EDEXCEL STUDENT CONFERENCE 2006

AS MATHEMATICS

STUDENT NOTES

South: Thursday 23rd March 2006, London

EXAMINATION HINTS

Before the examination

- 🕮 Obtain a copy of the formulae book – and use it!
- 🕮 Write a list of and LEARN any formulae not in the formulae book
- 🕮 Learn basic definitions
- 🕮 Make sure you know how to use your calculator!
- 🕮 Practise all the past papers - TO TIME!

At the start of the examination

- ✏ Read the instructions on the front of the question paper and/or answer booklet
- ✏ Open your formulae book at the relevant page

During the examination

- 🕑 Read the WHOLE question before you start your answer
- 🕛 Start each question on a new page (traditionally marked papers) or
- 🕐 Make sure you write your answer within the space given for the question (on-line marked papers)
- 🕛 Draw clear well-labelled diagrams
- 🕒 Look for clues or key words given in the question
- 🕖 Show ALL your working - including intermediate stages
- 🕗 Write down formulae before substituting numbers
- 🕔 Make sure you finish a ‘prove’ or a ‘show’ question – quote the end result
- 🕘 Don’t fudge your answers (particularly if the answer is given)!
- 🕚 Don’t round your answers prematurely
- 🕛 Make sure you give your final answers to the required/appropriate degree of accuracy
- 🕖 Check details at the end of every question (e.g. particular form, exact answer)
- 🕑 Take note of the part marks given in the question
- 🕐 If your solution is becoming very lengthy, check the original details given in the question
- 🕔 If the question says “hence” make sure you use the previous parts in your answer
- 🕚 Don’t write in pencil (except for diagrams) or red ink
- 🕒 Write legibly!
- 🕗 Keep going through the paper – go back over questions at the end if time

At the end of the examination

- 🗐 If you have used supplementary paper, fill in all the boxes at the top of every page

C1 KEY POINTS

C1 Algebra and functions

Surds (i) $\sqrt{xy} = \sqrt{x} \times \sqrt{y}$ (ii) $\sqrt{\frac{x}{y}} = \frac{\sqrt{x}}{\sqrt{y}}$ (iii) $a\sqrt{x} \pm b\sqrt{x} = (a \pm b)\sqrt{x}$

N.B. In general $\sqrt{x \pm y} \neq \sqrt{x} \pm \sqrt{y}$

Rationalising Given $\frac{1}{\sqrt{a}}$, multiply by $\frac{\sqrt{a}}{\sqrt{a}}$. Given $\frac{1}{a \pm \sqrt{b}}$, multiply by $\frac{a \mp \sqrt{b}}{a \mp \sqrt{b}}$

Indices 1. $a^m \times a^n = a^{m+n}$ 2. $\frac{a^m}{a^n} = a^{m-n}$ 3. $(a^m)^n = a^{mn}$ 4. $a^0 = 1$

5. $a^{-n} = \frac{1}{a^n}$ 6. $a^{\frac{1}{n}} = \sqrt[n]{a}$ 7. $a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$

Quadratic functions If $f(x) = ax^2 + bx + c$, the discriminant is $b^2 - 4ac$
For $f(x) = 0$, $b^2 - 4ac > 0 \Rightarrow$ two real, distinct roots, $b^2 - 4ac = 0 \Rightarrow$ two real, equal roots, $b^2 - 4ac < 0 \Rightarrow$ two unreal roots
Factorising, completing the square, using the formula
If $f(x) = 0$, then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Sketching quadratic functions
(a) To find the point of intersection with the y-axis: put $x = 0$ in $y = f(x)$
(b) To find the points of intersection with the x-axis: solve $f(x) = 0$
(c) To find the maximum/minimum point: use completing the square, symmetry or solve $f'(x) = 0$ [This latter method uses C2 techniques]

Other curves: reciprocal $\left(y = \frac{1}{x}\right)$, cubics
Expanding brackets, collecting like tems, factorising
Simultaneous equations (including one linear and one quadratic)
Linear and quadratic inequalities

Transformation	Description
$y = f(x) + a$ $a > 0$	Translation of $y = f(x)$ through $\begin{pmatrix} 0 \\ a \end{pmatrix}$
$y = f(x + a)$ $a > 0$	Translation of $y = f(x)$ through $\begin{pmatrix} -a \\ 0 \end{pmatrix}$
$y = af(x)$ $a > 0$	Stretch of $y = f(x)$ parallel to y-axis with scale factor a
$y = f(ax)$ $a > 0$	Stretch of $y = f(x)$ parallel to x-axis with scale factor $\frac{1}{a}$

C1 Coordinate geometry $P(x_1, y_1)$ and $Q(x_2, y_2)$

Gradient of $PQ = \frac{y_2 - y_1}{x_2 - x_1}$ Distance $PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Equation of a straight line

(i) Given the gradient, m and the vertical intercept $(0, c)$: $y = mx + c$

(ii) Given a point $P(x_1, y_1)$ on the line and the gradient, m: $y - y_1 = m(x - x_1)$

(iii) Given two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ on the line: $\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$

Mid-point of PQ $M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Gradient of line l_1 is m_1, gradient of line l_2 is m_2

If line l_1 is parallel to line l_2, then $m_1 = m_2$

If line l_1 is perpendicular to line l_2, then $m_1 \times m_2 = -1$

C1 Sequences and Series

Sigma notation, e.g. $\sum_{r=1}^{4} (2r + 5) = 7 + 9 + 11 + 13$

$u_{n+1} = 3u_n + 5$, $n \geq 1$, $u_1 = -2$ The first 5 terms of this sequence are $-2, -1, 2, 11$ and 38

An arithmetic series is a series in which each term is obtained from the previous term by adding a constant called the common difference, d

nth term $= a + (n - 1)d$

$S_n = \frac{n}{2}[2a + (n - 1)d]$ or $S_n = \frac{n}{2}[a + l]$ where last term $l = a + (n - 1)d$

Sum of the first n natural numbers: $1 + 2 + 3 + 4 + \ldots + n$: $S_n = \frac{n}{2}(n + 1)$

C1 Differentiation

Notation: If $y = \mathrm{f}(x)$ then $\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{f}'(x)$ and $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = \mathrm{f}''(x)$

y	$\frac{\mathrm{d}y}{\mathrm{d}x}$
ax^n	anx^{n-1} (a is constant)
$\mathrm{f}(x) \pm \mathrm{g}(x)$	$\mathrm{f}'(x) \pm \mathrm{g}'(x)$

Equation of tangents and normals: Use the following facts:

(a) Gradient of a tangent to a curve $= \frac{\mathrm{d}y}{\mathrm{d}x}$

(b) The normal to a curve at a particular point is perpendicular to the tangent at that point

(c) If two perpendicular lines have gradients m_1 and m_2 then $m_1 \times m_2 = -1$

(d) The equation of a line through (x_1, y_1) with gradient m is $y - y_1 = m(x - x_1)$

C1 Integration

$\int ax^n \, \mathrm{d}x = \frac{ax^{n+1}}{n+1} + c$ provided $n \neq -1$ $\int (\mathrm{f}'(x) + \mathrm{g}'(x)) \, \mathrm{d}x = \mathrm{f}(x) + \mathrm{g}(x) + c$

C2 KEY POINTS

C2 Algebra and functions

Algebraic division by $(x \pm a)$

Remainder theorem: When f(x) is divided by $(x - a)$, $f(x) = (x - a)Q(x) + R$ where $Q(x)$ is the quotient and R is the remainder

Factor theorem: If f(a) = 0 then $(x - a)$ is a factor of f(x)

C2 Coordinate geometry

Circle, centre (0, 0) radius r: $x^2 + y^2 = r^2$

Circle centre (a, b) radius r: $(x - a)^2 + (y - b)^2 = r^2$

Useful circle facts:

The angle between the tangent and the radius is 90°

Tangents drawn from a common point to a circle are equal in length

The centre of a circle is on the perpendicular bisector of any chord

The angle subtended by a diameter at the circumference is 90°

C2 Sequences and Series

A geometric series is a series in which each term is obtained from the previous term by multiplying by a constant called the common ratio, r

nth term = ar^{n-1}, $S_n = \dfrac{a(1-r^n)}{1-r}$, $S_\infty = \dfrac{a}{1-r}$ where $|r| < 1$.

The following expansions are valid for all $n \in$ N:

$$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-1}b^2 + \ldots + {}^nC_r a^{n-r}b^r + \ldots + b^n$$

$$(1+x)^n = 1 + nx + {}^nC_2 x^2 + \ldots + {}^nC_r x^r + \ldots + x^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!} + \ldots + x^n$$

where ${}^nC_r = \dfrac{n!}{r!(n-r)!}$

C2 Trigonometry

Sine rule $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$ and ambiguous case

Cosine rule $a^2 = b^2 + c^2 - 2bc\cos A$ or $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

Area of $\Delta ABC = \frac{1}{2}ab\sin C$

$\sin x° = \cos(90 - x)°$, $\cos x° = \sin(90 - x)°$, $\tan x° = \dfrac{1}{\tan(90° - x)}$

Graphs of trigonometric functions

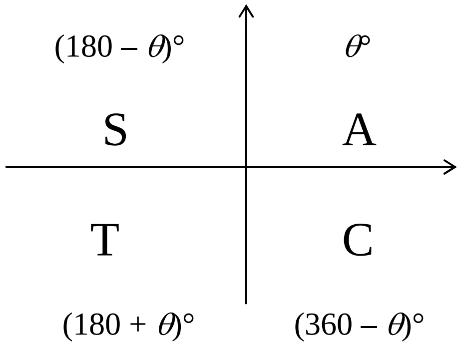
$\sin(-x) = -\sin x$, $\cos(-x) = \cos x$, $\tan(-x) = \tan x$

$\sin 30° = \cos 60° = \frac{1}{2}$, $\cos 30° = \sin 60° = \frac{\sqrt{3}}{2}$, $\tan 30° = \frac{1}{\sqrt{3}}$, $\tan 60° = \sqrt{3}$

$\cos 45° = \sin 45° = \frac{1}{\sqrt{2}}$, $\tan 45° = 1$

Degrees	360°	180°	90°	45°	60°	30°	270°	120°	135°	etc
Radians	2π	π	$\frac{\pi}{2}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{6}$	$\frac{3\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	

Arc length = $r\theta$, Area of sector = $\frac{1}{2}r^2\theta$ (θ in radians)



$\cos^2\theta + \sin^2\theta = 1$, $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

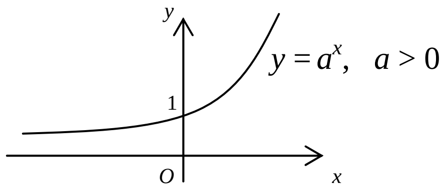
C2 Exponentials and Logarithms

If $y = a^x$ then $\log_a y = x$

Laws of logarithms: $\log_a pq = \log_a p + \log_a q$, $\log_a \dfrac{p}{q} = \log_a p - \log_a q$, $\log_a x^n = n.\log_a x$

Other useful results: $\log_a x = \dfrac{\log_b x}{\log_b a}$, $\log_a 1 = 0$, $\log_a a = 1$

f: $x \to a^x$ $x \in$ R $a > 0$ (a is constant)
is an exponential function, e.g. 7^{2x+4}



Solve equations of the form $a^x = b$

C2 Differentiation

If $\dfrac{dy}{dx} = 0$ and $\dfrac{d^2y}{dx^2} > 0$ the stationary point is a minimum turning point

If $\dfrac{dy}{dx} = 0$ and $\dfrac{d^2y}{dx^2} < 0$ the stationary point is a maximum turning point

For an increasing function, $\dfrac{dy}{dx} > 0$, for a decreasing function, $\dfrac{dy}{dx} < 0$

Maxima and minima problems: (a) Find the point at which $f'(x) = 0$. (b) Find the nature of the turning point to confirm that the value is a maximum or minimum as required. (c) Make sure that all parts of the question have been answered (e.g. finding the maximum/minimum as well as the value of x at which it occurs).

C2 Integration

If $\int f(x)\,dx = F(x) + c$ then $\int_a^b f(x)\,dx = \left[F(x)\right]_a^b = F(b) - F(a)$

If $y > 0$ for $a \le x \le b$, then area is given by $A = \int_a^b y\,dx$

Trapezium rule

$\int_a^b f(x)\,dx \approx \frac{1}{2}h[y_0 + y_n + 2(y_1 + ... + y_{n-1})]$ where $y_i = f(a + ih)$ and $h = \dfrac{b-a}{n}$