

# A3 • Creating and solving harder equations

## Mathematical goals

To enable learners to:

- create and solve equations, where the unknown appears more than once.

To help learners to:

- recognise that there may be more than one way of solving such equations;
- teach and learn from each other.

## Starting points

Before embarking on this session, we strongly suggest that you use **A2 Creating and solving equations**. This will ensure that learners are able to solve equations where the unknown appears only once.

## Materials required

For each learner you will need:

- Sheet 1 – *Creating equations*;
- Sheet 2 – *Solving an equation in different ways*.

It is helpful to make OHTs of Sheets 1 and 2 for use in the whole group introduction.

Optional:

- computer programs *Balance 1*, *Balance 2* and *Cover up*;
- Sheet 3 – *Instructions for using the program Balance 1*;
- Sheet 4 – *Instructions for using the program Balance 2*;
- Sheet 5 – *Instructions for using the program Cover up*.

## Time needed

About 1 hour.

## Suggested approach Beginning the session

### 1. Build an equation

Write a letter and its value (e.g.  $x = 2$ ) on the board. You may wish to use an OHT of Sheet 1 – *Creating equations*.

Build up an equation, step by step, as in **A2 Creating and solving equations**. This time, however, include a step that uses the letter itself. For example, you might add  $3x$  to both sides:

$$\begin{array}{l}
 x = 2 \\
 \text{Add } 3x \quad \curvearrowright \\
 4x = 2 + 3x \\
 \text{Add } 6 \quad \curvearrowright \\
 4x + 6 = 8 + 3x
 \end{array}$$

### 2. Check the equation

Ask learners to check that the original value for  $x$  still fits the final equation.

Thus:  $4 \times 2 + 6 = 14$ ;  $8 + 3 \times 2 = 14$ .

### 3. Solve the equation

Now, starting with  $4x + 6 = 8 + 3x$ , ask how can we go back and discover the value of  $x$ . Ask learners to suggest alternative methods and explore each one. Follow up dead ends as necessary and ask them to explain why they do not lead to a solution.

For example:

$$\begin{array}{ll}
 \text{Subtract } 6 \quad \curvearrowright & 4x + 6 = 8 + 3x \\
 & 4x = 2 + 3x \\
 \text{Subtract } 3x \quad \curvearrowright & \\
 & x = 2
 \end{array}
 \qquad
 \begin{array}{ll}
 \text{Subtract } 3x \quad \curvearrowright & 4x + 6 = 8 + 3x \\
 & x + 6 = 8 \\
 \text{Subtract } 6 \quad \curvearrowright & \\
 & x = 2
 \end{array}$$

$4x + 6 = 8 + 3x$ <p>Subtract 8</p> $4x - 2 = 3x$ <p>Subtract <math>3x</math></p> $x - 2 = 0$ <p>Add 2</p> $x = 2$	$4x + 6 = 8 + 3x$ <p>Subtract <math>4x</math></p> $6 = 8 - x$ <p>Subtract 8</p> $-2 = -x$ <p>Multiply by <math>-1</math></p> $2 = x$
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#### 4. Create your own equation

Give each learner Sheet 1 and Sheet 2 – *Solving an equation in different ways*.

Ask learners to use Sheet 1 to:

- build an equation, operating on both sides with the unknown at some stage;
- check the equation, by substitution;
- solve the equation using a different method from the one used to create it.

### Working in groups

Learners should now copy their equations onto Sheet 2 and give this sheet to their partner. The partner should try to solve the equation in different ways. The originators should try to help the partners when they get stuck.

Encourage learners to help each other as much as possible. As learners get the idea, the structured Sheets 1 and 2 can be discarded and learners may enjoy creating more challenging equations.

This activity enables learners at all levels to create equations of varying degrees of difficulty. However, all learners should be encouraged to be adventurous.

## Reviewing and extending learning

Ask learners to write their 'favourite creations' on the board and invite other learners to solve them. See if several ways can be found for solving each equation.

Ask learners to use their mini-whiteboards to write down algebraic equations that correspond to some 'think of a number' problems. This time, include some that involve 'the number you first thought of'.

Think of a number, call it  $n$ .

Multiply it by 7.

Add 4.

Subtract the number you first thought of.

The result is 34.

Show me the equation.

Now solve it.

## What learners might do next

Further, individual practice at solving equations may be done using the computer software programs *Balance 1*, *Balance 2* and *Cover up* (available on the DVD-ROM/CD).

*Balance 1* asks learners to say what operations they must do in order to solve an equation. The computer then carries out these operations. Learners do the strategic thinking, while the computer does the manipulation.

*Balance 2* is the same as *Balance 1*, except that learners have to identify appropriate operations to perform, and then carry out these operations themselves.

*Cover up* uses a different strategy for tackling equations. At each step, the learner 'covers up' part of the equation and reasons what number must be hidden.

All three programs also allow the user to create equations.

Instructions for using these programs are given in Sheets 3, 4 and 5.

Learners may like to create examples where both sides of the equation are multiplied or divided by some expression containing the unknown.

For example:

Diagram illustrating two paths of algebraic manipulation starting from  $x = 4$ :

Left path:

- Start:  $x = 4$
- Operation: Divide by  $x$  →  $1 = \frac{4}{x}$
- Operation: Add 8 →  $9 = \frac{4}{x} + 8$

Right path:

- Start:  $x = 4$
- Operation: Subtract 5 →  $x - 5 = -1$
- Operation: Square →  $(x - 5)^2 = 1$

When unpicking the second equation, learners should notice that the natural thing to do is to square root both sides and obtain the positive root, thus obtaining  $x = 6$  as a solution. Check that  $x = 4$  and  $x = 6$  both satisfy the equation. Point out that squaring may introduce a second solution because, when we square root, we can either follow the positive or negative root. We have to do both to find all the solutions.

You may find that learners quickly begin to generate quadratic equations that they cannot yet solve. This is a good springboard for a future session.

## BLANK PAGE FOR NOTES

### A3 Sheet 1 – Creating an equation

Name .....

1. Create an equation in the space below. Remember to show every step.

<b>Operations</b>		
.....		$x =$
.....		.....
.....		.....
.....		.....

2. Check that your equation works by substituting the original value of  $x$ .

<b>Check</b>
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3. Solve your own equation. Use a different method from the one you used to create it.

<b>Solution</b>
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### A3 Sheet 2 – Solving an equation in different ways

1. Write your equation from Sheet 1 in the space below.

<b>Equation</b>
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2. Give this sheet to your partner and ask them to solve your equation in as many different ways as possible.

Name of partner . . . . .

Solution 1	Solution 2
Solution 3	Solution 4

3. Correct your partner's work. Did they do it correctly? Talk to your partner about any errors you found and write some comments to help.

## A3 Sheet 3 – Instructions for using the program Balance 1

Solve the equation on the right, using the "balance-strategy".

Use the red buttons to select an operation that will be performed on both sides of the equation. Simplify the equation at each step, so you end with "x = number". The equation is then solved.

Next, choose another equation from the list at the bottom.

Make an equation yourself

$5x - 6 = 2x - 21$   
 $5x = 2x - 15$   
 $3x = -15$   
 $x = -5$  ✓

+ 6  
- 2x  
÷ 3

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

### Purpose

To give practice at solving linear equations with one unknown in which the unknown appears on either side or on both sides of the equation, using the balance method. The interesting feature in *Balance 1* is that the user does the strategic thinking, while the computer does the manipulation.

### Running the program

You are given (or you create) a series of equations. You must decide what has to be done to each side of an equation in order to simplify and eventually solve it. Your decisions are entered using the red buttons. From left to right, these are: *add*, *subtract*, *multiply*, *divide*, *expand brackets* (use the distributive law), and *combine alike terms*. If you change your mind at any stage, you can use the *Back* button.

For example, here is one way to solve the equation  $5x - 6 = 2x - 21$  using this program.

- Click on the red [+] button.
- Type '6' in the rectangular box on the right and hit 'Enter'.
- Click on the red [-] button.
- Type '2x' in the rectangular box on the right and hit 'Enter'.
- Click on the red [÷] (divide) button.
- Type '3' in the rectangular box on the right and hit 'Enter'.
- The solution  $x = -5$  appears on the screen followed by a tick indicating that the solution is correct.

### A3 Sheet 3 – Instructions for using the program Balance 1 (continued)

Along the bottom of the screen there are 20 pink circles indicating that there are twenty equations that you can solve. After you have correctly solved an equation, the colour of the circle changes to green.

If you wish, you can make your own equations. Click the 'Make an equation yourself' button, then enter your equation in the small drop-down window. Click the red [+ ] button. Your equation will now appear on the right hand side of the screen, where it can be solved as before. Equations that cannot be solved will not appear.

The screenshot shows a software interface for solving equations. On the left, there is instructional text: "Solve the equation on the right, using the 'balance-strategy'." followed by "Use the red buttons to select an operation that will be performed on both sides of the equation. Simplify the equation at each step, so you end with 'x = number'. The equation is then solved." and "Next, choose another equation from the list at the bottom." Below this text is a button labeled "Make an equation yourself".

On the right, a calculator-style interface shows the steps of solving the equation  $2(3x - 4) = 20$ . The steps are:
 

- $2(3x - 4) = 20$
- $3x - 4 = 10$  (achieved by dividing both sides by 2)
- $3x = 14$  (achieved by adding 4 to both sides)
- $x = 4\frac{2}{3}$  (achieved by dividing both sides by 3)

 A green checkmark is next to the final answer. The calculator interface includes buttons for  $\sqrt{\quad}$ ,  $\square^\square$ ,  $\frac{\square}{\square}$ ,  $(\square)$ ,  $+$ ,  $-$ ,  $\times$ ,  $\div$ ,  $\frac{1}{\square}$ ,  $\square\square$ , and  $\text{Back}$ .

At the bottom of the interface is a row of 20 numbered circles. Circle 1 is green, and circles 2 through 20 are pink.

## A3 Sheet 4 – Instructions for using the program Balance 2

Solve the equations, using the "balance strategy".

Use the red buttons to select an operation that will be performed on both sides of the equation. For each step, fill in the new equation. You can get help with the help-button, but you get less points.

A correct solution gets you at least 5 points. Without help this is 10 points per problem.

Make an equation yourself

Calculator window:

$$2(5x - 1) - 4 = -2x + 3(2x - 6)$$

distributive law

$$10x - 2 - 4 = -2x + 6x - 18$$

combine alike terms

$$10x - 6 = 4x - 18$$

equal with:

$$6x = -12$$

÷ 6

$$x = -2$$

Score: 10

### Purpose

To give practice at solving linear equations with one unknown in which the unknown appears on either side or on both sides of the equation, using the balance method.

*Balance 2* is more demanding than *Balance 1*, in that you not only need to decide what must be done to each side at each step, you must also type in the result of doing this.

### Running the program

When you have decided what to do for a particular step in solving the equation, but have difficulty writing the correct result of doing this step, click the *Help* button. The computer will now give the result of doing the step, as in *Balance 1*. A scoring system is used to discourage you from using this help facility too often. The more often you click the *Help* button, the lower your score will be.

Note: the computer will not mark the solution as correct unless the operations and the results at each step are both correct.



Solve the equations, using the "balance strategy".

Use the red buttons to select an operation that will be performed on both sides of the equation.

For each step, fill in the new equation. You can get help with the help-button, but you get less points.

A correct solution gets you at least 5 points. Without help this is 10 points per problem.

Make an equation yourself

$2(5x - 1) - 4 = -2x + 3(2x - 6)$  } distributive law  
 $10x - 2 - 4 = -2x + 6x - 18$  } combine alike terms  
 $10x - 6 = 4x - 18$  } equal with:  
 $6x = -12$  } ÷ 6  
✓  $x = -2$

- 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

Scores:

10

### A3 Sheet 5 – Instructions for using the program Cover up

Solve the equation on the right, using the "plate strategy".

Select a part of the equation on the left side of the equation sign, and find the value of this part. Go on till you have found the value of  $x$ .

When you have found the value for  $x$ , click on another problem from the list below.

Make an equation yourself

Calculator interface showing the equation  $3 + \frac{44}{\sqrt{87+x}} = 7$  and the steps of solving it using the 'cover up' method:

$$3 + \frac{44}{\sqrt{87+x}} = 7$$

$$\frac{44}{\sqrt{87+x}} = 4$$

$$\sqrt{87+x} = 11$$

$$87+x = 121$$

$$x = 34$$

Buttons 1 through 20 are visible at the bottom of the program window.

### Purpose

To give practice at solving linear equations with one unknown in which the unknown appears on one side of the equation. This uses the 'cover up' method. Though not as generalisable as the *Balance* method, it is simple to use and helps you to interpret equations. At each step you 'cover up' part of the equation and decide what number must be hidden.

### Running the program

The above screen shows how the 'cover up' method may be used to solve the equation:

$$3 + \frac{44}{\sqrt{87+x}} = 7$$

- 'Cover up' the  $\frac{44}{\sqrt{87+x}}$ .

You can see that the equation takes the form  $3 + \dots = 7$ .

The missing number must therefore be 4.

Drag the cursor over the term  $\frac{44}{\sqrt{87+x}}$  and it appears on the line below. Type 4 in the

box on the right then hit 'Enter'.

A green (or red) symbol indicates that the step is correct (or incorrect). If you make an incorrect step, click the red button again.

### A3 Sheet 5 – Instructions for using the program Cover up (continued)

- Cover up the  $\sqrt{87 + x}$ .  
Now you have an equation of the form  $44 \div \dots = 4$ .  
The missing number must be 11.  
Drag the cursor over the  $\sqrt{87 + x}$  and it appears on the line below.  
Enter 11 in the box on the right.

And so on...

Along the bottom of the screen there are 20 pink circles indicating that there are twenty equations that you can solve. After you have correctly solved an equation, the colour of the circle changes to green.

If you wish, you can make an equation yourself. Click the *Make an equation yourself* button, then enter your equation in the small drop-down window. Click the red [+] button. Your equation appears in the larger screen, where it can be solved.

Solve the equation on the right, using the "plate strategy".

Select a part of the equation on the left side of the equation sign, and find the value of this part. Go on till you have found the value of  $x$ .

When you have found the value for  $x$ , click on another problem from the list below.

Make an equation yourself

Calculator interface showing the solution process:

$$12 = 6$$

$$\sqrt{6-x} = 6$$

$$\sqrt{6-x} = 2$$

$$6-x = 4$$

$$x = 2$$

Progress indicator: 20 pink circles numbered 1 to 20.