SS4 • Evaluating statements about length and area

Mathematical goals

To help learners to:

- understand concepts of length and area in more depth;
- revise the names of plane shapes;
- develop reasoning through considering areas of plane compound shapes;
- construct their own examples and counter-examples to help justify or refute conjectures.

Starting points

Learners should have already encountered the concepts of area and perimeter before beginning this activity. This activity will consolidate their understanding and overcome some common misconceptions, such as the notion that perimeter and area are in some way interrelated.

Learners are given a number of mathematical statements on cards. They have to try to justify or refute these statements using their own examples and arguments. They create posters showing their collaborative reasoning.

Materials required

For each small group of learners you will need:

- Card set A – *Statements*;
- Card set B – *Hints*;
- blank A4 sheets on which to make mini-posters;
- glue stick.

Time needed

This will depend on the number of *Statements* cards you give to each learner. One hour is probably the minimum time needed.

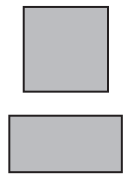
Suggested approach

Beginning the session

This activity is best introduced with a whole group discussion.

Choose one of the statements from Card set A and write it on the board:

> If a square and a rectangle have the same area, the square has the smaller perimeter.



Demonstrate the process that learners should adopt when tackling this activity, by working through this example together.

Step 1. Understand the problem

Explain that the learners have to decide whether the statement is always true, sometimes true, or never true. What does this mean? Well, we need to decide whether the statement is true or not for all possible squares and rectangles that have the same area.

What does the word 'area' mean? How do we calculate it?
What does the word 'perimeter' mean? How do we calculate it?

Step 2. Try some examples

Give me some dimensions for a square.	(6 cm × 6 cm)
What is the area of that square?	(36 cm^2)
What is its perimeter?	(24 cm)
Now give me some dimensions for a rectangle with the same area as the square.	(12 cm × 3 cm)
What is the perimeter of the rectangle?	(30 cm)
So is the statement true for this example?	(Yes)
Can you give me a different rectangle with the same area?	(9 cm × 4 cm)
What is the perimeter of the rectangle?	(26 cm)
So is the statement true for this example?	(Yes)

In this way, help learners to generate examples and then to offer conjectures.

Step 3. Make conjectures

Learners may begin to notice, for example, that, for a given area, as the dimensions of the rectangle become more nearly equal, so the perimeter reduces. This might lead to the conjecture that the square has the smallest perimeter for a given area. Thus the statement appears to be always true.

Step 4. Try to disprove or justify the conjectures

Can we see why the statement must be true? This statement might be too difficult for learners to prove algebraically, but they may be able to test particular cases in an organised way. So, when the area of the rectangle is 36 cm^2, possible perimeters are:

One side of the rectangle	1	2	3	4	5	**6**	7	8	9	10	11	12
Other side (to 1 d.p.)	36	18	12	9	7.2	**6**	5.1	4.5	4	3.6	3.3	3
Perimeter (to 1 d.p.)	74	40	30	26	24.4	**24**	24.2	25	26	27.2	28.6	30

This table (which might be supported by a graph) suggests that the statement is always true, except when the rectangle takes the same shape as the square (i.e. 6×6) when of course their areas are equal.

Working in groups

Ask learners to work in pairs. Give Card set A – *Statements* to each pair and ask them to choose a statement to work on.

Learners who you think may struggle should be guided towards one of the earlier statements.

Learners should try to decide whether their chosen statement is always, sometimes or never true, using the process described above. Remind them of the need to test specific examples in an organised way. In many cases they may find it possible to prove their ideas using reasoning that doesn't depend on numbers at all.

Learners should choose a statement, stick it in the middle of a sheet of A4 paper and write their reasoning around it. If a different A4 poster is made for each statement, these can be gathered and displayed for later discussion and critical comment.

If a pair of learners get stuck, offer the appropriate card from Set B – *Hints*.

When two pairs have completed the same problem, ask them to exchange posters and comment on the reasoning of the other group.

Reviewing and extending learning

Invite pairs of learners to describe one problem that they think they have solved successfully and the reasoning that they employed. Then ask other pairs who have solved the same problem to show their reasoning. Ask the remaining learners to say which reasoning they found most clear and convincing and why.

What learners might do next

Session **SS5 Evaluating statements about enlargement** is a good follow-up to this session. It develops the ideas of perimeter and area and includes some discussion of 3D solids.

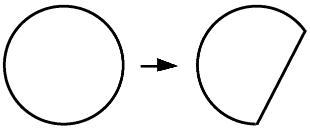
Further ideas

This activity is about examining a mathematical statement and deciding on its truth or falsehood. This idea may be used in many other topics and levels. Examples in the pack include:

- **N2** **Evaluating statements about number operations;**
- **A4** **Evaluating algebraic expressions;**
- **S2** **Evaluating probability statements.**

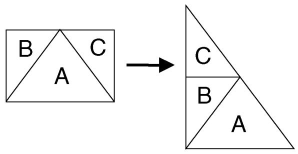
SS4 Card set A – *Statements*

A



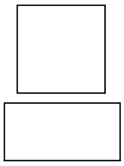
When you cut a piece off a shape you reduce its area and perimeter.

B



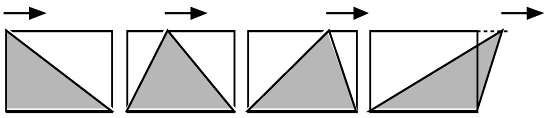
When you cut a shape and rearrange the pieces, the area and perimeter stay the same.

C



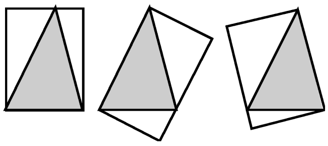
If a square and a rectangle have the same perimeter, the square has the smaller area.

D



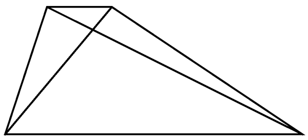
Slide the top corner of a triangle from left to right. The area of the triangle stays the same.

E



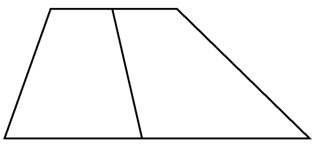
Draw a triangle. There are three ways of drawing a rectangle so that it passes through all three vertices and shares an edge with the triangle. The areas of the three rectangles are equal.

F



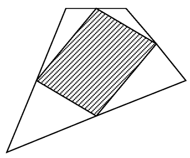
Draw a trapezium and draw its diagonals. The shape is now split into four triangles. Exactly two of these triangles are equal in area.

G



If you join the mid points of the opposite sides of a trapezium, you split the trapezium into two equal areas.

H

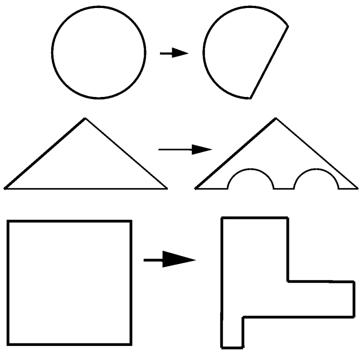


If you join the mid points of the sides of a quadrilateral, you get a parallelogram with one half the area of the original quadrilateral.

SS4 Card set B – *Hints*

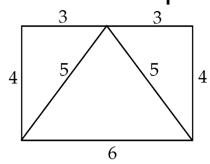
A

What happens to the area and perimeter with these cuts?



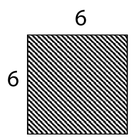
B

Draw a 6 cm by 4 cm rectangle and cut it into three pieces.



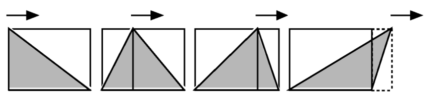
What are the area and the perimeter?
Use all three pieces to make each of the following shapes: *Rhombus, Trapezium, Parallelogram, Pentagon, Kite, Hexagon.*
Write down the area and the perimeter of each shape.
What areas and perimeters are possible?

C



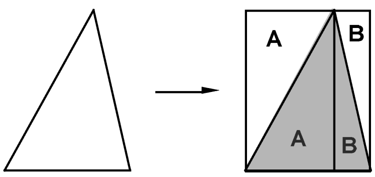
Draw some rectangles with the same perimeter as this square.
What is the area in each case?
Does the square have the smaller area?
Will this be true when you start with different squares?

D



Divide the shaded triangle into smaller triangles as shown.
What fraction of the rectangle is the shaded area in each case?
Will this work in extreme cases?

E



What fraction of each rectangle is the triangle?
What happens when the triangle contains an obtuse angle?

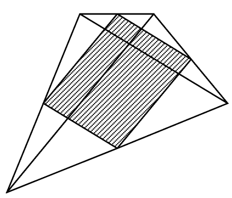
F

Begin by looking for triangles with a common base and the same height.
Try this for different trapezia.
Does it always work?
Can you get more than two equal areas? When?

G

Divide the diagram into triangles.
Can you see why some have the same base and height?
Try this for different trapezia.
Does it always work?
Can you get more than two equal areas? When?

H



Draw the diagonals of the quadrilateral.