Model Solutions

# Edexcel GCSE

## Mathematics (Linear)

A* Paper (not for the faint hearted)

# Higher Tier

Time: 2 hours

**Materials required for examination**
Ruler graduated in centimetres and millimetres, protractor, compasses, pen, HB pencil, eraser.
Tracing paper may be used.

**Items included with question papers**
Nil

## Instructions to Candidates

Check that you have the correct brain power required to attempt this question paper.
Answer ALL the questions. Write your answers in the spaces provided in this question paper.
**You must NOT phone a friend or ask the audience.**
**Anything you write on the formulae page will gain NO credit.**
If you need more space to complete your answer to any question, write smaller.

## Information for Candidates

The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 23 questions in this question paper. The total mark for this paper is 110.

**Calculators must not be used unless the**  **symbol appears**

## Advice to Candidates

Show all stages in any calculations – A* questions often require you to **explain** or **prove** something.
Work steadily through the paper. Do not spend too long on one question.
If you cannot answer a question, leave it, attempt the next one and try not to cry.
Return at the end to those you have left out.
Have a lie down afterwards to help recover.

# GCSE A* Questions

**Skill:** Manipulate expressions containing surds

## Question 1

(a) Rationalise $\frac{1}{\sqrt{7}}$ $\times \frac{\sqrt{7}}{\sqrt{7}} = \frac{\sqrt{7}}{7}$

$\frac{\sqrt{7}}{7}$ ....................................

**(2)**

(b)(i) Expand and simplify

$$(\sqrt{3} + \sqrt{15})^2$$

Give your answer in the form $n + m\sqrt{5}$, where $n$ and $m$ are integers.

$(\sqrt{3} + \sqrt{15})(\sqrt{3} + \sqrt{15}) = 3 + \sqrt{3}\sqrt{15} + \sqrt{15}\sqrt{3} + 15$

$\sqrt{3}\sqrt{15} = \sqrt{45} = \sqrt{9}\sqrt{5}$ $= 18 + 6\sqrt{5}$

$= 3\sqrt{5}$

....$18 + 6\sqrt{5}$....................

(ii)

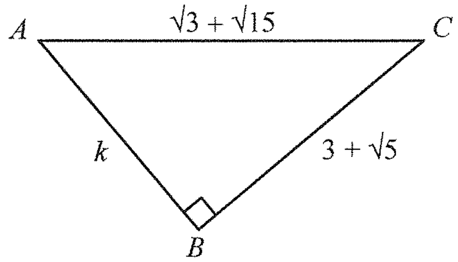



Diagram **NOT** accurately drawn

*ABC* is a right-angled triangle. *k* is a positive integer.

Find the value of *k*.

Pythagoras $\Rightarrow k^2 + (3+\sqrt{5})^2 = (\sqrt{3} + \sqrt{15})^2$ using (bi)

$\Rightarrow k^2 + 14 + 6\sqrt{5} = 18 + 6\sqrt{5}$

$(3+\sqrt{5})^2$ $\Rightarrow k^2 + 14 = 18$

$= 9 + 3\sqrt{5} + 3\sqrt{5} + 5$ $\Rightarrow k^2 = 4$

$= 14 + 6\sqrt{5}$ $\Rightarrow k = 2$ as k positive

$k =$ ....2..............................

**(5)**

**(Total 7 marks)**

# GCSE A* Questions

**Skill:** Solve direct and inverse variation problems

## Question 2

The force, $F$, between two magnets is inversely proportional to the square of the distance, $x$, between them.

When $x = 3$, $F = 4$.

(a) Find an expression for $F$ in terms of $x$.

told $F = \frac{k}{x^2} \Rightarrow k = Fx^2$

so $F = \frac{36}{x^2}$

when $x=3$, $F=4$ this gives $k = 4 \times 3^2 = 36$

$F =$ ....$36/x^2$....

**(3)**

(b) Calculate $F$ when $x = 2$.

$x = 2 \Rightarrow F = \frac{36}{2^2} = \frac{36}{4} = 9$

....9....

**(1)**

(c) Calculate $x$ when $F = 64$.

$F = \frac{36}{x^2}$

so $F=64 \Rightarrow x = \frac{6}{\sqrt{64}} = \frac{6}{8} = \frac{3}{4}$ simplified

$\Rightarrow Fx^2 = 36$

$\Rightarrow x^2 = \frac{36}{F}$

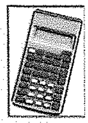
$\Rightarrow x = \sqrt{\frac{36}{F}} = \frac{6}{\sqrt{F}}$

....$3/4$....

**(2)**

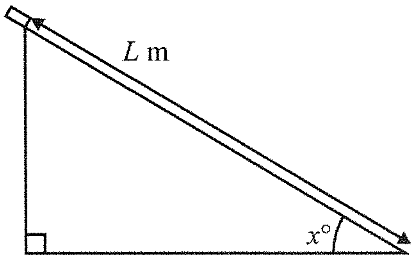
**(Total 6 marks)**

# GCSE A* Questions



**Skill:** Calculate the limits of compound measures

## Question 3




Elliot did an experiment to find the value of $g$ m/s$^2$, the acceleration due to gravity.
He measured the time, $T$ seconds, that a block took to slide $L$ m down a smooth slope of angle $x°$.

He then used the formula $g = \dfrac{2L}{T^2 \sin x°}$ to calculate an estimate for $g$.

$T = 1.3$ correct to 1 decimal place. $L = 4.50$ correct to 2 decimal places. $x = 30$ correct to the nearest integer.

(a) Calculate the lower bound and the upper bound for the value of $g$. Give your answers correct to 3 decimal places.

$g_{max} = \dfrac{2 \times L_{max}}{T_{min}^2 \sin_{min} x} = \dfrac{2 \times 4{\cdot}55}{1{\cdot}25^2 \sin 29{\cdot}5} = 11{\cdot}8272\ldots = 11{\cdot}827$ (3dp)

$g_{min} = \dfrac{2 \times L_{min}}{T_{max}^2 \sin_{max} x} = \dfrac{2 \times 4{\cdot}45}{1{\cdot}35^2 \sin 30{\cdot}5} = 9{\cdot}6217\ldots = 9{\cdot}622$ (3dp)

Lower bound ....9·622....

Upper bound ....11·827....

**(4)**

(b) Use your answers to part (a) to write down the value of $g$ to a suitable degree of accuracy. Explain your reasoning.

....$g = 10$ m/s$^2$ (1sf) as this is the highest level of accuracy to which both bounds agree....

**(1)**

**(Total 5 marks)**

# GCSE A* Questions

**Skill:** Solve a pair of simultaneous equations where one is linear and the other is non-linear

## Question 4

Solve the simultaneous equations

$$x^2 + y^2 = 29 \qquad (1)$$

$$y - x = 3 \qquad (2)$$

(2) $\Rightarrow y = x + 3$

sub $y = x+3$ in (1) $\Rightarrow x^2 + (x+3)^2 = 29$

expanding $\Rightarrow x^2 + x^2 + 6x + 9 = 20$

simplifying $\Rightarrow 2x^2 + 6x - 20 = 0$

halving $\Rightarrow x^2 + 3x - 10 = 0$

factorising $\Rightarrow (x-2)(x+5) = 0$

solving $\Rightarrow x = 2, -5$

then sub these values in (2) $\Rightarrow y = 5, -2$

Check values work in (1): $2^2 + 5^2 = 29$ ✓

$(-5)^2 + (-2)^2 = 29$ ✓

$x = 2, y = 5$ and $x = -5, y = -2$ ……………………

**(Total 7 marks)**

# GCSE A* Questions

**Skill:** Rearrange more complicated formulae where the subject may appear twice or as a power

## Question 5

$$P = \frac{n^2 + a}{n + a}$$

Rearrange the formula to make $a$ the subject.

cross-multiplying $\Rightarrow P(n+a) = n^2 + a$

expanding $\Rightarrow Pn + Pa = n^2 + a$

putting terms in a on 1 side $\Rightarrow Pa - a = n^2 - Pn$ or $\Rightarrow Pn - n^2 = a - Pa$

removing factor of a $\Rightarrow a(P-1) = n^2 - Pn$ $\qquad \Rightarrow Pn - n^2 = a(1-P)$

making a the subject $\Rightarrow a = \frac{n^2 - Pn}{P-1}$ $\qquad \Rightarrow \frac{Pn - n^2}{1-P} = a$

$a =$ $\frac{n^2 - Pn}{P-1}$ or $\frac{Pn - n^2}{1-P}$

**(Total 4 marks)**

# GCSE A* Questions

**Skill:** Simplify algebraic fractions by factorisation and cancellation

## Question 6

Simplify $$\frac{4x^2 - 9}{2x^2 - 5x + 3}$$

difference of squares $a^2 - b^2 = (a+b)(a-b)$

$$= \frac{(2x+3)(2x-3)}{(2x-3)(x-1)}$$

given nature of question, either $(2x-3)$ or $(2x+3)$ is going to be a factor of the denominator

$$= \frac{2x+3}{x-1}$$ cancelling common factor

$\frac{2x+3}{x-1}$ ....................................

**(Total 3 marks)**

# GCSE A* Questions

**Skill:** Solve a quadratic equation obtained from manipulating algebraic fractions where the variable appears in the denominator

## Question 7

Solve the equation

$$\frac{7}{x+2} + \frac{1}{x-1} = 4$$

cross-multiplying to clear fractions $\Rightarrow 7(x-1) + 1(x+2) = 4(x-1)(x+2)$

expanding $\Rightarrow 7x - 7 + x + 2 = 4x^2 + 4x - 8$

Simplifying $\Rightarrow 4x^2 - 4x - 3 = 0$

find 2 numbers with product of $-12$ and sum of $-4$

$\Rightarrow$ 2 and $-6$

splitting x term $\Rightarrow 4x^2 + 2x - 6x - 3 = 0$

factorising in pairs $\Rightarrow 2x(2x+1) - 3(2x+1) = 0$

factorising to get double bracket $\Rightarrow (2x-3)(2x+1) = 0$

solving $\Rightarrow x = \frac{3}{2}, -\frac{1}{2}$

$\frac{3}{2}, -\frac{1}{2}$ ..............................

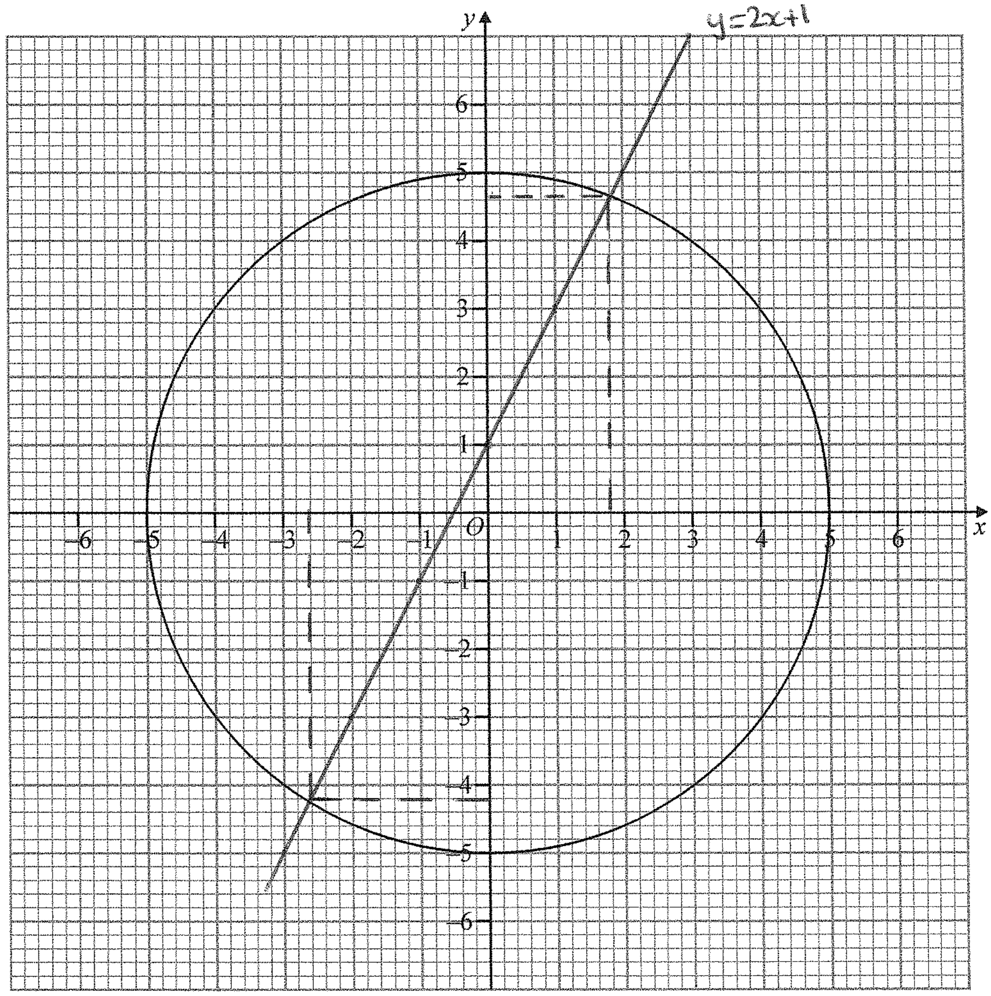
**(Total 7 marks)**

# GCSE A* Questions

**Skill:** Solve equations using the intersection of two graphs

## Question 8

The diagram shows a circle of radius 5 cm, centre the origin.




Draw a suitable straight line on the diagram to find estimates of the solutions to the pair of equations

find intersections

$$x^2 + y^2 = 25 \text{ and } y = 2x + 1$$

intercept 1, gradient 2

x = ......1·8......, $y$ = ......4·6......

$x$ = ......−2·6......, $y$ = ......−4·2......

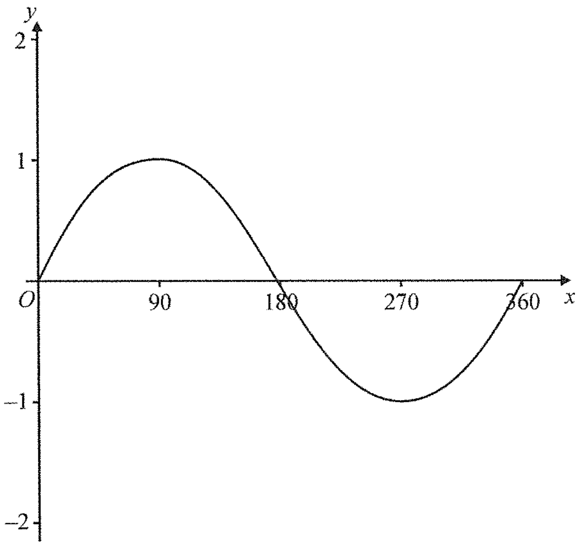
**(Total 3 marks)**

# GCSE A* Questions

**Skill:** Identify the equation of a function from its graph, which has been formed by a transformation on a known function

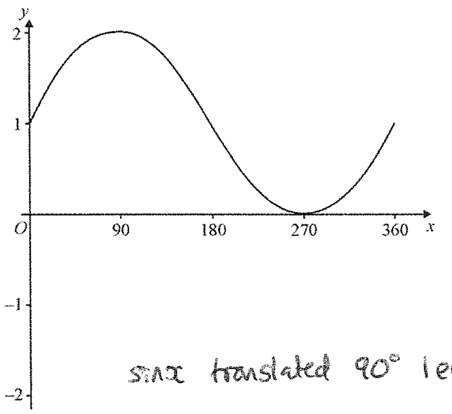
## Question 9

A sketch of the curve $y = \sin x°$ for $0 \leq x \leq 360$ is shown below.



Using the sketch above, or otherwise, find the equation of each of the following two curves.

(i)

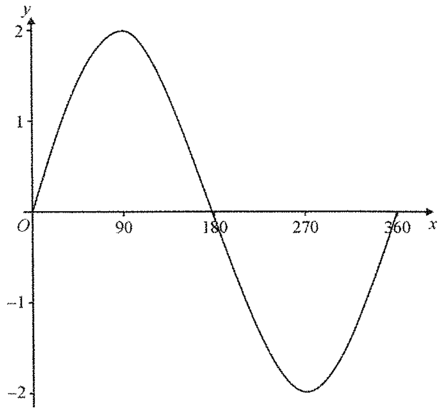


sinx translated 90° left

Equation $y =$ ......sin (x+90).........

(ii)

sin stretched vertically ×2



Equation $y =$ ......2 sinx.........

**(Total 2 marks)**

# GCSE A* Questions

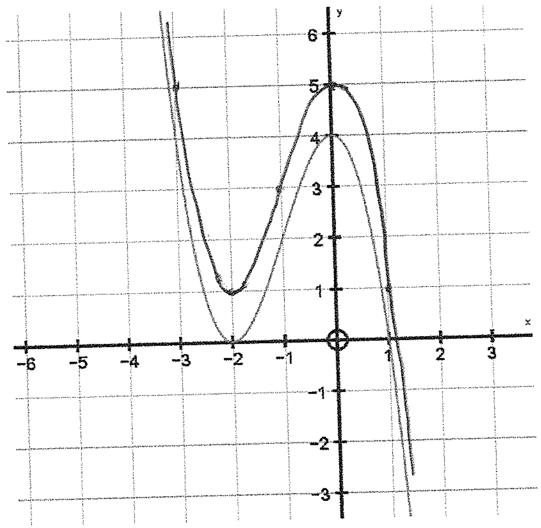
**Skill:** Transform the graph of a given function

## Question 10

The graph of $y = f(x)$ is shown on the grids.

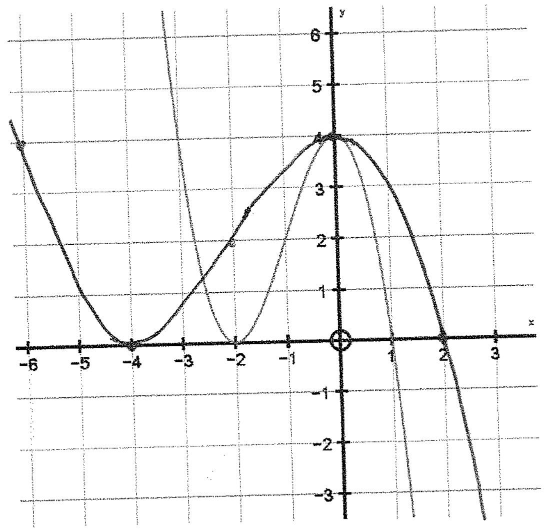
translate 1 up

(a) On this grid, sketch the graph of $y = f(x) + 1$




**(2)**

(b) On this grid, sketch the graph of $y = f\left(\frac{x}{2}\right)$ ← stretch horizontally ×2




**(2)**

**(Total 4 marks)**

# GCSE A* Questions

**Skill:** Prove algebraic & geometric results with rigorous and logical mathematical arguments

## Question 11

(a) Show that $(2a - 1)^2 - (2b - 1)^2 = 4(a - b)(a + b - 1)$

$(2a-1)^2 - (2b-1)^2 = (2a-1)(2a-1) - (2b-1)(2b-1)$

expanding $= 4a^2 - 2a - 2a + 1 - (4b^2 - 2b - 2b + 1)$

simplifying $= 4a^2 - 4b^2 - 4a + 4b$

factorising in pairs $= 4(a+b)(a-b) - 4(a-b)$

factorising into double bracket $= 4(a-b)(a+b-1)$ as required

**(3)**

(b) Prove that the difference between the squares of any two odd numbers is a multiple of 8.
(You may assume that any odd number can be written in the form $2r - 1$, where $r$ is an integer).

$2n-1$ is odd for all values of $n$, so 'the difference between the squares of two odd numbers' can be expressed as

$(2a-1)^2 - (2b-1)^2$ for some values of $a$ and $b$

using (a), this is equal to $4(a-b)(a+b-1)$

if $a$ and $b$ are even, then this $= 4 \times$ even $\times$ odd $=$ multiple of 8 ✓
if $a$ even, $b$ odd, this $= 4 \times$ odd $\times$ even $=$ " ✓
if $a$ odd, $b$ even, this $= 4 \times$ odd $\times$ even $=$ " ✓
if $a$ and $b$ are odd, this $= 4 \times$ even $\times$ odd $=$ " ✓

**(3)**

Therefore it is always a multiple of 8

**(Total 6 marks)**

# GCSE A* Questions

**Skill:** Solve real life problems that lead to constructing & solving a quadratic equation

## Question 12

The diagram below shows a 6-sided shape.
All the corners are right angles.
All measurements are given in centimetres.

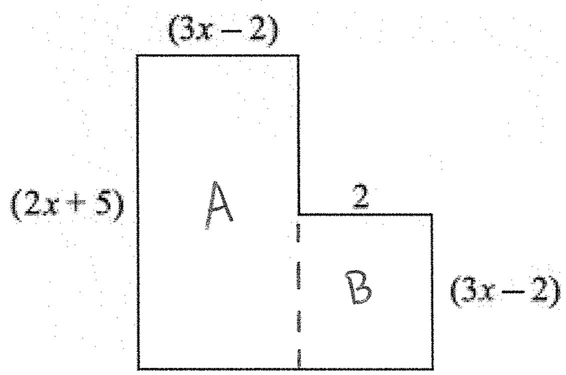



Diagram **NOT** accurately drawn

The area of the shape is 25 cm$^2$.

Show that $6x^2 + 17x - 39 = 0$

area of A $= (3x-2)(2x+5) = 6x^2 + 11x - 10$

area of B $= 2(3x-2) = 6x - 4$ +

so Total area $= 6x^2 + 17x - 14$

but area is given as 25 cm$^2$,

so $6x^2 + 17x - 14 = 25$

(−25) $\Rightarrow 6x^2 + 17x - 39 = 0$ as required.

**(Total 3 marks)**

# GCSE A* Questions

**Skill:** Complete the square to solve problems with quadratics

## Question 13

The expression $8x - x^2$ can be written in the form $p - (x - q)^2$, for all values of $x$.

(a) Find the value of $p$ and the value of $q$.

$8x - x^2 = -(x^2 - 8x)$ to make completing the square easier

$= -[(x-4)^2 - 16]$ completing the square

$= 16 - (x-4)^2$ putting into required form

$p =$ ....16..........

$q =$ ....4...........

**(3)**

(b) The expression $8x - x^2$ has a maximum value.

(i) Find the maximum value of $8x - x^2$ $= 16 - (x-4)^2$

$(x-4)^2$ is always $\geq 0$, so if subtracting this, the best thing to maximise your answer is make it $= 0$

so $(x-4)^2 = 0$ $\Rightarrow x = 4$
$\Rightarrow$ expression $= 16$ ..............16..............

(ii) State the value of $x$ for which this maximum value occurs.

..............4..............

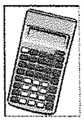
**(3)**

**(Total 6 marks)**

# GCSE A* Questions

**Skill:** Solve 3-D problems using Pythagoras' theorem and trigonometric ratios

## Question 14



The diagram represents a cuboid *ABCDEFGH*

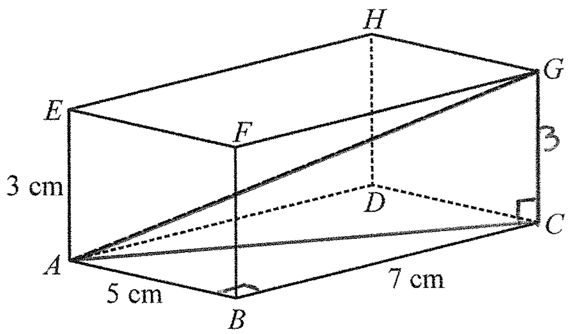


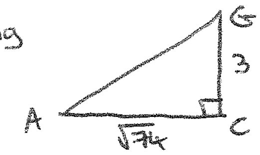

Diagram **NOT** accurately drawn

. $AB = 5$ cm. $BC = 7$ cm. $AE = 3$ cm.

(a) Calculate the length of *AG*.
Give your answer correct to 3 significant figures.

Pythagoras $\Rightarrow AC = \sqrt{5^2+7^2} = \sqrt{74}$

giving

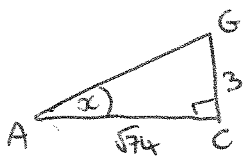



repeating gives $AG = \sqrt{74+3^2} = \sqrt{83} = 9{\cdot}1104\ldots = 9{\cdot}11$ (3sf

..........9·11.......... cm

**(2)**

(b) Calculate the size of the angle between *AG* and the face *ABCD*.
Give your answer correct to 1 decimal place.




T$^{O}$A question $\Rightarrow \tan x = \frac{O}{A}$

$\Rightarrow x = \tan^{-1}\left(\frac{3}{\sqrt{74}}\right) = 19{\cdot}225\ldots = 19{\cdot}3$ (1dp)

..........19·3..........°

**(2)**

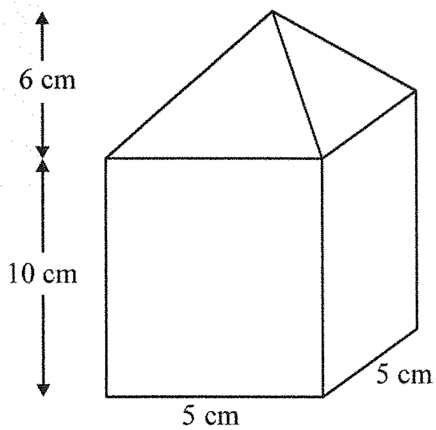
**(Total 4 marks)**

# GCSE A* Questions

**Skill:** Solve related problems using area and volume scale factors

## Question 15

Diagram **NOT** accurately drawn




The diagram shows a model.

The model is a cuboid with a pyramid on top.

(a) Calculate the volume of the model.

pyramid $= \frac{1}{3}x^2h = \frac{1}{3} \times 5^2 \times 6 = \frac{150}{3} = 50$

cuboid $= abc = 5 \times 5 \times 10 = 250$ +

Total volume $= 300 \text{ cm}^3$

..........300.......... $\text{cm}^3$

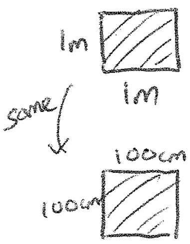
**(3)**

The model represents a concrete post.
The model is built to a scale of 1:30 ⇒ scale factor $k = 30$
The surface area of the model is 290 $\text{cm}^2$. ⇒ area factor $k^2$ needed, so $k^2 = 30^2 = 900$

(b) Calculate the surface area of the post.
Give your answer in square metres.

$290 \times 900 = 261000 \text{ cm}^2$

$= 26{\cdot}1 \text{ m}^2$




$1 \times 1 = 1\text{m}^2$

$100 \times 100 = 10{,}000 \text{ cm}^2$

so $10{,}000 \text{ cm}^2 = 1\text{m}^2$

..........26·1.......... $\text{m}^2$

**(3)**

**(Total 6 marks)**

# GCSE A* Questions

**Skill:** Use circle theorems to prove geometrical results

**Question 16**

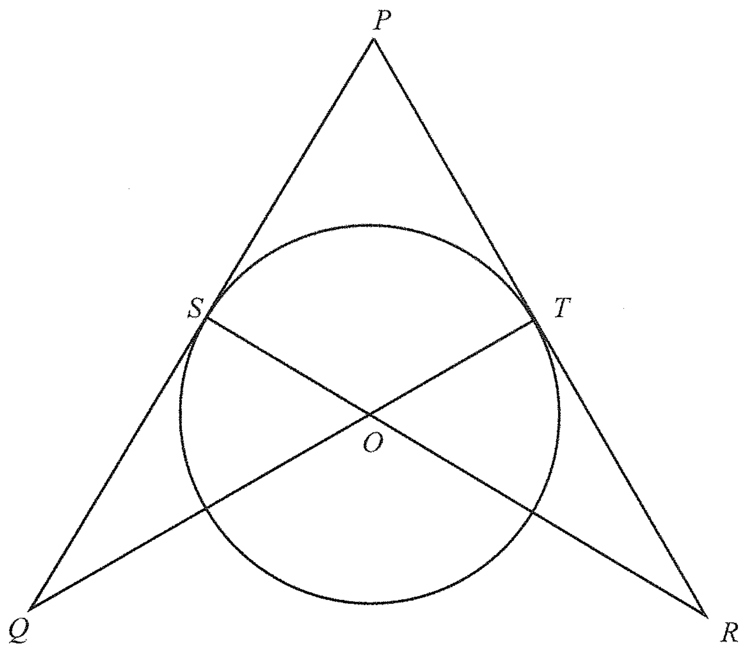



Diagram **NOT** accurately drawn

*S* and *T* are points on a circle, centre *O*.
*PSQ* and *PTR* are tangents to the circle.
*SOR* and *TOQ* are straight lines.

Prove that triangle *PQT* and triangle *PRS* are congruent.

PS = PT (tangents from a point are equal)

$\angle$SPT same (shared angle)

$\angle$PTQ = $\angle$PSR = 90° (tangents at 90° to centre of circle)

$\therefore$ congruent by <u>ASA</u>

**(Total 3 marks)**

# GCSE A* Questions

**Skill:** Solve more complex geometrical problems using vectors

## Question 17

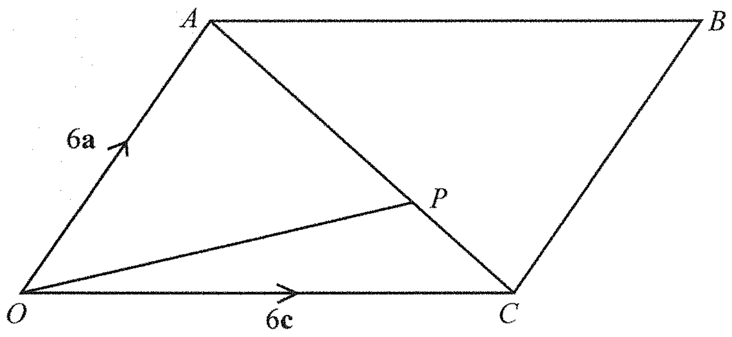



Diagram **NOT** accurately drawn

*OABC* is a parallelogram.

*P* is the point on *AC* such that $AP = \frac{2}{3}AC$.

$\overrightarrow{OA} = 6\mathbf{a}$. $\overrightarrow{OC} = 6\mathbf{c}$.

(a) Find the vector $\overrightarrow{OP}$.
Give your answer in terms of **a** and **c**.

$\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP}$

$= 6a + \frac{2}{3}(-6a + 6c)$

$= 6a - 4a + 4c$

$= 2a + 4c$

$2a + 4c$ ..............................

**(3)**

The midpoint of *CB* is *M*.

(b) Prove that OPM is a straight line. $\Rightarrow$ show that $\overrightarrow{OM} = k\overrightarrow{OP}$ for some value k

$\overrightarrow{OM} = \overrightarrow{OC} + \frac{1}{2}\overrightarrow{CB}$ as parallelogram, can assume $\overrightarrow{AB} = \overrightarrow{OC}$

$= 6c + \frac{1}{2}(-6c + 6a + 6c)$

$= 3a + 6c$ and $\overrightarrow{OP} = 2a + 4c$ from part (a)

$\therefore \overrightarrow{OM} = \frac{3}{2}\overrightarrow{OP}$

$\Rightarrow$ OPM is a straight line ✓

**(2)**

**(Total 5 marks)**

# GCSE A* Questions

**Skill:** Solve simple equations where the trigonometric ratio is the subject

**Question 18**

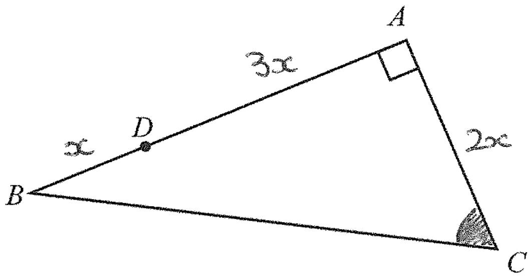



Diagram **NOT** accurately drawn

*ABC* is a right angled triangle.
*D* is the point on *AB* such that $AD = 3DB$.
$AC = 2DB$ and angle $A = 90°$.

Show that $\sin C = \frac{k}{\sqrt{20}}$, where *k* is an integer.

$\Rightarrow$ need $S\frac{O}{H}$

Write down the value of *k*.

H hypotenuse $= \sqrt{(4x)^2 + (2x)^2}$ using Pythagoras

$= \sqrt{20x^2}$

$= \sqrt{20}\,x$ as $x \geq 0$

then $\sin x = \frac{O}{H} = \frac{4x}{\sqrt{20}\,x} = \frac{4}{\sqrt{20}} \Rightarrow k = 4$

$k = $ ........4..........................

**(Total 4 marks)**

# GCSE A* Questions

**Skill:** Use the cyclic properties of the graphs of sine and cosine to solve problems

## Question 19

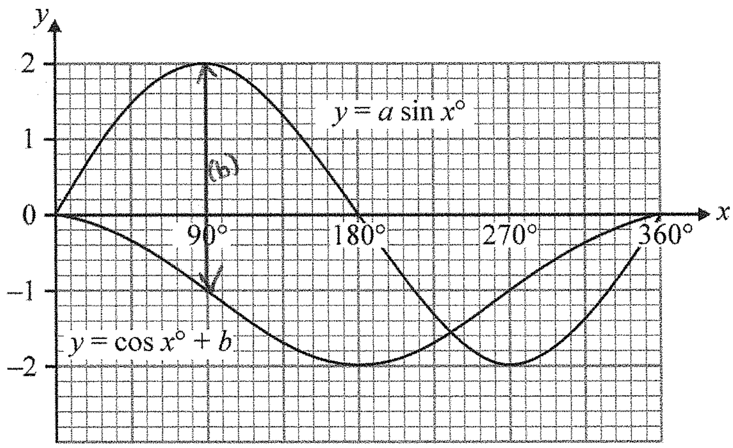



Diagram **NOT** accurately drawn

The diagram shows part of two graphs.

The equation of one graph is $y = a \sin x°$
The equation of the other graph is $y = \cos x° + b$

(a) Use the graphs to find the value of $a$ and the value of $b$.

$a =$ ..........2..........

$b =$ ..........-1..........

**(2)**

(b) Use the graphs to find the values of $x$ in the range $0° \leq x \leq 720°$ when $a \sin x° = \cos x° + b$. => graphs intersect

intersection at 0°, 360° will also repeat at 720°
intersection at ≈ 234° will repeat at 234+360 = 594°

$x =$ ......0, 234, 360, 594, 720

**(2)**

(c) Use the graphs to find the value of $a \sin x° - (\cos x° + b)$ when $x = 450°$. will be same at 450−360 = 90°     => difference in 'height' between curves

..........3..........

so from graph, x=90 gives 2−(−1) = 3

**(2)**

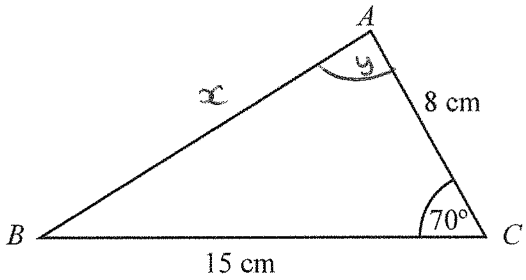
**(Total 6 marks)**

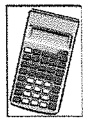
# GCSE A* Questions

**Skill:** Use the sine & cosine rules to solve more complex problems involving non right-angled triangles

**Question 20**

Diagram **NOT** accurately drawn



In triangle $ABC$, $AC = 8$ cm, $BC = 15$ cm, Angle $ACB = 70°$.

(a) Calculate the length of $AB$.
Give your answer correct to 3 significant figures.

cosine rule $\Rightarrow x^2 = 8^2 + 15^2 - 2 \times 8 \times 15 \cos 70 = 206.91\ldots$

$\Rightarrow x = \sqrt{206.91\ldots} = 14.384\ldots = 14.4$ (3sf)

..........14.4.......... cm

**(3)**

(b) Calculate the size of angle $BAC$.
Give your answer correct to 1 decimal place.

Sine rule $\Rightarrow \dfrac{\sin y}{15} = \dfrac{\sin 70}{x}$

$\Rightarrow y = \sin^{-1}\left(\dfrac{15 \sin 70}{x}\right) = 78.492\ldots = 78.5$ (1dp)

use exact value
14.384...

..........78.5..........°

**(2)**

**(Total 5 marks)**

# GCSE A* Questions

## Skill: solve problems involving more complex shapes & solids

### Question 21

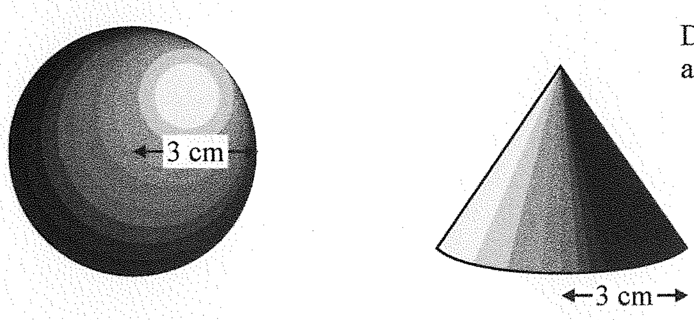



Diagram **NOT** accurately drawn

The radius of a sphere is 3 cm.
The radius of the base of a cone is also 3 cm.
The volume of the sphere is 3 times the volume of the cone.

Work out the curved surface area of the cone.
Give your answer as a multiple of $\pi$.

From formula sheet, curved area of cone $= \pi r l$ where $r =$ radius of base
$l =$ slant height

and


is a cross-section of the cone.

so need to know $h$ to work out $l$ to work out curved area

Volume of sphere $= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi \times 3^3 = 36\pi$

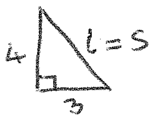
Volume of cone $= \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \times 3^2 \times h = 3\pi h$

But you are told $V_{sphere} = 3 \times V_{cone}$

$\Rightarrow 36\pi = 9\pi h$

$\Rightarrow h = 4$

so




using Pythagoras and so curved area $= \pi r l$

$= \pi \times 3 \times 5$

$= 15\pi$

..................$15\pi$.................. cm$^2$

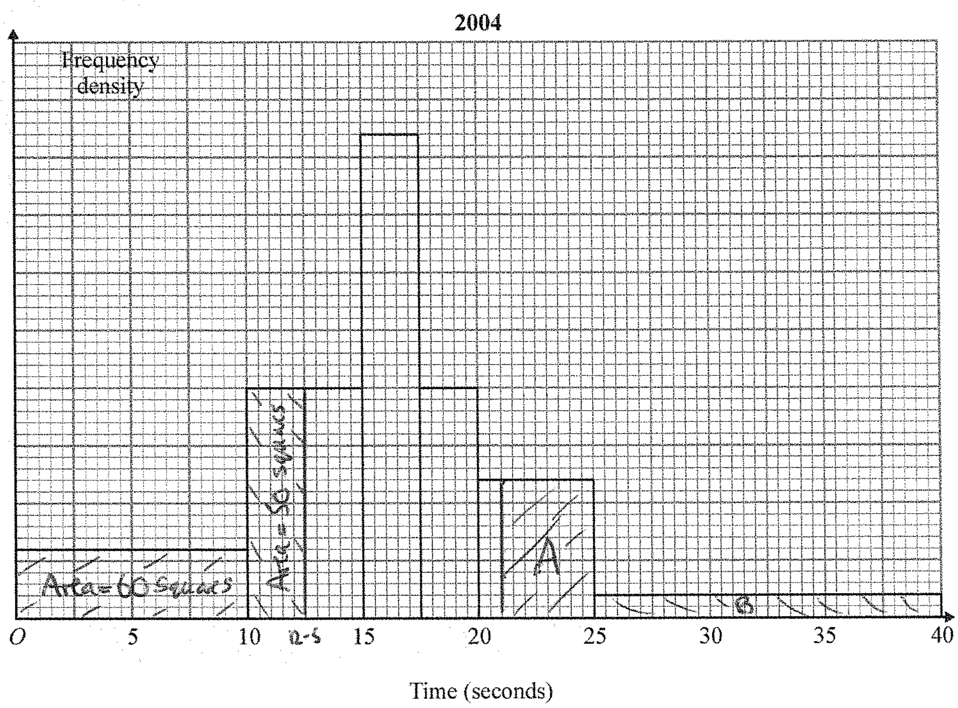
**(Total 7 marks)**

# GCSE A* Questions

**Skill:** Estimate statistics from a histogram

## Question 22

Area of 2 bars = 110
=> 1 square = 1 child




The histogram shows information about the time it took some children to connect to the internet.
None of the children took more than 40 seconds to connect to the internet.

110 children took up to 12.5 seconds to connect to the internet.

(c) work out an estimate for the number of children who took 21 seconds or more to connect to the internet.

= area of A + area of B

= 4 × 12 + 15 × 2

= 48 + 30

= 78 children

........78........

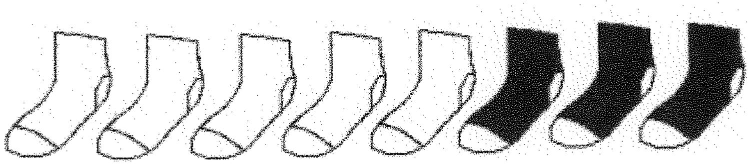
**(Total 3 marks)**

# GCSE A* Questions

**Skill:** Work out the probabilities of combined events when the probability of each event changes depending on the outcome of the previous event

**Question 23**

5 white socks and 3 black socks are in a drawer.



Stefan takes out two socks at random.

Work out the probability that Stefan takes out two socks of the same colour.

$P(WW) = \frac{5}{8} \times \frac{4}{7} = \frac{20}{56}$

+

$P(BB) = \frac{3}{8} \times \frac{2}{7} = \frac{6}{56}$

$P(\text{same colour}) = \frac{26}{56} = \frac{13}{28}$ simplified

$\frac{13}{28}$ ..........................

**(Total 4 marks)**