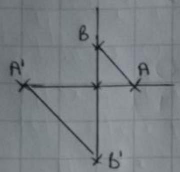
Further Maths GCSE
Matrices Answers

1. As $M = \begin{pmatrix} -2, & 0 \\ 0, & -2 \end{pmatrix} \Rightarrow (1,0) \to (-2,0)$
and $(0,1) \to (0,-2)$



Enlargement s.f. -2 centre $(0,0)$

Note this is same as $\begin{pmatrix} -2, & 0 \\ 0, & -2 \end{pmatrix} \begin{array}{cc} A & B \\ \hline \end{array}\!\!\!\!\!\!\!\!\!\!\!\! \begin{pmatrix} 1, & 0 \\ 0, & 1 \end{pmatrix} = \begin{array}{cc} A' & B' \end{array}\!\!\!\!\!\!\!\!\!\!\!\! \begin{pmatrix} -2, & 0 \\ 0, & -2 \end{pmatrix}$

2. $PQ = \begin{pmatrix} \sin x, & \cos x \\ -\cos x, & \sin x \end{pmatrix} \begin{pmatrix} \sin x, & -\cos x \\ \cos x, & \sin x \end{pmatrix}$

$= \begin{pmatrix} \sin^2 x + \cos^2 x, & -\sin x \cos x + \sin x \cos x \\ -\cos x \sin x + \sin x \cos x, & \cos^2 x + \sin^2 x \end{pmatrix}$

Because $\sin^2 x + \cos^2 x = 1$ we get $PQ = \begin{pmatrix} 1, & 0 \\ 0, & 1 \end{pmatrix}$

3. $\begin{pmatrix} 2, & a \\ 1, & -3 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 2a + ab \\ a - 3b \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$

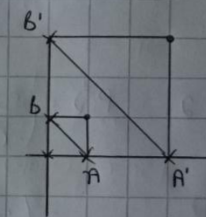
so $2a + ab = -1$ and $a - 3b = 2$ so $a = 2 + 3b$.
Therefore $2(2+3b) + (2+3b)b = -1$
so $3b^2 + 8b + 5 = 0$ $\quad (3b+5)(b+1) = 0$
so $b = -1$ giving $a = -1$
$b = -5/3$ $\quad a = -3$.

4. $\begin{pmatrix} -7, & 4 \\ 5, & -3 \end{pmatrix} \begin{pmatrix} -3, & -4 \\ -5, & t \end{pmatrix} = \begin{pmatrix} 1, & 0 \\ 0, & 1 \end{pmatrix}$

so $\begin{pmatrix} 21 - 20, & 28 + 4t \\ -15 + 15, & -20 - 3t \end{pmatrix} = \begin{pmatrix} 1, & 0 \\ 0, & 1 \end{pmatrix}$

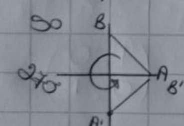
so $28 + 4t = 0 \Rightarrow t = -7$
Check $-20 - 3t = 1$ ✓

5. $\begin{pmatrix} 3, & 0 \\ 0, & 3 \end{pmatrix} \begin{array}{c} \begin{array}{cc} A & B \end{array} \\ \begin{pmatrix} 1, & 0 \\ 0, & 1 \end{pmatrix} \end{array} = \begin{array}{c} \begin{array}{cc} A' & B' \end{array} \\ \begin{pmatrix} 3, & 0 \\ 0, & 3 \end{pmatrix} \end{array}$



6. A positive rotation means anti-clockwise.

so



so matrix is $\begin{array}{c} \begin{array}{cc} A' & B' \end{array} \\ \begin{pmatrix} 0 & 1 \\ -1, & 0 \end{pmatrix} \end{array}$

7. $\begin{pmatrix} a, & b \\ -a, & 2b \end{pmatrix} \begin{pmatrix} 5 \\ 4 \end{pmatrix} = \begin{pmatrix} 5a + 4b \\ -5a + 8b \end{pmatrix} = \begin{pmatrix} 1 \\ 17 \end{pmatrix}$

so
$$\begin{array}{r} 5a + 4b = 1 \\ -5a + 8b = 17 \; + \\ \hline 12b = 18 \end{array}$$
so $b = 1.5$ and $a = -1$

8. $PQ = \begin{pmatrix} 2, & 3 \\ a, & b \end{pmatrix}\begin{pmatrix} 1, & 1 \\ 0, & 1 \end{pmatrix} = \begin{pmatrix} 2, & 5 \\ a, & a+b \end{pmatrix}$

$QP = \begin{pmatrix} 1, & 1 \\ 0, & 1 \end{pmatrix}\begin{pmatrix} 2, & 3 \\ a, & b \end{pmatrix} = \begin{pmatrix} 2+a, & 3+b \\ a, & b \end{pmatrix}$

So $\begin{pmatrix} 2, & 5 \\ a, & a+b \end{pmatrix} = \begin{pmatrix} 2+a, & 3+b \\ a, & b \end{pmatrix}$

top left $\Rightarrow a = 0$ top right $\Rightarrow b = 2$
bottom row also works.

9. $\begin{pmatrix} a, & 2 \\ -1, & 1 \end{pmatrix}\begin{pmatrix} 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 3a+8 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ b \end{pmatrix}$

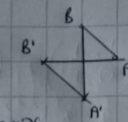
$3a+8 = 2 \Rightarrow a = -2$
$1 = b \Rightarrow b = 1.$

10. $M^2 = \begin{pmatrix} -2, & -1 \\ 3, & 1 \end{pmatrix}\begin{pmatrix} -2, & -1 \\ 3, & 1 \end{pmatrix} = \begin{pmatrix} 1, & 1 \\ -3, & -2 \end{pmatrix}$

$M^3 = \begin{pmatrix} -2, & -1 \\ 3, & 1 \end{pmatrix}\begin{pmatrix} 1, & 1 \\ -3, & -2 \end{pmatrix} = \begin{pmatrix} 1, & 0 \\ 0, & 1 \end{pmatrix}$ QED

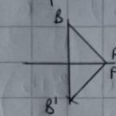
11. $\begin{pmatrix} 2, & -1 \\ 1/3, & 0 \end{pmatrix}\begin{pmatrix} 0, & b \\ a, & c \end{pmatrix} = \begin{pmatrix} -a, & 2b-c \\ 0, & 1/3\,b \end{pmatrix}$

12. $\begin{array}{cc} A' & B' \end{array}$ $\begin{pmatrix} 0, & -1 \\ -1, & 0 \end{pmatrix}$



reflection in line $y = -x$

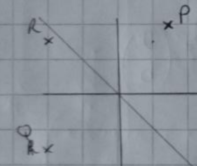
$\begin{array}{cc} A' & B' \end{array}$ $\begin{pmatrix} 1, & 0 \\ 0, & -1 \end{pmatrix}$



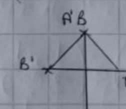
reflection in x-axis.

So reversing the transformation

$R = (-4,3) \Rightarrow Q = (-4,-3) \Rightarrow P = (3,4)$

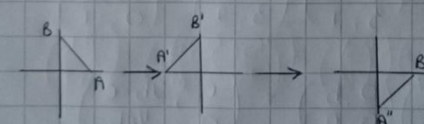


13. $\begin{array}{cc} A' & B' \end{array}$ $\begin{pmatrix} 0, & -1 \\ 1, & 0 \end{pmatrix}$



rotation $90°$ anticlockwise centre $(0,0)$

14.



$\begin{pmatrix} 0, & 1 \\ -1, & 0 \end{pmatrix}$

or $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} -1, & 0 \\ 0, & 1 \end{pmatrix} = \begin{pmatrix} 0, & 1 \\ -1, & 0 \end{pmatrix}$

2nd $\leftarrow$ 1st

15. $\begin{pmatrix} 1, & a \\ 0, & 2 \end{pmatrix}\begin{pmatrix} b \\ 5 \end{pmatrix} = \begin{pmatrix} 5 \\ b \end{pmatrix}$

$\Rightarrow b + 5a = 5$
$10 = b \Rightarrow a = -1.$