e$$
$$9ey - 7e - 5y = 1$$
$$+5y \qquad \qquad +5y$$
$$9ey - 7e = 1 + 5y$$

4. Factorise ‘e’

$$e(9y-7) = 1+5y$$

5. List the order of operations for the unknown.
   e → $\times\,(9y-7)$ → $(1+5y)$
6. Inverse the order of operations in the right order
   $(1+5y)$ → $\div\,(9y-7)$ → e

$$e(9y-7) = 1+5y$$

$$\div (9y-7) \qquad \qquad \div (9y-7)$$

$$e = \frac{1+5y}{(9y-7)}$$

Mini-quiz

Make ‘e’ the subject		
(a)	(b)	(c)
$7(e+a) = n(\,e+b)$	$2ae + 9 = 7 + ke + ae$	$y = \dfrac{2e+1}{4e-5}$

Practice Exercises

Question 1: Make ‘y’ the subject

(a)	(b)	(c)	(d)
$6(y-p) = 2(y+m)$	$9(y-p) = 3(y+n)$	$9(y-p) = 8(y+n)$	$7(p-y) = 8(y+n) + y$
(e)	(f)	(g)	(h)
$6(y-p) = 3(y+x)$	$7(y+a) = n(\,y+b)$	$4(y-b) = y+d$	$8(a+y) = 3(y+b-a)$
(i)	(j)	(k)	(l)

$my + 5 = 6 - ky$	$ay + 9 = 7 + ky$	$ab - p = z + by$	$2ay + 9 = 7 + ky + ay$
(m)	(n)	(o)	(p)
$db + 3 = m + eb$	$db + 3 = 3 + m + eb$	$by + 9 = 9y - q$	$by + 9 = 9y - q - 4y$

Question 2:

Make ‘x’ the subject		
(a)	(b)	(c)
$y = \dfrac{2x - 1}{4x + 5}$	$y = \dfrac{2x + 1}{4x + 5}$	$y = \dfrac{2x + 1}{4x - 5}$

Question 3:

Make ‘y’ the subject		
(d)	(e)	(f)
$b = \dfrac{a - y}{a + y}$	$b = \dfrac{m - y}{m + y}$	$b = \dfrac{2m - y}{2m + y}$
(g)	(h)	(i)
$4 = \dfrac{2m - y}{2m + y}$	$y - a = \dfrac{y + 2}{a}$	$y = \dfrac{n^2 + 2}{n - 1}$
(j)	(k)	(l)
$\sqrt{\left(\dfrac{y + x}{y - x}\right)} = 2$	$\sqrt{\left(\dfrac{z + y}{z - y}\right)} = \dfrac{1}{3}$	$\sqrt{\left[\dfrac{m(y + n)}{y}\right]} = p$

Question 4:

Make ‘p’ the subject		
(a)	(b)	(c)
$7(p + a) = n(p + b)$	$2ap + 9 = 7 + kp + ap$	$y = \dfrac{2p + 1}{4p - 5}$