C1 • Linking the properties and forms of quadratic functions

Mathematical goals

To enable learners to:

- identify different forms and properties of quadratic functions;
- connect quadratic functions with their graphs and properties, including intersections with axes, maxima and minima.

For learners working towards a lower level qualification, you can focus on the graphs and their intersections with the axes.

Starting points

Learners will need to be familiar with the following forms of quadratic functions and their interpretation:
$y = ax^2 + bx + c$,
$y = (x + a)(x + b)$,
$y = a(x + b)^2 + c$.

Learners working towards a lower level qualification will not need to know the form $y = a(x + b)^2 + c$

Materials required

For each learner you will need:

- mini-whiteboard.

For each small group of learners you will need:

- Card set A – *Properties of quadratic graphs* (two pages);
- Card set B – *Graphs of quadratic functions*;
- large sheet of paper for making a poster;
- glue stick;
- felt tip pens;

and possibly

- access to computer with graph-drawing software;
- graphic calculators.

Time needed

At least 1 hour.

Suggested approach

Beginning the session

Explain that there are seven sets of cards. Each set is made up of a quadratic function written in different forms, some associated properties, and a graph.

Learners working towards a lower level qualification should leave out cards in the form $y = a(x + b)^2 + c$ and those that refer to maxima or minima.

Working in groups

Ask learners to work in pairs. Issue Card set A and Card set B to each pair. Invite each pair to match cards into the seven sets. As they do this, encourage them to explain their reasoning both to you, as you move round the room, and to each other.

Learners may like to create posters by sticking a set of cards onto a large sheet of paper and writing their reasoning around each card.

If some learners are likely to find the task easy, you may wish to give them reduced sets of cards and ask them to create the missing ones. For example, give them Card set A without the y =. . . cards and ask them to create their own.

Learners who struggle may be helped by using some graph-drawing software or graphic calculators. They could focus on completing four or five sets rather than all seven.

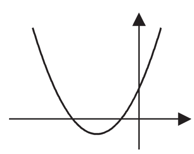
Learners who finish quickly may enjoy devising their own sets of cards at a more challenging level, e.g. when the coefficient of x^2 is not ±1.

Reviewing and extending learning

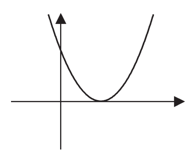
Ask a representative from each pair of learners to come to the board and explain how they decided that a set of cards belonged together.

Ask learners questions using mini-whiteboards. For example:

Give me a possible equation for this graph:



Give me a possible equation for this graph:



What are the x and y intercepts of $y = (x + 3)(x – 2)$? How can you tell?

What are the x and y intercepts of $y = (x + 3)^2$? How can you tell?

Show me the equation of a quadratic that intersects the y axis at –10. Now show me the same equation in a different form.

Show me the equation of a quadratic that intersects the x axis at –5 and +7. Now show me the same equation in a different form.

Where are the intercepts and where is the minimum of the function $y = (x – 4)^2 – 9$. How can you tell?

Show me the equation of a quadratic function with a minimum at (4, 6). Now show me the same equation in a different form.

Ask learners to produce generalisations of these results, such as:

Show me the equation of a quadratic with a minimum at (a, b). Now show me the same equation in a different form.

Learners working towards a lower level qualification should focus on the earlier questions.

Show me the equation of a quadratic with x intercepts at $(a, 0)$ and $(b, 0)$. Now show me the same equation in a different form.

What learners might do next

Learners could be asked to produce a graph from a given quadratic function, with intercepts and stationary points marked. Different levels of challenge could be available for learners to choose from, e.g. quadratics with a coefficient of x^2 that is not ±1.

Completed square form could be linked to translations of the graphs.

Finding stationary points using calculus could be introduced and linked with the completed square form of a quadratic function, to show that they both give the same result.

Further ideas

This idea could be extended and used with other types of functions, e.g. linear or trigonometric.

This session could be used to introduce the idea of quadratic functions. Learners would need access to a computer with graph-drawing software or a graphic calculator. They could use the computer or calculator to match up the properties with their functions and then work out for themselves how the different forms and properties link together.

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C1 Card set A – *Properties of quadratic graphs* (page 1)

$y = x^2 + 6x - 16$	$y = x^2 - 8x + 16$
$y = 8 - x^2 + 2x$	$y = 6x - x^2 - 8$
$y = x^2 - 10x + 16$	$y = x^2 + 6x + 8$
$y = x^2 - 6x - 16$	$y = (x - 8)(x + 2)$
$y = (x + 4)(x + 2)$	$y = (x + 2)(4 - x)$
$y = (x - 4)(2 - x)$	$y = (x - 8)(x - 2)$
$y = (x - 4)(x - 4)$	$y = (x + 8)(x - 2)$
$y = (x + 3)^2 - 25$	$y = (x - 4)^2$
$y = (x - 5)^2 - 9$	$y = -(x - 3)^2 + 1$
$y = -(x - 1)^2 + 9$	$y = (x + 3)^2 - 1$
$y = (x - 3)^2 - 25$	**Minimum at (3, –25)**
Minimum at (–3, –1)	**Maximum at (1, 9)**

C1 Card set A – *Properties of quadratic graphs* (page 2)

Maximum at (3, 1)	**Minimum at (5, –9)**
Minimum at (4, 0)	**Minimum at (–3, –25)**
$x = 0, y = -16$	$x = 0, y = 16$
$x = 0, y = 16$	$x = 0, y = -8$
$x = 0, y = 8$	$x = 0, y = 8$
$x = 0, y = -16$	$y = 0, x = 8$ or -2
$y = 0, x = -4$ or -2	$y = 0, x = -2$ or 4
$y = 0, x = 4$ or 2	$y = 0, x = 8$ or 2
$y = 0, x = 4$	$y = 0, x = -8$ or 2

C1 Card set B – *Graphs of quadratic functions*