Further Maths GCSE
Differentiation Answers

1. $y = x^2 + 3x + 4$

$\frac{dy}{dx} = 2x + 3$

At the stationary point $\frac{dy}{dx} = 0$

So $2x+3 = 0 \Rightarrow x = -3/2 = -1.5$

$y = x^2+3x+4 \Rightarrow y = \frac{9}{4} - \frac{9}{2} + 4 = 1.75$

So coordinates are $\underline{\underline{(-1.5, 1.75)}}$

2. $y = (x+1)(2-x) = -x^2 + x + 2$.

So $A = (0,2)$

$\frac{dy}{dx} = -2x + 1$ so gradient of tangent at A

$\Rightarrow x = 0 \Rightarrow \frac{dy}{dx} = 1$

The gradient of the normal $= \frac{-1}{1} = -1$

$y = mx + c$ with $m = -1$ and passes through $(0,2)$

So $2 = -1 \times 0 + c \Rightarrow c = 2$

So $y = -1x + 2$

When $\cancel{x=0}$ $\Rightarrow y = 0 \Rightarrow x = 2$ which is point P.

$\left\{ \text{Alternatively gradient of AP} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-2}{2-0} = -1 \right\}$

3. $y = px^3 - 3x^2 + 8x + r$

$\frac{dy}{dx} = 3x^2p - 6x + 8$

(i) We know that when $x = 2$, $y = 10$

(ii) We also know that when $x = 2$; $\frac{dy}{dx} = 0$

(i) So $10 = p \times 2^3 - 3 \times 2^2 + 8 \times 2 + r$

$\Rightarrow 10 = 8p - 12 + 16 + r$

$\Rightarrow 6 = 8p + r$.

(ii)

Also $\frac{dy}{dx} = 0 \Rightarrow 3 \times 2^2 p - 6 \times 2 + 8 = 0$

$12p + -4 = 0$

$\underline{\underline{p = 1/3}}$.

As $p = 1/3$ $\quad 6 = 8 \times 1/3 + r$

$\Rightarrow \underline{\underline{r = 10/3}}$

4. $y = \frac{3x(2x^4 - 5x)}{x^2} = \frac{6x^5 - 15x^2}{x^2} = 6x^3 - 15$

So $\underline{\underline{\frac{dy}{dx} = 18x^2}}$

5. $y = (3x-4)(x+2) = 3x^2 + 2x - 8$

$\frac{dy}{dx} = 6x + 2$ when $x = 2$ $\underline{\underline{\frac{dy}{dx} = 14}}$

6. $y = 10 - 8x - x^3$

$\frac{dy}{dx} = -8 - 3x^2 = -(8 + 3x^2)$

$x^2 > 0$ for all values of x $\Rightarrow$ $-(8 + 3x^2) < 0$ $(\forall x)$

$\Rightarrow$ y is a decreasing function for all x $(\forall x)$

7. $\frac{dy}{dx} = 2x^2 - 7$ when $x = -3$ $\frac{dy}{dx} = 2(-3)^2 - 7 = 11.$

$\frac{dy}{dx} = 1$ $\Rightarrow$ $2x^2 - 7 = 1$

$\Rightarrow$ $2x^2 = 8$

$\Rightarrow$ $x^2 = 4$ so $x = +2$ or -2.

8. $y = x^{1/2}(x^{7/2} - x^{1/2}) = x^4 - x$

$\frac{dy}{dx} = 4x^3 - 1.$

9. $y = 4x^3 + 6x^2 + 3x + 5$

$\frac{dy}{dx} = 12x^2 + 12x + 3$

$dy/dx = 0$ $\Rightarrow$ $12x^2 + 12x + 3 = 0$

$(\div 3)$ $4x^2 + 4x + 1 = 0$

$\Rightarrow$ $(2x+1)(2x+1) = 0$

$\Rightarrow$ $2x + 1 = 0$ so $x = -1/2$.

as it is a cubic with a single stationary point : inflexion

~~$\frac{d^2y}{dx^2} = 12x^2$ when $x = -1/2$ $\frac{d^2y}{dx^2} > 0$ so minimum~~

[could check gradient at -1 and 0 both are +ve]

10. $P = (2, 0)$ $Q = (3, 0)$

$y = x^2 - 5x + 6$

$\frac{dy}{dx} = 2x - 5$

When $x = 2$ gradient $= dy/dx = -1$

$x = 3$ " " $= 1$

As $1 \times -1 = -1$ the two tangents are perpendicular.

11. $y = x^2(x - 2) = x^3 - 2x^2$

$\frac{dy}{dx} = 3x^2 - 4x$

When $x = 3$, gradient $= dy/dx = 3 \times 3^2 - 4 \times 3 = 15.$

12 $y = (5x - 3)^2 = 25x^2 - 30x + 9$

$dy/dx = 50x - 30$

$= 10(5x - 3)$

13. $y = x^3 + bx + c$ $dy/dx = 0$ when $x = -2$

$\frac{dy}{dx} = 3x^2 + b$ so $3(-2)^2 + b = 0$

$\Rightarrow$ $\underline{\underline{b = -12}}$

Also $(-2, 20)$ lies on the curve

so $20 = (-2)^3 + (-2)b + c$

$\Rightarrow 28 = -2b + c$ but $b = -12$

$\Rightarrow 28 = -2 \times -12 + c$

So $\underline{\underline{c = 4}}.$

14. An increasing function means when the gradient >0

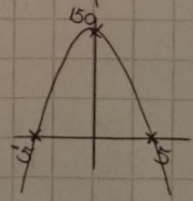
So $\frac{dy}{dx} = 150 - \cancel{8}6x^2 > 0$

$\Rightarrow$ If $150 = 6x^2$

$\frac{150}{6} = x^2$

$25 = x^2$

$\pm 5 = x$



Increasing function if $dy/dx > 0 \Rightarrow -5 < x < 5$

15. $\frac{dy}{dx} = -x(x-2)^2$

When $x=0$ and $x=2$ $\quad dy/dx = 0$

Let's check the gradient at $x=-1$, 1 and 3

So

x	-1	0	1	2	3
dy/dx	9	0	-1	0	-3
	+ve		-ve		-ve

So when $x=0$ it is a maximum point as the gradient changes from +ve to -ve

$\nearrow$ +ve $\quad$ 0 $\quad \searrow$ -ve

When $x=2$ it is a point of inflexion as the gradient is negative both before and after

$\searrow$ -ve $\quad$ 0 $\quad \searrow$ -ve

16. $y = (x^3-1)^2 + (\sqrt{x})^8$

$= x^6 - 2x^3 + 1 + (x^{1/2})^8$

$y = x^6 + x^4 - 2x^3 + 1$

$dy/dx = 6x^5 + 4x^3 - 6x^2$

17. $y = 2x^3 + ax$

$\frac{dy}{dx} = 6x^2 + a$

When $x=2$ $\quad dy/dx = 24 + a$

$x=-1$ $\quad dy/dx = 6 + a$

We are told $24 + a = 2(6+a)$

$\Rightarrow 24 + a = 12 + 2a$

$\underline{\underline{12 = a}}$