C5 • Finding stationary points of cubic functions

Mathematical goals

To enable learners to:

- find the stationary points of a cubic function;
- determine the nature of these stationary points;

and to discuss and understand these processes.

Starting points

Learners should have some knowledge of:

- differentiation of polynomials;
- finding stationary points of a quadratic function;
- using $\frac{d^2y}{dx^2}$ or $f''(x)$ to determine their nature.

Materials required

For each learner you will need:

- mini-whiteboard;
- Sheet 1 – *Matching*.

For each small group of learners you will need:

- Card set A – *Stationary points*;
- large sheet of paper for making a poster;
- glue stick;
- felt tip pens.

Time needed

At least 1 hour.

Suggested approach

Beginning the session

Use mini-whiteboards and whole group questioning to revise differentiation of polynomials and stationary points of a quadratic function by asking a range of questions such as:

If $y = 4x^2 + 5x - 1$ what is $\frac{dy}{dx}$?

If $y = x^3 - 8x^2 + 6x - 1$ what is $\frac{dy}{dx}$?

Find the x coordinate of the stationary point of $y = x^2 - 8x + 3$.

How do you know whether it is a maximum or a minimum?

Explain that, to find the stationary points on a cubic graph, exactly the same method is used. You may like to encourage learners to think about the likely shape of the graph.

Working in groups (1)

Ask learners to work in pairs.

Give out Card set A – *Stationary points* and ask learners to sort the cards into functions. Explain that the cards include three cubic functions and all the steps required to find their stationary points and to determine the nature of those stationary points. Learners have to sort the cards/steps into an appropriate order and stick them onto the poster paper in three columns, one for each function.

Learners who struggle can be given just one or two functions to sort.

Learners who find the task easy could be asked to find examples of cubic functions that have only one stationary point or no stationary points. You should encourage them to relate their answers to the derived quadratic functions.

Whole group discussion

When all learners have completed at least one function, discuss the order in which they have stuck down the steps. Ask learners to compare their orders with pairs sitting near them and to report any differences. Discuss how the orders can be checked and which orders are logically correct.

Working in groups (2)

Ask learners to write a comment beside each step of one of their functions, explaining its purpose. Finally, ask them to sketch a graph of the function, marking the stationary points.

Reviewing and extending the learning

Give individual learners Sheet 1 – *Matching* and ask them to match the equations to the graphs, justifying their choice as fully as they can. You may wish to ask them to match only two equations and graphs.

What learners might do next

Learners could be asked to explore and then sketch a given cubic function that factorises. This would connect work using the factor theorem with this session on stationary points.

Further ideas

This approach could be used to find stationary points of other types of functions (e.g. functions with fractional or negative indices).

This way of sorting the order of a solution can be used for solving multi-stage problems on any topic.

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C5 Card set A – *Stationary points* (page 1)

$y = x^3 - 4x^2 + 5x + 11$	$y = x^3 - x^2 - x + 5$
$y = x^3 - 7x^2 - 5x + 9$	$3x^2 - 8x + 5 = 0$
$\frac{d^2y}{dx^2} = 6x - 8$	$\frac{dy}{dx} = 3x^2 - 14x - 5$
$x = -\frac{1}{3},\ \frac{d^2y}{dx^2} = \ldots$	$x = \frac{5}{3},\ x = 1$
$\frac{dy}{dx} = 3x^2 - 2x - 1$	$\frac{d^2y}{dx^2} = 6x - 2$
$x = 1,\ \frac{d^2y}{dx^2} = \ldots$	$x = -\frac{1}{3},\ x = 5$
$(3x + 1)(x - 5) = 0$	$x = 1,\ \frac{d^2y}{dx^2} = \ldots$
$3x^2 - 14x - 5 = 0$	$3x^2 - 2x - 1 = 0$

C5 Card set A – *Stationary points* (page 2)

$(3x - 5)(x - 1) = 0$	$(3x + 1)(x - 1) = 0$
$x = 5, \quad \frac{d^2y}{dx^2} = \ldots$	$\frac{dy}{dx} = 3x^2 - 8x + 5$
$x = -\frac{1}{3}, \quad \frac{d^2y}{dx^2} = \ldots$	$x = -\frac{1}{3}, \quad x = 1$
$\frac{d^2y}{dx^2} = 6x - 14$	$x = \frac{5}{3}, \quad \frac{d^2y}{dx^2} = \ldots$
Maximum is at	**Minimum is at . . . .**
Minimum is at	**Maximum is at . . . .**
Maximum is at	**Minimum is at . . . .**

C5 Sheet 1 – *Matching*

Name:

Match these equations to their graph. Give as much evidence as you can to justify your matchings. The graphs are not drawn to scale.

Equation 1

$$y = x^3 + 2x^2 - 5x + 12$$

Equation 2

$$y = x^3 - 9x^2 + 22x - 10$$

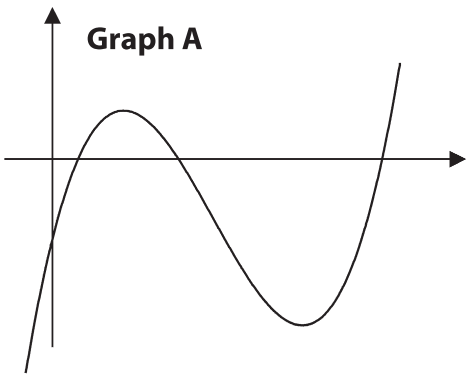
Equation 3

$$y = x^3 + 7x^2 + 15x + 12$$

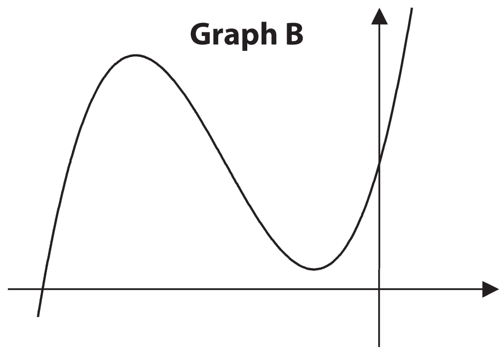
Equation 4

$$y = x^3 - 2x^2 - 13x - 10$$

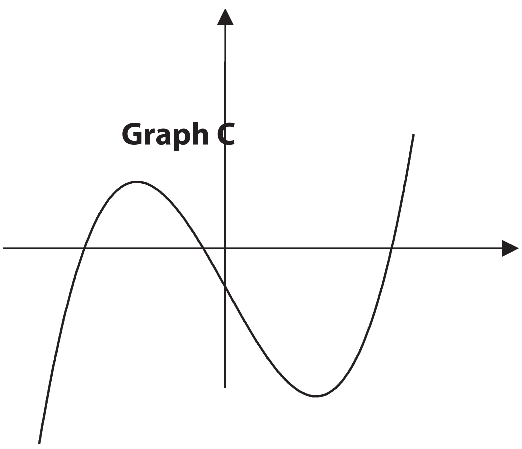
Graph A



Graph B



Graph C



Graph D