O1 • Moving from Eulerian graphs to the route inspection (Chinese postman) problem

Mathematical goals

To enable learners to:

- distinguish, by drawing, between Eulerian graphs, semi-Eulerian graphs and graphs that are neither;
- distinguish, by using the order of the vertices, between Eulerian graphs, semi-Eulerian graphs and graphs that are neither;
- find strategies for solving the route inspection or 'Chinese postman' problem.

Starting points

Learners should have some knowledge of what is meant by a graph, a vertex and an edge in the context of decision mathematics.

Materials required

For each learner you will need:

- mini-whiteboard.

For each small group of learners you will need:

- Card set A – *Graphs*;
- Card set B – *Properties*;
- Card set C – *Completing the route*;
- glue stick;
- large sheet of paper for making a poster.

Time needed

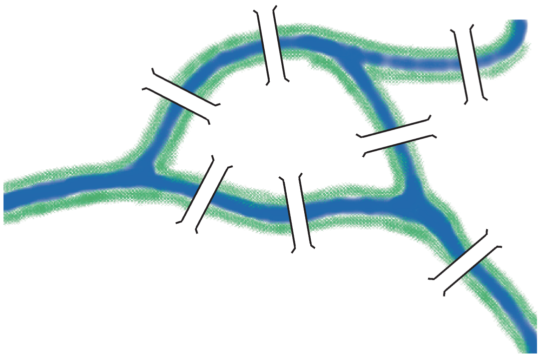
At least 1 hour 30 minutes.

Suggested approach

Beginning the session

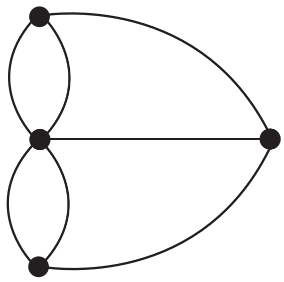
Tell the story of the bridges of Königsberg. Draw Figure 1 on the board.

Figure 1



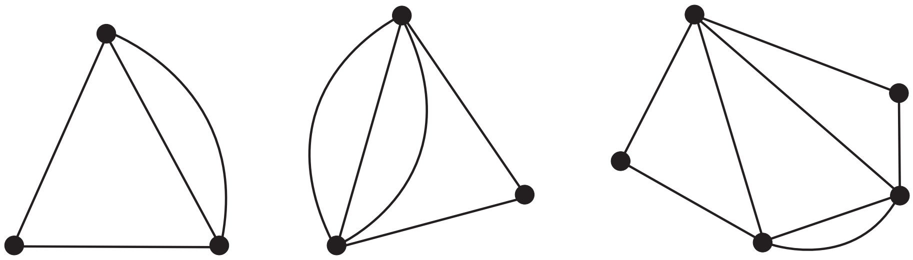
"In the town of Königsberg in the eighteenth century there was a river with an island and seven bridges, as shown in Figure 1. It is said that on Sunday afternoons the people of the town would try to walk across every bridge only once and end up back where they started from. Can it be done?"

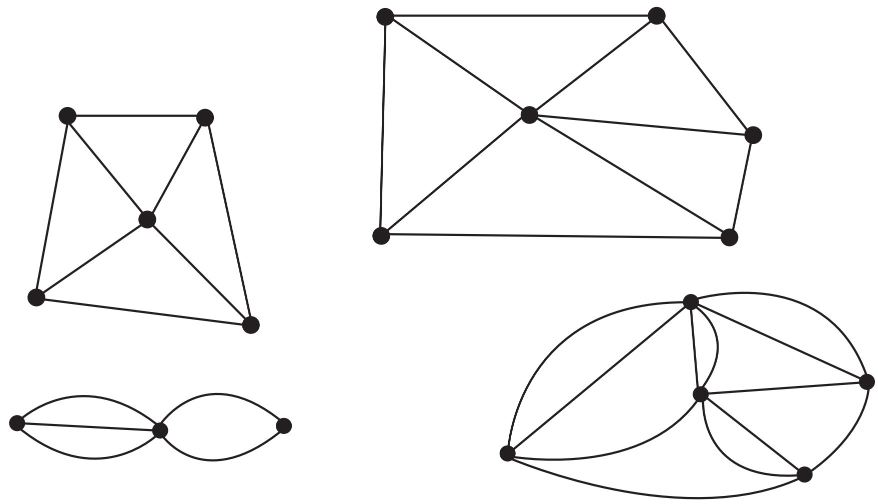
It is helpful to represent the problem as a graph. The vertices represent land and the edges represent the bridges.



Give out mini-whiteboards and ask learners to try to solve the problem for themselves. They should find that it is impossible. Tell them that Euler, a Swiss mathematician, eventually proved that it was impossible – but how did he do it?

Draw some simple graphs on the board, e.g.





Ask learners to decide which graphs can be drawn without going along any edge more than once and without lifting the pencil from the paper.

Next, ask them to divide the graphs that can be drawn in this way into two categories:

- those that start and finish at the same vertex;
- those that have to start and finish at different vertices.

You will need mini-whiteboards or lots of scrap paper so that learners can be encouraged to try different ways.

Whole group discussion

Come to a consensus about which graphs fit into each category. If there are any disagreements, ask learners to demonstrate on the board until a consensus is reached.

Explain that those that start and finish at the same vertex are called 'Eulerian graphs' and those that start and finish at different vertices are called 'semi-Eulerian graphs'. The remaining graphs are called 'non-Eulerian'. Also, explain that the number of edges leaving a vertex is called the 'order' of the vertex.

Ask learners to discuss, in small groups, what properties the vertices have to have in order for a graph to be assigned to each category. Help learners to reach the conclusion that all the vertices on Eulerian graphs have to be of even order, while two of the vertices on semi-Eulerian graphs have to be odd and the rest even. Encourage learners to explain why this must be so. Go back to the bridges of Königsberg and ask for an explanation as to why the townspeople's task is impossible.

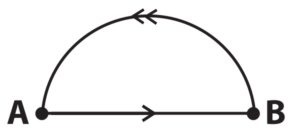
Working in groups

Ask learners to work in pairs. Give out a large sheet of paper and a glue stick to each pair. Ask them to divide the sheet into four. Give each pair Card set A – *Graph*s and ask them to stick one graph into each of the four sections. Then give out Card set B – *Properties* and Card set C – *Completing the route*. Ask them to stick an appropriate Properties card into each section. Learners should write out the reason as well as listing the odd vertices.

Explain the route inspection (Chinese postman) problem.

> Find a route that travels along all the edges. You may go down an edge more than once but you must keep the total distance travelled to a minimum.

You may need to remind learners that they should show any edges travelled more than once by adding additional edges to the diagram. Thus, travelling from A to B and then B to A would be shown as:



For each problem they should record what their solution is on one of the *Completing the route* cards and stick it on to the sheet next to its graph. At this stage they should be solving the problem using their own strategies.

Compare results and see which pair managed to obtain the shortest route for each graph. If none got the shortest possible on any of the graphs, challenge them to find a shorter one. Discuss strategies and come up with the rules for the Chinese postman algorithm.

Reviewing and extending the learning

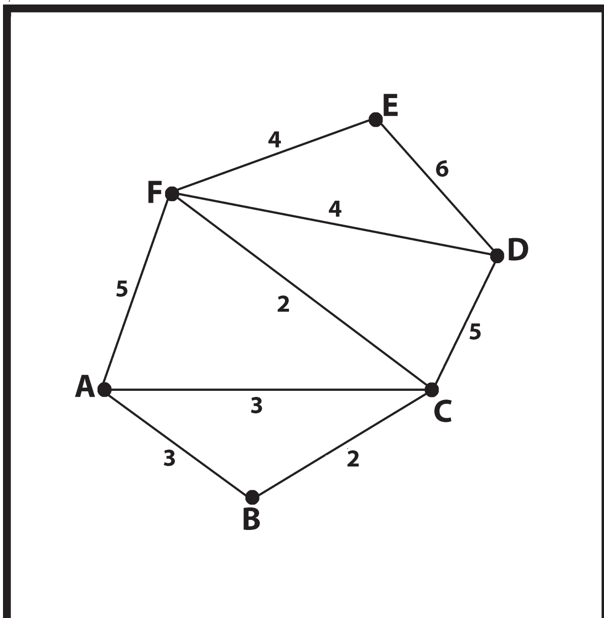
Ask each pair of learners to draw a graph of their own such that four of the vertices have an odd order. Ask them to write out the rules alongside a solution for their graph. When they have finished, they should give it to another pair for checking and comment.

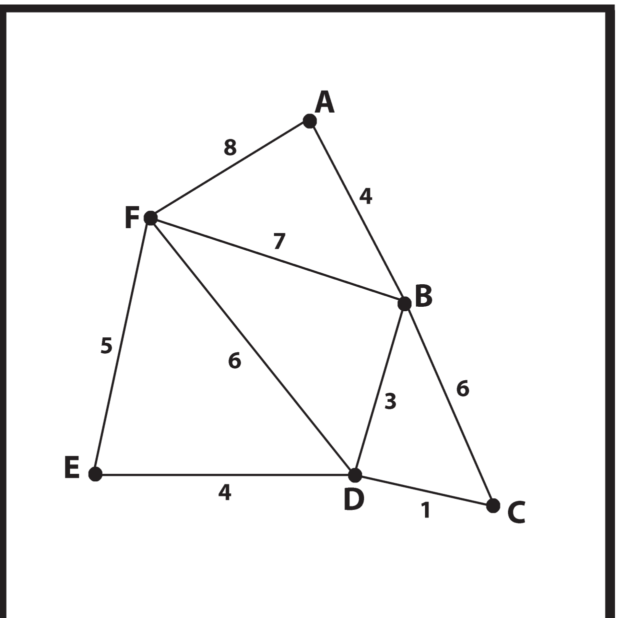
What learners might do next

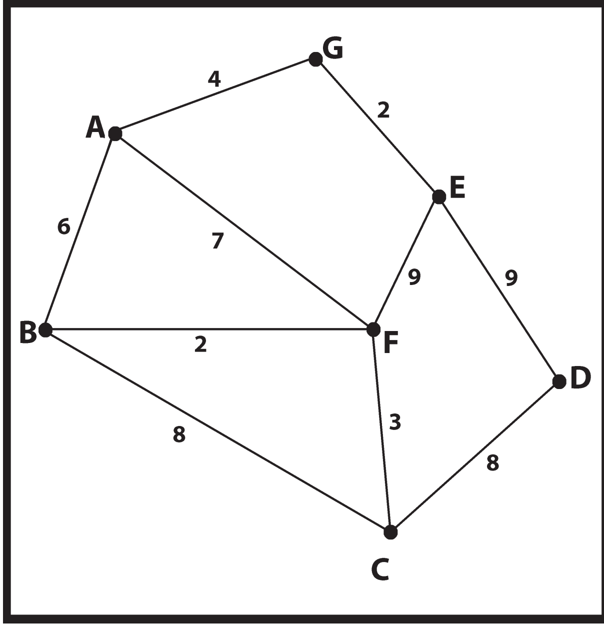
Explore how many possible pairings there are for 6, 8, 10 . . . n vertices that have an odd order.

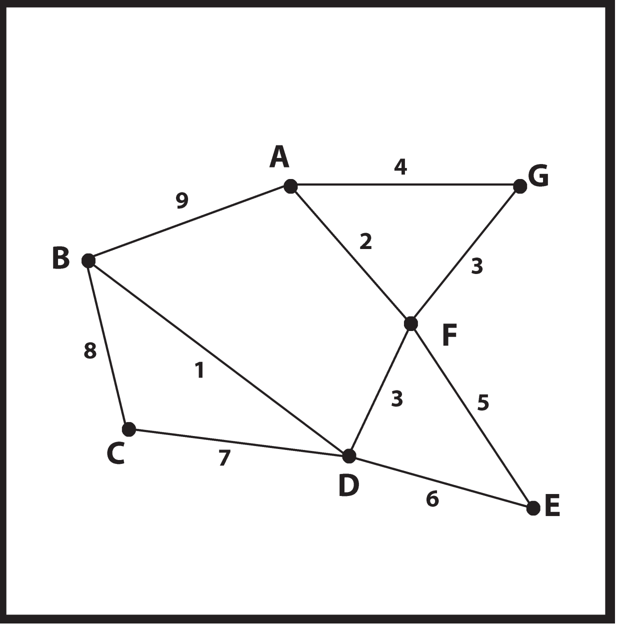
Investigate whether it is possible to have a graph that has an odd number of vertices that have an odd order.

O1 Card set A – *Graphs*









O1 Card set B – *Properties*

This is an Eulerian graph because… The odd vertices are:	This is a semi-Eulerian graph because… The odd vertices are:
This is a non-Eulerian graph because… The odd vertices are:	This is a semi-Eulerian graph because… The odd vertices are:

O1 Card set C – *Completing the route*

Extra edge(s) are: Possible route: Total length of route:	Extra edge(s) are: Possible route: Total length of route:
Extra edge(s) are: Possible route: Total length of route:	Extra edge(s) are: Possible route: Total length of route: