A2 • Creating and solving equations

Mathematical goals

To enable learners to:

- create and solve their own equations, where the unknown appears once;
- develop confidence with the notation used in equations.

To help learners to:

- teach and learn from each other.

Starting points

Most learners will have been taught rules for solving equations such as 'change the side, change the sign' or 'you always do the same to both sides'. When used without understanding, such rules result in many errors. For example:

$3x = 15 \Rightarrow x = \frac{15}{-3}$ (change the side, change the sign);

$3(x - 2) = 6 \Rightarrow x - 2 = 3$ (taking 3 from both sides).

'Doing the same to both sides' is the more meaningful method, but there are two difficulties:

- knowing how to change both sides of an equation so that equality is preserved;
- knowing which operations lead towards the desired goal.

Building equations is easier than solving them because it postpones the second difficulty and so is an easier place to start.

It is helpful if learners have already encountered the following ideas.

- Addition is the inverse of subtraction (and vice versa).
- Multiplication is the inverse of division (and vice versa).
- The use of brackets/fraction bars when multiplying and dividing.

Materials required

For each learner you will need:

- Sheet 1 – *Creating equations*;
- Sheet 2 – *Solving equations*.

It is also helpful to make OHTs of Sheets 1 and 2 for use in the whole group introduction.

Time needed

About 1 hour.

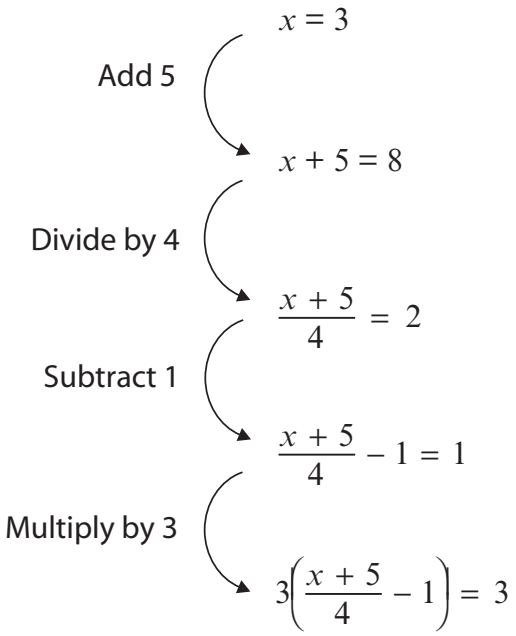
Suggested approach

Beginning the session

1. Build an equation

Write down a letter and its value on the board, e.g. $x = 3$. (This may be done on an OHT of Sheet 1.)

Using learners' suggestions for operations, build up an equation, step by step, using each of the four rules, +, –, ×, ÷ and whole numbers between 1 and 10.



As learners suggest each operation, you supply the notation and explain it carefully. For example, explain that we use brackets to show that a whole expression is being multiplied and that we use the fraction bar rather than the usual division symbol (÷).

During this process, experience has shown that it is better not to simplify the left hand side of the equation at any stage. For example, if learners suggest the four operations +5, –1, ÷4, ×3, then we write $3\left(\frac{(x + 5) - 1}{4}\right)$ rather than $3\left(\frac{x + 4}{4}\right)$.

2. Check the equation

Ask the group to check that the original value of x still satisfies the final equation.

$$3\left(\frac{3+5}{4}\right)-1=3\left(\frac{8}{4}-1\right)=3(2-1)=3\times 1=3$$

3. Solve the equation

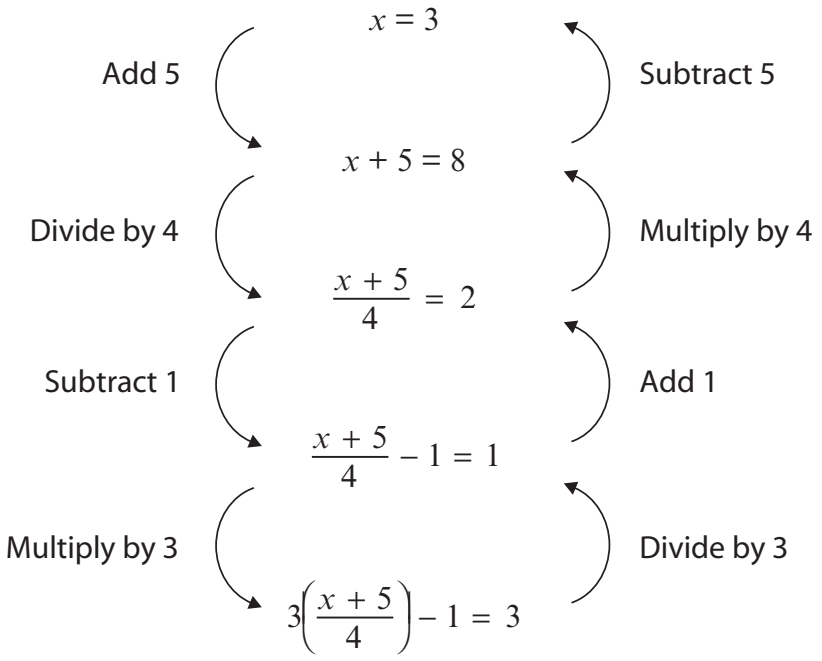
Hide all the steps except the final equation and ask the group to recall each operation in sequence.

> This equation tells the story of 'a day in the life of x'.
>
> What happened to it first? How can you tell by looking only at the equation?
>
> What then?
>
> What then?
>
> What was the last thing that happened?

In this way, show that the final equation tells the story of the operations used.

> Suppose you had started with this equation and you wanted to find the value of x.
>
> How could you do this?
>
> How can you undo what we have just done?
>
> You take your socks and boots off in the reverse order to the order you put them on.
>
> It's the same here.

Gradually get the group to unpick each step in reverse order. As they do this, uncover the preceding equations one by one and write the corresponding operation to the right of each equation (with upward arrows):



You will probably need to work through one or two more examples like this with the group, until they get the idea. It is worth changing the letter used (from x) each time, just to make the point that there is nothing special about it.

4. Create your own equation

Ask learners to create two equations of their own in a similar way. Sheet 1 – *Creating equations* provides a structure for this. After creating each equation, learners should check that it works by substituting the answer back into it.

Learners who struggle may be asked to restrict themselves to fewer steps and operations to start with.

When learners are satisfied that their equations work (and maybe when they have checked them with you), ask them to write the equations on Sheet 2 – *Solving equations*.

Working in groups

Each learner should then give their Sheet 2 to a partner. The partner should try to 'undo' the operations, step by step. Partners may call on originators for help if they get stuck. Encourage learners to help each other as much as possible.

As learners get the idea, the structured sheets may be discarded and learners may enjoy creating more challenging equations. Encourage them to do this by having more steps to the equation, rather than by using harder numbers.

Reviewing and extending learning

Ask learners to write their 'favourite creations' on the board and ask other learners, working in pairs, to solve them.

Ask learners to use their mini-whiteboards to write down algebraic equations that correspond to some 'think of a number' problems. For example, you might say:

> Think of a number, call it n.
> Double it.
> Add 4.
> Divide your answer by 7.
> Multiply your answer by 2.
> The result is 4.
> Show me the equation.

And your learners might respond: $2\left(\frac{2n+4}{7}\right) = 4$

What learners might do next

Learners may enjoy introducing further operations, such as $\frac{1}{x}$ (the inverse operation) and +/– (the 'change the sign' operation). Both are self-inverses.

Using these, learners may create more complex equations, such as $1 - \frac{1}{n-3} = \frac{1}{2}$.

This was created by starting with $n = 5$ and then operating as follows:

–3, $\frac{1}{x}$ (invert), +/– (change the sign), +1.

To undo this sequence, we simply do:

–1, +/– (change the sign), $\frac{1}{x}$ (invert), +3.

In addition, you may like to use **A3 Creating and solving harder equations**. This considers equations where the unknown appears more than once.

Further ideas

This kind of activity may be used for other 'doing' and 'undoing' activities.

For example:

- Learners might draw a rectangle and then calculate the area and perimeter. Partners must then attempt to 'find' the original rectangle from the area and perimeter.

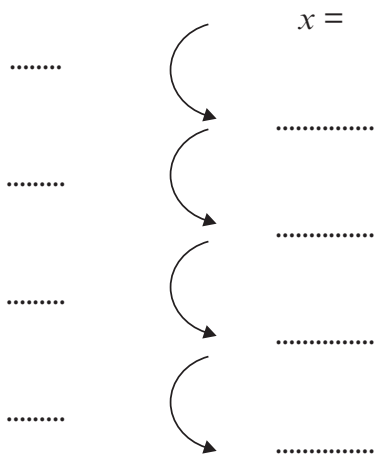
- Learners might write down an expression involving brackets and then expand them. Partners must then attempt to factorise the resulting expansion to get back to the original expression.
- Learners might sketch a distance–time graph and then write a story describing what happened. Partners must then try to reconstruct the graph from the story.

A2 Sheet 1 – *Creating equations*

Name . . . . . .

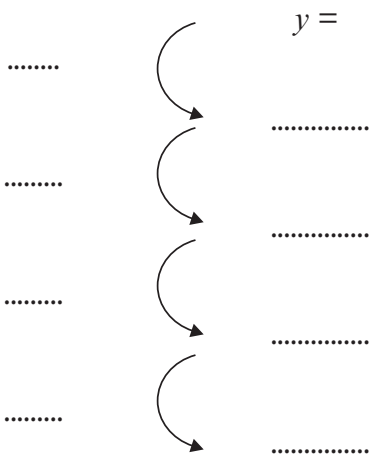
1. Create two equations in the spaces below.
   Show, step by step, how you make your equations, by writing an operation next to each arrow.

Operations



This is equation 1

Operations



This is equation 2

2. Check that your equations work by substituting the original value.

Check equation 1

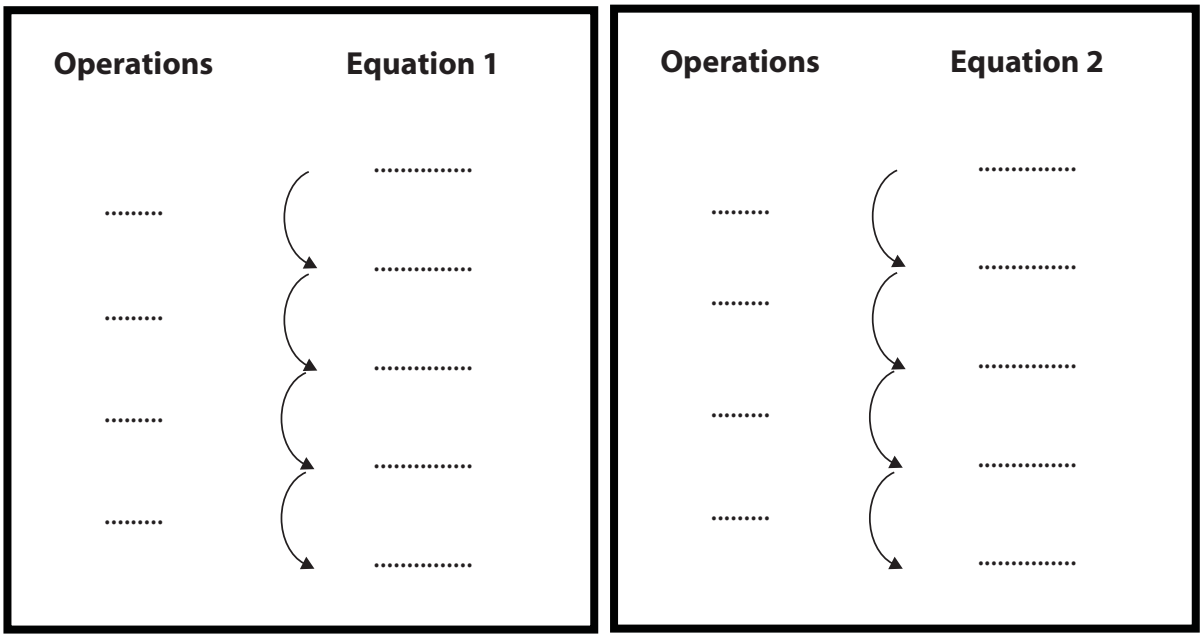
Check equation 2

A2 Sheet 2 – *Solving equations*

3. Rewrite your final equations in the spaces below.
   Give this sheet to your partner and ask them to solve your equations.

Name of partner . . . . . .

4. Solve the equations, showing what you do at each step.



5. Correct your partner's work. Did they follow the steps and solve the equation correctly? Describe any difficulties that they had.

Comments