MR BARTON'S ANSWERS

Centre Number		Candidate Number	
Surname			
Other Names			
Candidate Signature			

For Examiner's Use	
Examiner's Initials	
Pages	Mark
3	
4 – 5	
6 – 7	
8 – 9	
10 – 11	
12 – 13	
14 – 15	
16 – 17	
TOTAL	

AQA

Level 2 Certificate in Further Mathematics
January 2013

Further Mathematics 8360/1
Level 2

Paper 1 Non-Calculator

Monday 28 January 2013 1.30 pm to 3.00 pm

For this paper you must have:
- mathematical instruments.

You may **not** use a calculator.

Time allowed
- 1 hour 30 minutes

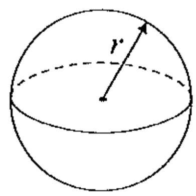
Instructions
- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- In all calculations, show clearly how you work out your answer.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 70.
- You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer book.

Formulae Sheet

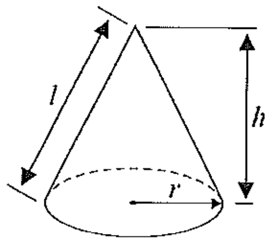
Volume of sphere $= \frac{4}{3}\pi r^3$

Surface area of sphere $= 4\pi r^2$

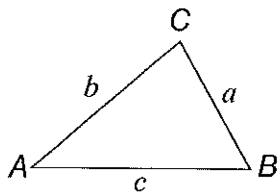


Volume of cone $= \frac{1}{3}\pi r^2 h$

Curved surface area of cone $= \pi r l$



In any triangle ABC



Area of triangle $= \frac{1}{2}ab\sin C$

Sine rule $\quad \dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Cosine rule $\quad a^2 = b^2 + c^2 - 2bc\cos A$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

The Quadratic Equation

The solutions of $ax^2 + bx + c = 0$, where $a \neq 0$, are given by

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

Trigonometric Identities

$\tan\theta \equiv \dfrac{\sin\theta}{\cos\theta} \qquad \sin^2\theta + \cos^2\theta \equiv 1$

Do not write outside the box

Answer **all** questions in the spaces provided.

1 The line $y = mx + c$ passes through the point (4, 3).
It is parallel to the line $y = 5x + 6$

Work out the values of m and c.

$x_1 = 4$ $\qquad y - y_1 = m(x - x_1)$

$y_1 = 3$ $\qquad y - 3 = 5(x - 4)$

$m = 5$ $\qquad y - 3 = 5x - 20$

$+3 \; \{ \; y = 5x - 17$

$m =$ 5, $c =$ -17 *(3 marks)*

2 The matrix $\begin{pmatrix} 5 & b \\ 4 & -1 \end{pmatrix}$ maps the point (a, 2) onto the point (28, 18),

such that $\begin{pmatrix} 5 & b \\ 4 & -1 \end{pmatrix} \begin{pmatrix} a \\ 2 \end{pmatrix} = \begin{pmatrix} 28 \\ 18 \end{pmatrix}$

Work out the values of a and b.

$\begin{pmatrix} a \\ 2 \end{pmatrix}$

$\begin{pmatrix} 5 & b \\ 4 & -1 \end{pmatrix} \begin{pmatrix} 28 \\ 18 \end{pmatrix}$

① $5a + 2b = 28$

② $4a - 2 = 18$
$4a = 20$
$a = 5$

using ① $5a + 2b = 28$
$5(5) + 2b = 28$
$25 + 2b = 28 \rightarrow b = 1.5$

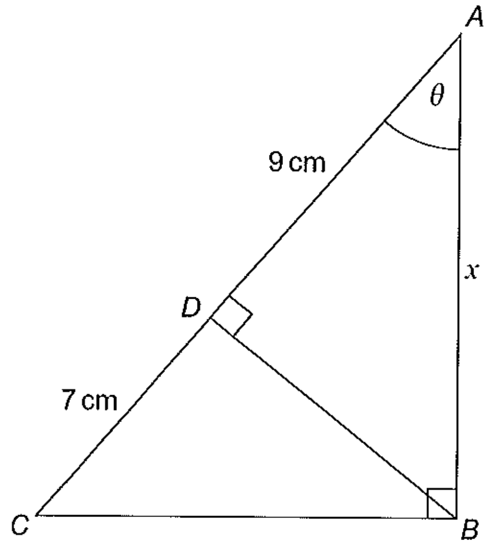
$a =$ 5, $b =$ 1.5 *(4 marks)*

7

Turn over ▶

Do not write outside the box

3 ABC is a right-angled triangle.
D is a point on AC.
BD is perpendicular to AC.



Not drawn accurately

3 (a) Use triangle ABC to write $\cos\theta$ in terms of x.

$\cos\theta = ADJ/HYP = x/16$

$\cos\theta =$ $x/16$ *(1 mark)*

3 (b) By writing another expression for $\cos\theta$ in terms of x, or otherwise, work out the value of x.

Using triangle ABD → $\cos\theta = Adj/Hyp = 9/x$

So, $\cos\theta = x/16$ & $\cos\theta = 9/x$

→ $x/16 = 9/x$

$\times 16$ } $x = 144/x$

$\times x$ } $x^2 = 144$

$\sqrt{\ }$ } $x = 12$

$x =$ 12 cm *(2 marks)*

Do not write outside the box

4 $w \blacktriangledown h$ is defined as $5w^2 - 8w + h^2 - 2h$

For example $1 \blacktriangledown 6 = 5 \times 1^2 - 8 \times 1 + 6^2 - 2 \times 6$
$= 5 - 8 + 36 - 12$
$= 21$

4 (a) Work out $2 \blacktriangledown 4$

$5 \times (2)^2 - 8(2) + (4)^2 - 2(4)$
$= 20 - 16 + 16 - 8$
$= 12$

Answer 12 *(2 marks)*

4 (b) Solve $x \blacktriangledown 3 = 0$

$5(x)^2 - 8(x) + 3^2 - 2(3) = 0$
$5x^2 - 8x + 9 - 6 = 0$
$5x^2 - 8x + 3 = 0$

$(5x - 3)(x - 1) = 0$

$5x - 3 = 0$ $x = 1$
$5x = 3$
$x = 3/5$

Answer $x = 3/5$ and $x = 1$ *(4 marks)*

9

Turn over ▶

Do not write outside the box

5 (a) n is a positive integer.

Write down the **next** odd number after $2n - 1$

Answer (+2) $2n + 1$ *(1 mark)*

5 (b) Prove that the product of two consecutive odd numbers is **always** one less than a multiple of 4.

$(2n - 1)(2n + 1)$

$= 4n^2 + 2n - 2n - 1$

$= 4n^2 - 1$

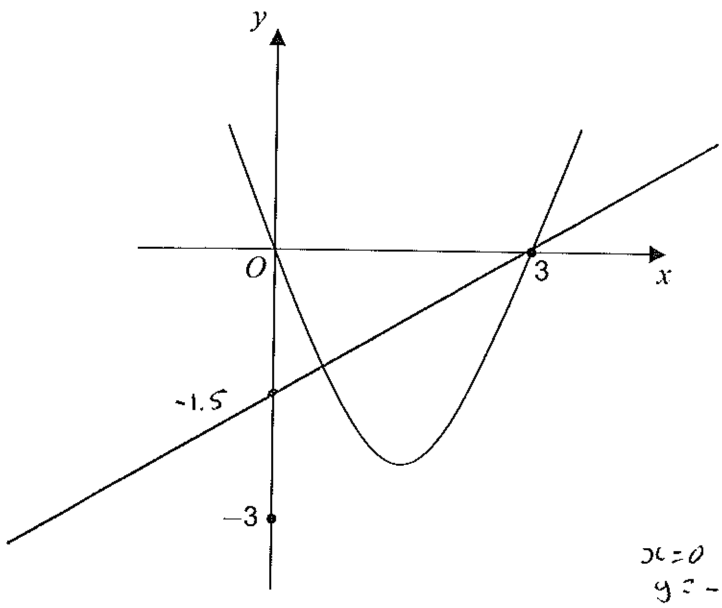
$4n^2$ is always a multiple of 4

So, $4n^2 - 1$ is always 1 less than a multiple of 4

(3 marks)

Do not write outside the box

6 The diagram shows a sketch of $y = x^2 - 3x$



$x=0$
$y=-1.5$

$y=0$
$x=3$

6 (a) Sketch the line $y = \frac{1}{2}(x-3)$ on the diagram. $= y = \frac{1}{2}x - 1.5$

Mark the value where this line crosses the y-axis. *(2 marks)*

6 (b) By factorising $x^2 - 3x$, or otherwise, work out the smaller solution of

$$x^2 - 3x = \frac{1}{2}(x-3)$$

$\rightarrow \quad x(x-3) = \frac{1}{2}(x-3)$

$\div (x-3) \quad x = \frac{1}{2}$

$x =$ $\frac{1}{2}$ *(2 marks)*

8

Turn over ▶

Do not write outside the box

7 $y = \dfrac{2x^2(3x^3 - 7x)}{x}$

Work out $\dfrac{dy}{dx}$

$$y = \frac{6x^5 - 14x^3}{x}$$

$$\Rightarrow y = 6x^4 - 14x^2$$

$$\Rightarrow \frac{\partial y}{\partial x} = 24x^3 - 28x$$

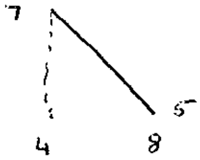
$\dfrac{dy}{dx} =$ .... *(4 marks)*

Do not write outside the box

8 f(x) is a decreasing function.

$$f(x) = b - ax \quad \text{for} \quad 4 \leqslant x < 8$$

The range of f(x) is $5 < f(x) \leqslant 7$

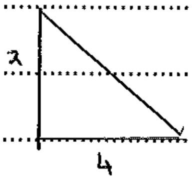


Work out the values of a and b.

As it decreases, when $x = 4$, $f(x) = 7$

when $x = 8$, $f(x) = 5$

Find gradient:



$= -2/4 = -0.5$

Use point $(4, 7)$

$x_1 = 4$ $\qquad y - y_1 = m(x - x_1)$

$y_1 = 7$ $\qquad y - 7 = -0.5(x - 4)$

$m = -0.5$ $\qquad y - 7 = -0.5x + 2$

$y \cancel{+7} = -0.5x + 9$

$\rightarrow y = 9 - 0.5x$

$a =$ 0.5, $b =$ 9 *(4 marks)*

8

Turn over ▶

Do not write outside the box

9 Bag A contains $7x$ counters.

Bag B contains $2x$ counters.

Five counters are taken from bag A and put in bag B.

9 (a) Write an expression, in terms of x, for the number of counters now in bag B.

Answer $2x + 5$ *(1 mark)*

9 (b) The ratio of counters in bag A to bag B is now 8 : 3

Use algebra to work out the **total** number of counters in the bags.

3 × counters in A = 8 × counters in B

→ $3(7x - 5) = 8(2x + 5)$

→ $21x - 15 = 16x + 40$

$5x - 15 = 40$

$5x = 55$

$x = 11$

Total counters = $7x + 2x$

$= 9x = 9(11) = 99$

Answer 99 *(4 marks)*

Do not write outside the box

10 Solve the simultaneous equations

$$① \quad \frac{x-1}{y-2}=3 \qquad \frac{x+6}{y-1}=4 \quad ②$$

Do **not** use trial and improvement.
You **must** show your working.

$×(y-1)$

① $×(y-2)$ $x-1 = 3(y-2)$ ② $x+6 = 4(y-1)$

$x-1 = 3y-6$ $x+6 = 4y-4$

$+1$ $x = 3y-5$ -6 $x = 4y-10$

So, $x = 3y-5$ AND $x = 4y-10$

$\therefore 3y-5 = 4y-10$

$-3y$ $-5 = y-10$

$+10$ $5 = y$

use ①: $x = 3y-5$

$x = 3(5)-10 = 10$

$x =$ 10, $y =$ 5 *(5 marks)*

10

Turn over ▶

Do not write outside the box

11 Write $\sqrt{500} - 2\sqrt{45}$ in the form $a\sqrt{5}$ where a is an integer.

$= \sqrt{100} \times \sqrt{5} - 2(\sqrt{9} \times \sqrt{5})$

$= 10\sqrt{5} - 2(3\sqrt{5})$

$= 10\sqrt{5} - 6\sqrt{5}$

Answer $4\sqrt{5}$ *(2 marks)*

12 Simplify fully $\dfrac{4x^2 + 19x - 5}{9x^2 - 16} \div \dfrac{x+5}{3x-4}$

$= \dfrac{(4x-1)(x+5)}{(3x-4)(3x+4)} \div \dfrac{(x+5)}{(3x-4)}$

$= \dfrac{(4x-1)(x+5)}{(3x-4)(3x+4)} \times \dfrac{(3x-4)}{(x+5)}$

$= \dfrac{4x-1}{3x+4}$ cancel $(x+5)$ + $(3x-4)$

Answer .... *(5 marks)*

Do not write outside the box

13 $y = 2x^3 - 12x^2 + 24x - 11$

13 (a) Work out $\frac{dy}{dx}$

Give your answer in the form $\frac{dy}{dx} = a(x - b)^2$, where a and b are integers.

$\frac{dy}{\partial x} = 6x^2 - 24x + 24$

$= 6(x^2 - 4x + 4)$

$= 6(x - 2)(x - 2)$

$= 6(x-2)^2$

$\frac{dy}{dx} =$ .... **(3 marks)**

13 (b) Hence, or otherwise, work out the coordinates of the stationary point of

$y = 2x^3 - 12x^2 + 24x - 11$

As at st points, $dy/dx = 0 \rightarrow 6(x-2)^2 = 0$

$\div 6 \quad (x-2)^2 = 0$

$\sqrt{} \quad x - 2 = 0$

$+2 \quad x = 2$

$y = 2(x)^3 - 12(x)^2 + 24(x) - 11$

$= 2(2)^3 - 12(2)^2 + 24(2) - 11$

$= 16 - 48 + 48 - 11$

$= 5$

Answer (2 , 5) **(2 marks)**

13 (c) Explain how you know that this stationary point is a point of inflection.

Test point either side: $x = 1$ $\quad dy/dx = 6(1-2)^2 = 6$

$x = 3$ $\quad dy/dx = 6(3-2)^2 = 6$

gradient positive either side

so point of inflection.

(1 mark)

13

Do not write outside the box

14 $x^2 - 2x + y^2 - 6y = 0$ is the equation of a circle.

By writing the equation in the form $(x-a)^2 + (y-b)^2 = r^2$
work out the centre and radius of the circle.

$(x-1)^2 - 1 + (y-3)^2 - 9 = 0$

$\Rightarrow (x-1)^2 + (y-3)^2 - 10 = 0$

$\Rightarrow (x-1)^2 + (y-3)^2 = 10$

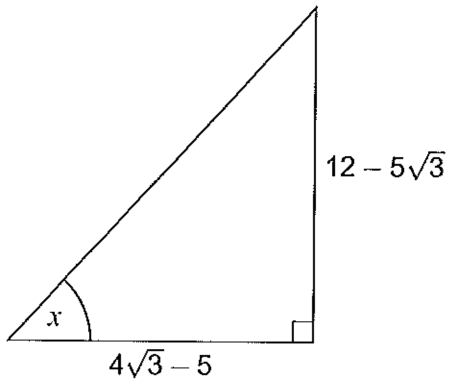
Centre = (1 , 3)

Radius = $\sqrt{10}$

(5 marks)

Do not write outside the box

15 Show that angle $x = 60°$



Not drawn accurately

You **must** show your working.

$\text{Tan}(x) = {}^{opp}/_{adj}$

$\text{Tan}(x) = \dfrac{12 - 5\sqrt{3}}{4\sqrt{3} - 5}$

Rationalise denominator : $\dfrac{12 - 5\sqrt{3}}{4\sqrt{3} - 5} \times \dfrac{4\sqrt{3} + 5}{4\sqrt{3} + 5}$

$= \dfrac{48\sqrt{3} + 60 - 20\sqrt{9} - 25\sqrt{3}}{16\sqrt{9} + 20\sqrt{3} - 20\sqrt{3} - 25} = \dfrac{23\sqrt{3}}{23} = \sqrt{3}$

$\Rightarrow \tan x = \sqrt{3}$

$\Rightarrow x = 60°$

(4 marks)

9

Turn over ▶

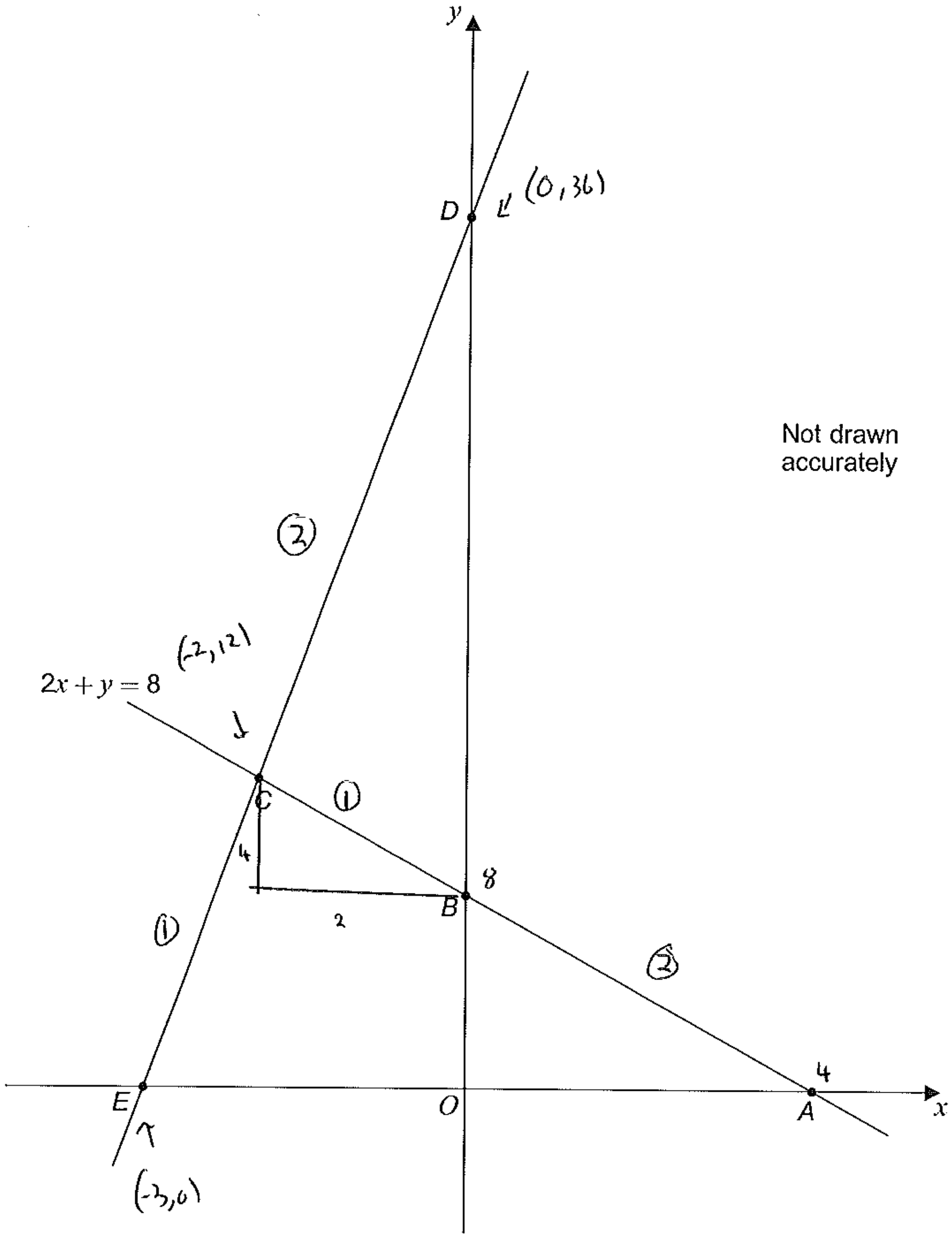
Do not write outside the box

16 A, B and C are points on the line $2x + y = 8$

DCE is a straight line.

$AB : BC = 2 : 1$

$EC : CD = 1 : 2$



Do not write outside the box

Work out the ratio Area of triangle *AEC* : Area of triangle *BCD*

Give your answer in its simplest form.

At B, $x = 0 \rightarrow 2(0) + y = 8 \rightarrow y = 8$

At A, $y = 0 \rightarrow 2x + 0 = 8 \rightarrow x = 4$

Point C must be another 2 along and 4 up due to the ratio 2 : 1 $\rightarrow C = (-2, 12)$

Point D must be $3 \times 12 = 36$ high due to ratio 2 : 1 $\rightarrow D = (0, 36)$

Point E must be at $(-3, 0)$ as C has an x value of -2 $\rightarrow E = (-3, 0)$

Area of AEC = ½ base × height = $\frac{1}{2}(7)(12) = 42$

Area of BCD = ½ base × height = $\frac{1}{2}(28)(2) = 28$

Ratio = 42 : 28

= 6 : 4

3 : 2

Answer 3 : 2 *(6 marks)*

END OF QUESTIONS

6