

MR BARTON'S SOLUTIONS

Centre Number								Candidate Number					
Surname													
Other Names													
Candidate Signature													

For Examiner's Use	
Examiner's Initials	
Pages	Mark
3	
4 - 5	
6 - 7	
8 - 9	
10 - 11	
12 - 13	
14 - 15	
TOTAL	



Level 2 Certificate in Further Mathematics

**Further Mathematics  
Level 2**

**8360/1**

**Practice Paper Set 3**

**Paper 1**

**Non-Calculator**

<p><b>For this paper you must have:</b></p> <ul style="list-style-type: none"> <li>mathematical instruments.</li> </ul> <p>You may <b>not</b> use a calculator.</p>	
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**Time allowed**  
1 hour 30 minutes

**Instructions**

- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the space provided. Do not write outside the box around each page or on blank pages.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- In all calculations, show clearly how you work out your answer.

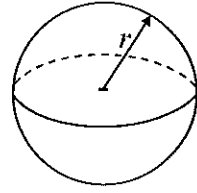
**Information**

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 70.
- You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer booklet.

## Formulae Sheet

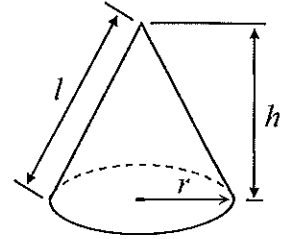
$$\text{Volume of sphere} = \frac{4}{3} \pi r^3$$

$$\text{Surface area of sphere} = 4\pi r^2$$



$$\text{Volume of cone} = \frac{1}{3} \pi r^2 h$$

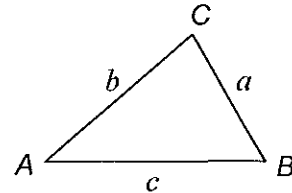
$$\text{Curved surface area of cone} = \pi r l$$



In any triangle  $ABC$

$$\text{Area of triangle} = \frac{1}{2} ab \sin C$$

$$\text{Sine rule} \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$



$$\text{Cosine rule} \quad a^2 = b^2 + c^2 - 2bc \cos A$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

### The Quadratic Equation

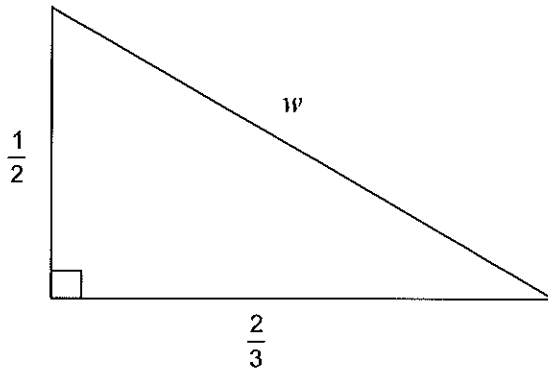
The solutions of  $ax^2 + bx + c = 0$ , where  $a \neq 0$ , are given by  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

### Trigonometric Identities

$$\tan \theta \equiv \frac{\sin \theta}{\cos \theta} \quad \sin^2 \theta + \cos^2 \theta \equiv 1$$

Answer all questions in the spaces provided.

- 1 Work out the exact value of  $w$ .



Not drawn  
accurately

Pythagoras

$$c^2 = a^2 + b^2$$

$$w^2 = \left(\frac{1}{2}\right)^2 + \left(\frac{2}{3}\right)^2$$

$$\rightarrow w^2 = \frac{1}{4} + \frac{4}{9}$$

$$\rightarrow w^2 = \frac{9}{36} + \frac{16}{36} = \frac{25}{36}$$

$$\rightarrow w = \sqrt{\frac{25}{36}}$$

$$w = \frac{5}{6} \dots \dots \dots (4 \text{ marks})$$

- 2  $5(3x - 2) - 3(x - h) \equiv 4(kx + 2)$

Expand Brackets!

Work out the values of  $h$  and  $k$ .

$$15x - 10 - 3x + 3h \equiv 4kx + 8$$

$$12x - 10 + 3h \equiv 4kx + 8$$

$$\boxed{x} \quad 12x \equiv 4kx \rightarrow k = 3$$

$$\boxed{\text{NUMBERS}} \quad -10 + 3h \equiv 8$$

$$\rightarrow 3h = 18 \rightarrow h = 6$$

$$h = 6, k = 3 \dots \dots \dots (4 \text{ marks})$$

3 Solve  $\frac{y-2}{5} + \frac{2y+1}{4} = 3$

$$\frac{4(y-2)}{20} + \frac{5(2y+1)}{20} = 3$$

$$\rightarrow \frac{4y-8+10y+5}{20} = 3$$

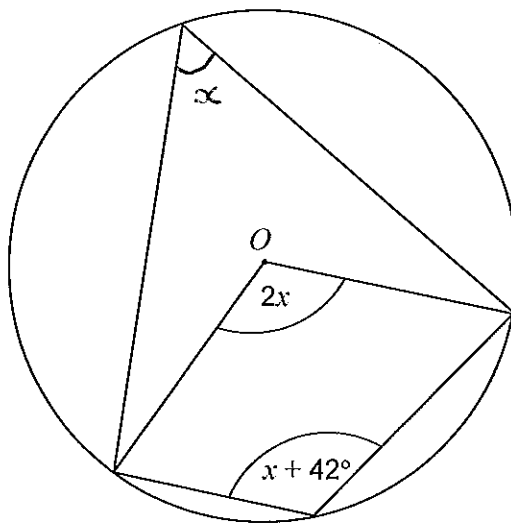
$$4y-8+10y+5 = 60$$

$$14y-3 = 60$$

$$14y = 63$$

$$y = \frac{63}{14} \text{ or } \frac{9}{2} \text{ or } 4\frac{1}{2} \dots (4 \text{ marks})$$

4  $O$  is the centre of this circle.



Not drawn  
accurately

Work out the value of  $x$ .

$$\text{Angle at circumference} = \frac{1}{2} \text{ angle at centre} = x$$

$$x + x + 42 = 180^\circ \quad (\text{Cyclic quadrilateral})$$

$$\rightarrow 2x + 42 = 180$$

$$\rightarrow 2x = 138$$

$$x = 69$$

$$x = 69 \text{ degrees } (3 \text{ marks})$$

5 Simplify fully  $20a^9b^6 \div 4a^3b^2$

$$\frac{20a^9b^6}{4a^3b^2}$$

.....

.....

Answer  $5a^6b^4$  ..... (2 marks)

6 The matrix  $\begin{pmatrix} a & b \\ -a & 2b \end{pmatrix}$  maps the point (5, 4) onto the point (1, 17).

Work out the values of  $a$  and  $b$ .

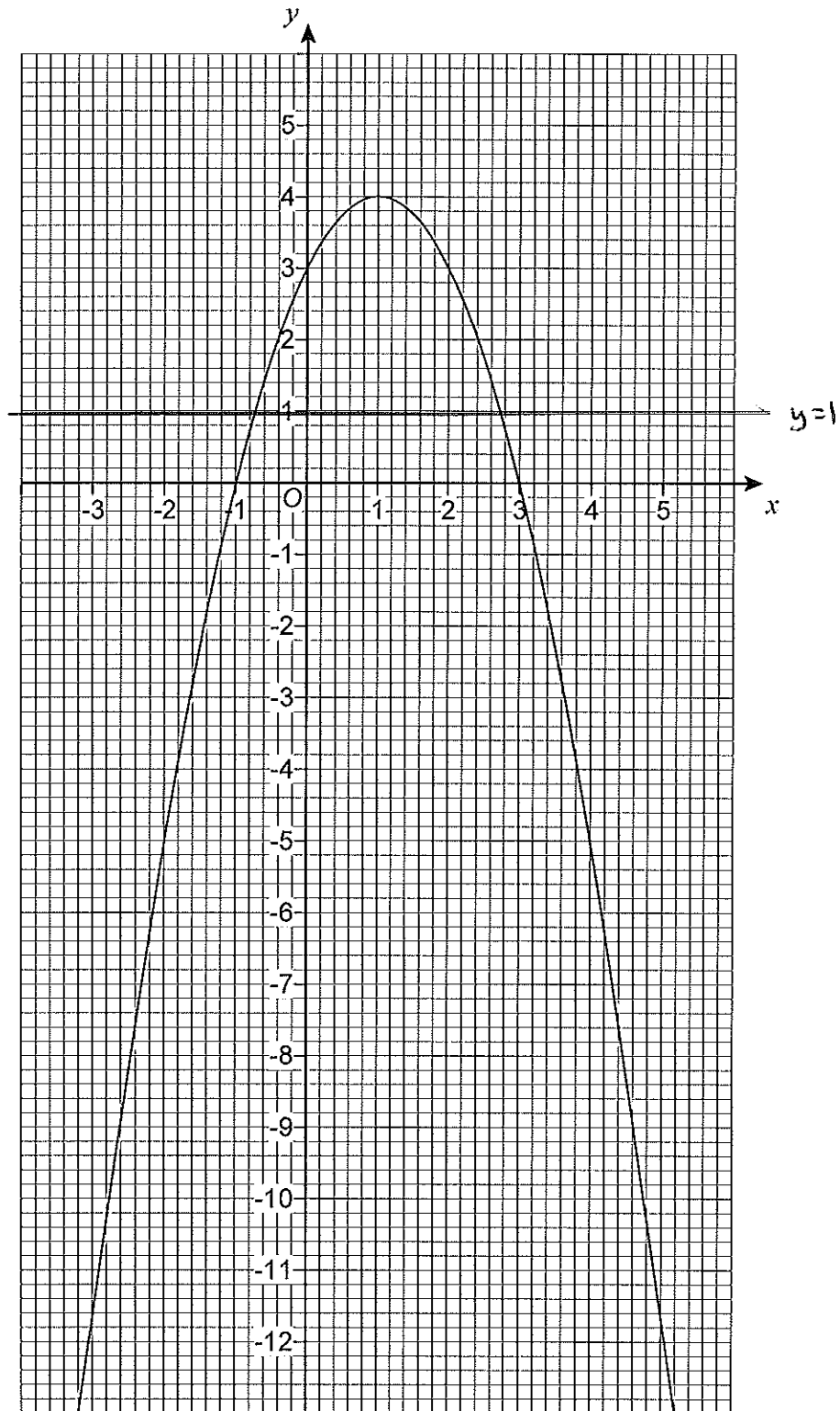
$$\begin{pmatrix} a & b \\ -a & 2b \end{pmatrix} \begin{pmatrix} 5 \\ 4 \end{pmatrix} = \begin{pmatrix} 1 \\ 17 \end{pmatrix}$$

$$\begin{array}{r} 5a + 4b = 1 \\ -5a + 8b = 17 \\ \hline 12b = 18 \\ \rightarrow b = \frac{18}{12} = 1.5 \end{array}$$

$$\begin{array}{r} 5a + 4b = 1 \\ 5a + 4(1.5) = 1 \\ 5a + 6 = 1 \\ 5a = -5 \\ \rightarrow a = -1 \end{array}$$

$a = \dots -1 \dots$ ,  $b = \dots 1.5 \dots$  (5 marks)

- 7  $f(x)$  is a quadratic function with domain all real values of  $x$ .  
Part of the graph of  $y = f(x)$  is shown.



- 7 (a) Write down the range of  $f(x)$ .

Answer .....  $f(x) \leq 4$  ..... (1 mark)

- 7 (b) Use the graph to find solutions of the equation  $f(x) = 1$   $\leftarrow$  crosses  $y = 1$   
Give your solutions to one decimal place.

Answer .....  $-0.7$  &  $2.8$  ..... (2 marks)

- 7 (c) Use the graph to solve  $f(x) < 0$   $\leftarrow$  below  $x$ -axis

Answer .....  $x < -1$  and  $x > 3$  ..... (2 marks)

8

$$x^{\frac{1}{2}} = 6 \text{ and } y^{-3} = 64$$

Work out the value of  $\frac{x}{y}$

$$\begin{aligned} x^{\frac{1}{2}} &= 6 \\ \rightarrow x &= 36 \end{aligned}$$

$$\begin{aligned} y^{-3} &= 64 \\ \rightarrow \frac{1}{y^3} &= 64 \\ \rightarrow y^3 &= \frac{1}{64} \\ \rightarrow y &= \frac{1}{4} \end{aligned}$$

$$\therefore \frac{x}{y} = \frac{36}{\frac{1}{4}}$$

$$= 36 \times 4 = 144$$

Answer .....  $144$  ..... (4 marks)

10

 $y = f(x)$  is a continuous graph.

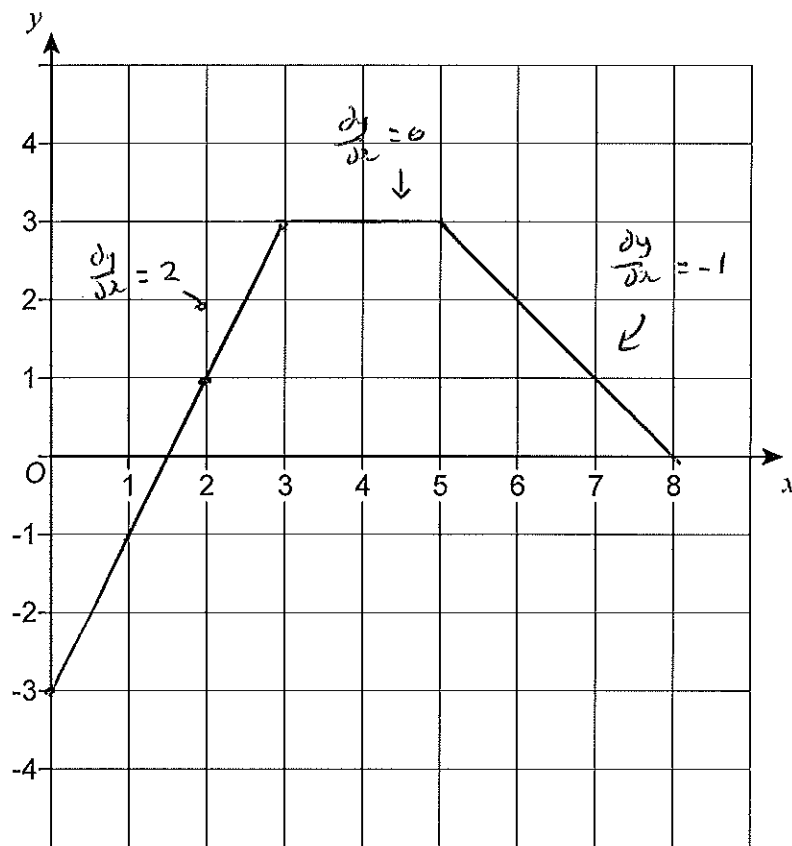
When  $0 \leq x < 3$   $\frac{dy}{dx} = 2$  and this part of the graph passes through  $(2, 1)$ .

When  $3 \leq x < 5$   $\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \text{gradient!}$

When  $5 \leq x \leq 8$   $\frac{dy}{dx} = -1$

Draw the graph of  $y = f(x)$  for  $0 \leq x \leq 8$



(3 marks)

Turn over for the next question

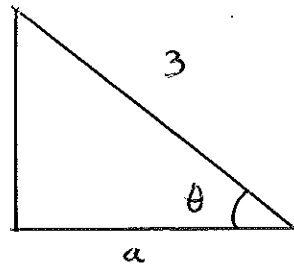


- 11 Angle  $\theta$  is obtuse and  $\sin \theta = \frac{\sqrt{5}}{3}$

Work out the values of  $\cos \theta$ .

$$\sin \theta = \frac{\text{Opp}}{\text{Hyp}} = \frac{\sqrt{5}}{3}$$

$\sqrt{5}$



Need Adjacent by Pythagoras:

$$a^2 = 3^2 - (\sqrt{5})^2$$

$$\rightarrow a^2 = 9 - 5$$

$$\cos \theta = \frac{\text{Adj}}{\text{Hyp}}$$

$$\rightarrow a^2 = 4$$

$$\rightarrow a = 2 \text{ or } -2$$

as  $\theta$  is obtuse

$$\rightarrow \cos \theta = -\frac{2}{3}$$

$$\cos \theta = -\frac{2}{3} \dots \dots \dots (4 \text{ marks})$$

12 (a)  $(x+3)$  is a factor of  $x^3 + 6x^2 + ax - 12 = f(x)$

Show that the value of  $a$  is 5

by factor theorem,

$$f(-3) = 0$$

$$\rightarrow (-3)^3 + 6(-3)^2 + ax - 12 = 0$$

$$\rightarrow -27 + 54 + ax - 12 = 0$$

$$\rightarrow ax - 3x + 15 = 0$$

$$\rightarrow 3x = 15 \rightarrow a = 5$$

(2 marks)

12 (b) Hence, factorise fully  $x^3 + 6x^2 + 5x - 12$

$$f(1) = 1^3 + 6 + 5 - 12 = 0$$

$\therefore (x-1)$  is a factor

$$\rightarrow (x-1)(x+3)(x+4) = x^3 + 6x^2 + 5x - 12$$

must be +4 to give us  $(-12)$

$$(x+3)(x+4)(x-1) \quad (3 \text{ marks})$$

Turn over for the next question

13 Expand and simplify  $(\sqrt{5} + 3)(\sqrt{5} - 2)(\sqrt{5} + 1)$

1st 2 brackets  $5 - 2\sqrt{5} + 3\sqrt{5} - 6$

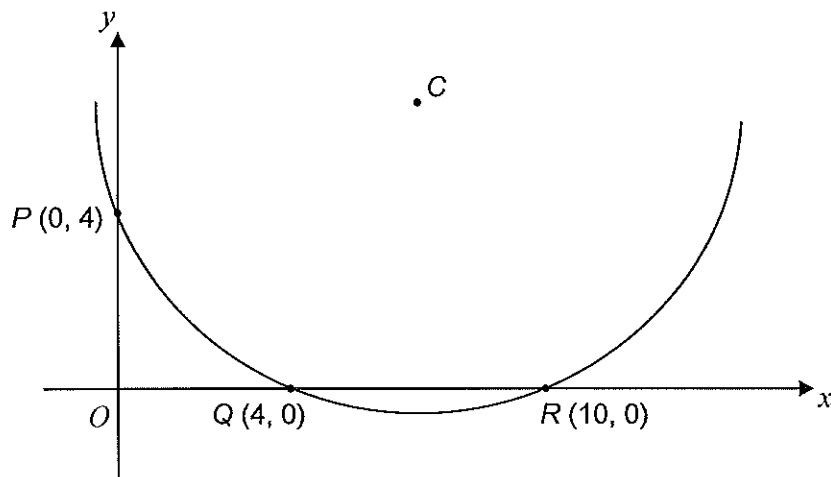
$\rightarrow 5 + \sqrt{5} - 6 \rightarrow -1 + \sqrt{5}$

2nd x 3rd  $(-1 + \sqrt{5})(\sqrt{5} + 1)$

$\rightarrow -\sqrt{5} - 1 + 5 + \sqrt{5} = 4$

Answer ..... 4 ..... (4 marks)

14 The sketch shows part of a circle, centre  $C$ , that intersects the axes at points  $P$ ,  $Q$  and  $R$ .



14 (a) Explain why the centre of the circle lies on the line  $x = 7$ .

Perpendicular bisector of chord passes through

centre of circle

(1 mark)

- 14 (b) Show that the line  $y = x$  is the perpendicular bisector of the line  $PQ$ .

$$\text{Midpoint} = (2, 2)$$

$$\text{Gradient} = -\frac{4}{4} = -1$$

$$\rightarrow \text{gradient of perpendicular} = 1$$

$$x_1 = 2$$

$$y_2 - y_1 = m(x_2 - x_1)$$

$$y_1 = 2$$

$$\rightarrow y - 2 = x - 2$$

$$m = 1$$

$$\rightarrow y = x$$

(2 marks)

- 14 (c) Work out the equation of the circle.

Centre must be where  $y = x$  <sup>AND</sup> ~~AND~~  $x = 7$  (2.5)

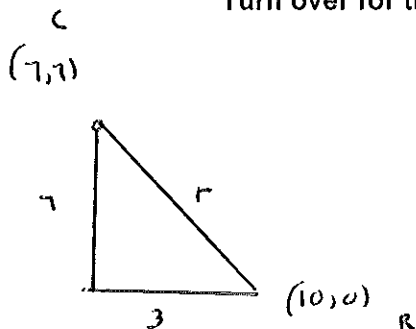
$$\rightarrow (7, 7)$$

See below for radius =  $\sqrt{58}$

$$\rightarrow (x - 7)^2 + (y - 7)^2 = 58$$

Answer ..... (4 marks)

Turn over for the next question



$$r^2 = 7^2 + 3^2$$

$$r^2 = 49 + 9 = 58$$

$$r = \sqrt{58}$$

15

$$2x^2 - 4x + 5 \equiv a(x + b)^2 + c$$

Complete the square!

Work out the values of  $a$ ,  $b$  and  $c$ .

$$2 [x^2 - 2x + 2.5]$$

$$2 [(x-1)^2 + 1.5]$$

$$\rightarrow 2(x-1)^2 + 3$$

$$a = \dots\dots\dots 2 \dots\dots\dots, b = \dots\dots\dots -1 \dots\dots\dots, c = \dots\dots\dots 3 \dots\dots\dots \quad (4 \text{ marks})$$

16

A curve has equation  $y = 4x^3 + 6x^2 + 3x + 5$

Work out the coordinates of any stationary points on this curve and determine their nature.

$$\frac{dy}{dx} = 12x^2 + 12x + 3$$

At stationary point,  $\frac{dy}{dx} = 0$

$$\Rightarrow 12x^2 + 12x + 3 = 0$$

$$\Rightarrow 4x^2 + 4x + 1 = 0$$

$$\Rightarrow (2x + 1)(2x + 1) = 0$$

$\Rightarrow x = -\frac{1}{2}$  is only stationary point

Try  $x = 0 \Rightarrow \frac{dy}{dx} = 3$  (+ve)

Try  $x = -1 \Rightarrow \frac{dy}{dx} = 12(-1)^2 - 12 + 3 = 3$  (+ve)

$x$	-1	-1/2	0
(grad)	/	-	/

∴ point of inflection

Find  $y$  coordinate:

$$y = 4\left(-\frac{1}{2}\right)^3 + 6\left(-\frac{1}{2}\right)^2 + 3\left(-\frac{1}{2}\right) + 5$$

$$= -\frac{4}{8} + \frac{6}{4} - \frac{3}{2} + 5$$

$$= 4.5 \text{ or } 4\frac{1}{2}$$

Answer  $\left(-\frac{1}{2}, 4\frac{1}{2}\right)$  (6 marks)

10

END OF QUESTIONS