(MR. BARTON'S ANSWERS)

Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Pages	Mark
3	
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12 - 13	
14 - 15	
TOTAL	

AQA Level 2 Certificate in Further Mathematics

Further Mathematics Level 2 8360/1

Practice Paper Set 1

Paper 1

Non-Calculator

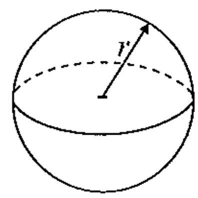
For this paper you must have: • mathematical instruments. You may **not** use a calculator.	

Time allowed
1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the space provided. Do not write outside the box around each page or on blank pages.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- In all calculations, show clearly how you work out your answer.

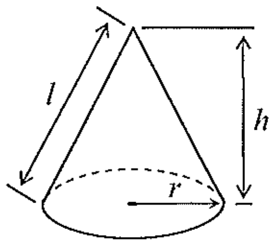
Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 70.
- You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer booklet.

Formulae Sheet



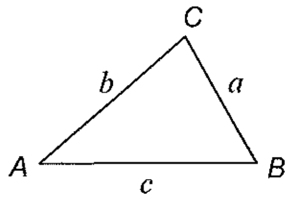
Volume of sphere $= \frac{4}{3}\pi r^3$

Surface area of sphere $= 4\pi r^2$



Volume of cone $= \frac{1}{3}\pi r^2 h$

Curved surface area of cone $= \pi r l$



In any triangle ABC

Area of triangle $= \frac{1}{2}ab \sin C$

Sine rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine rule $a^2 = b^2 + c^2 - 2bc \cos A$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

The Quadratic Equation

The solutions of $ax^2 + bx + c = 0$, where $a \neq 0$, are given by $x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$

Trigonometric Identities

$\tan\theta \equiv \frac{\sin\theta}{\cos\theta}$ $\qquad \sin^2\theta + \cos^2\theta \equiv 1$

Do not write outside the box

Answer **all** questions in the spaces provided.

1 (a) x is an integer such that $6 < 3x \leqslant 20$

Write down all the possible values of x.

$\div 3$ $\frac{6}{3} < x \leq \frac{20}{3}$

$2 < x \leq 6\frac{2}{3}$

Answer 3, 4, 5, 6 *(3 marks)*

1 (b) Given that $-1 < n < 2$ state a value of n for which

1 (b) (i) $n^2 > 1$

Anything between 1 + 2

Answer 1.5 *(1 mark)*

1 (b) (ii) $\frac{1}{n} > 1$

Anything between 0 + 1

Answer 0.5 *(1 mark)*

1 (b) (iii) $1 - n > 1$

Anything between -1 + 0

Answer -0.5 *(1 mark)*

Turn over for the next question

6

Do not write outside the box

2 (a) A function $f(x)$ is defined as

$$\begin{aligned} f(x) &= 3 & 0 \leqslant x < 2 \\ &= x + 1 & 2 \leqslant x < 4 \\ &= 9 - x & 4 \leqslant x \leqslant 9 \end{aligned}$$

$y = 3$

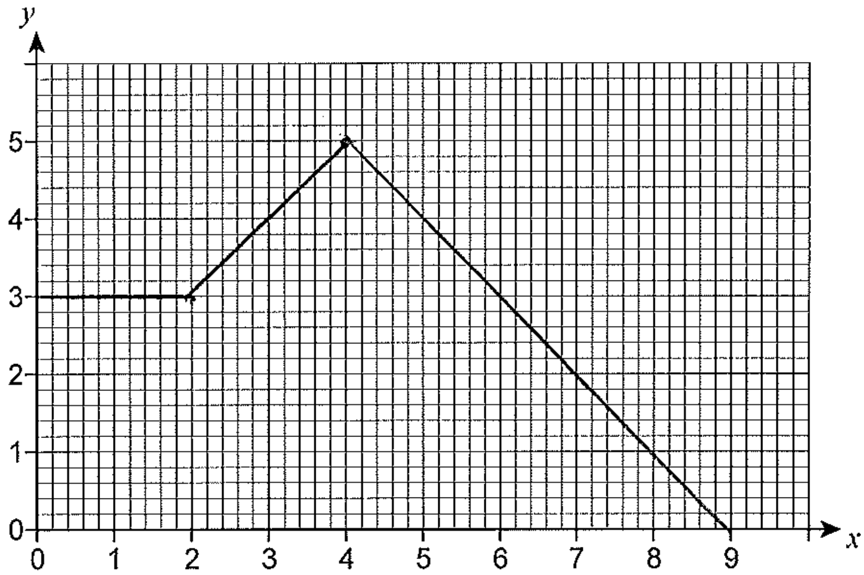
$y = x + 1$

$y = 9 - x$

Draw the graph of $y = f(x)$ on the grid below for values of x from 0 to 9.

....

....



(3 marks)

2 (b) Write down the range of $f(x)$.

Range = y values

Answer .... $0 \leq y \leq 5$ .... *(1 mark)*

T

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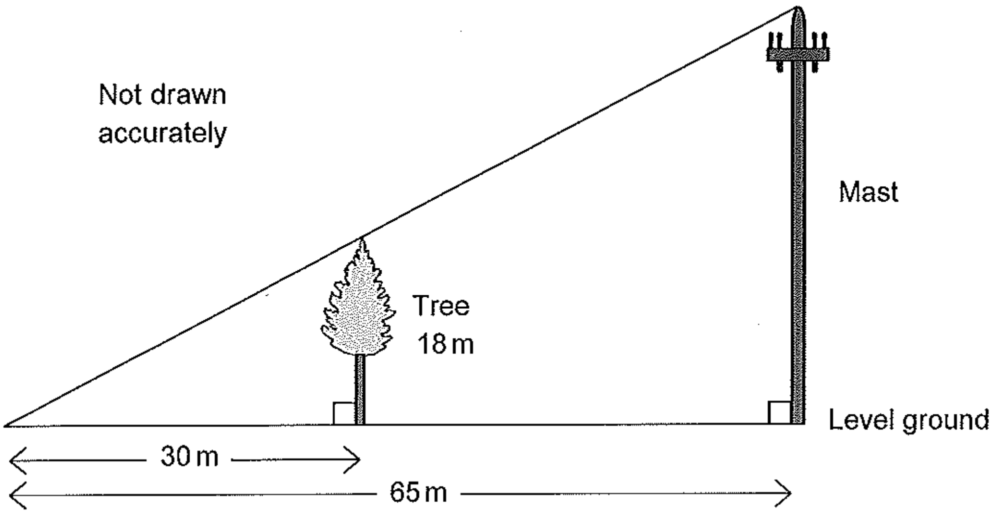
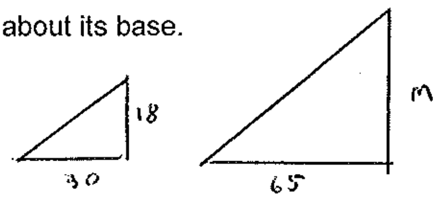
3 The diagram shows a tree of height 18 metres and a mast on level ground.



The mast is about to fall over, pivoting about its base.

Could it hit the tree?

Show clearly how you decide.



Similar Triangles

$m = \frac{65}{30} \times 18$

Scale factor

$= \frac{13}{6} \times 18 = 13 \times 3 = 39$

The mast is 39 m tall

(4 marks)

The tree is (65 − 30), 35 m from the mast

Turn over for the next question

So, the mast could hit the tree.

8

Do not write outside the box

4 (a) Simplify fully $\frac{\sqrt{12}}{\sqrt{48}}$ $= \sqrt{\frac{12}{48}} = \frac{\sqrt{1}}{\sqrt{4}} = \frac{1}{2}$

$\sqrt{\frac{1}{4}}$

Answer ½ *(3 marks)*

4 (b) Write down the value of $(\sqrt{6} - \sqrt{2})(\sqrt{6} + \sqrt{2})$

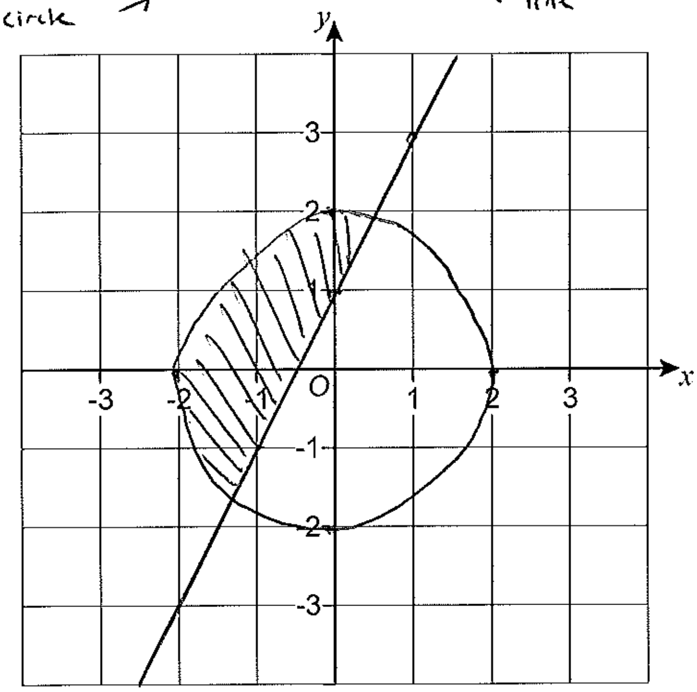
$= 6 + \sqrt{12} - \sqrt{12} - 2 = 4$

Answer 4 *(2 marks)*

5 (a) Sketch the graphs of $x^2 + y^2 = 4$ and $y = 2x + 1$

circle ↗ ↖ line

x	1	2	0
y	3	5	1



(2 marks)

5 (b) On your sketch, shade the region that satisfies both of the inequalities

$x^2 + y^2 \leqslant 4$ and $y \geqslant 2x + 1$ *(1 mark)*

↑ ↑

Inside circle "above" line

////

T

Do not write outside the box

6 Given that $\sin\theta = \frac{3}{5}$, work out the **two** possible values of $\cos\theta$.

$\sin^2\theta + \cos^2\theta = 1$

$\left(3/5\right)^2 + \cos^2\theta = 1$

$\Rightarrow \cos^2\theta = 1 - 9/25 = 16/25$

$\Rightarrow \cos\theta = \pm\sqrt{16/25} = \pm 4/5$

Answer $4/5$ and $-4/5$ *(3 marks)*

7 Here are the equations of three lines.

(1) $y = \frac{1}{2}x + 11$ (2) $y = \frac{1}{3}x + 14$ (3) $y = 2x - 16$

Do all three lines meet at a common point?

Show how you decide.

Find where (1) & (2) cross:

$1/2x + 11 = 1/3x + 14$

$(\times 6)$ $3x + 66 = 2x + 84$

$-2x$ $x + 66 = 84$

-66 $x = 18$

use (1) to find y: $y = 1/2(18) + 11$

$= 20$

(5 marks)

check (3) to see if (18, 20) lies on line:

$y = 2(18) - 16 = 20$ ✓

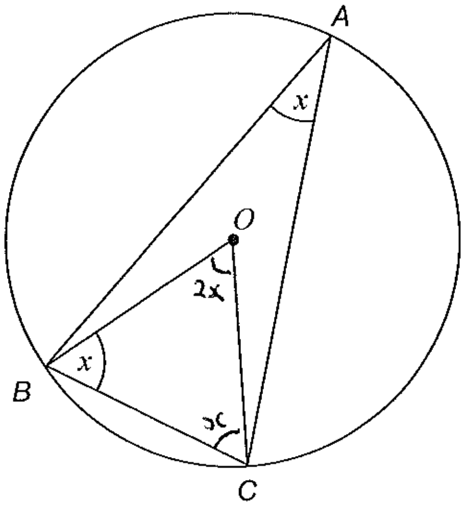
Turn over for the next question

So all 3 lines meet at (18, 20)

16

8 A, B and C are points on a circle, centre O.

Angle BAC = angle $OBC = x$.



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Prove that angle $BOC = 90°$

BCO = x (isosceles triangle)

BOA = 2x (angle at the centre = twice angle at circumference)

x + 2x + x = 180 (angles in a triangle = 180°)

4x = 180

→ x = 45°

→ BOC = 2(45) = 90°

(4 marks)

T

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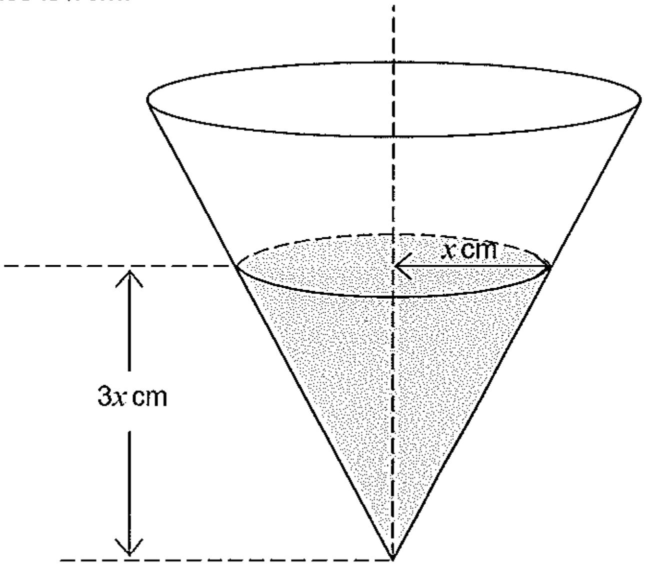
9 A right circular cone is being filled with water.

The volume of the water is y cm ^{3} when the depth of water is $3x$ cm and the radius of the surface is x cm.



9 (a) Show that $y = \pi x^3$

Volume $= \frac{1}{3}\pi r^2 h$

$\rightarrow \quad y = \frac{1}{3}\pi x^2 \times 3x$

$\rightarrow \quad y = \pi x^3$

(1 mark)

9 (b) Work out $\frac{dy}{dx}$ π is just a number!

Give your answer in terms of π.

$\frac{dy}{dx} = 3\pi x^2$

Answer .... *(1 mark)*

$\frac{dy}{dx}$

9 (c) Work out the rate of change of y with respect to x when $x = 5$.

Give your answer in terms of π.

when $x = 5$,

$dy/dx = 3\pi (5)^2$

$= 75\pi$

Answer .... *(2 marks)*

8

10 Simplify fully $\dfrac{3x^2 - x - 14}{9x^2 - 4} \div \dfrac{x + 2}{3x^2 + 2x}$

$= \dfrac{3x^2 - x - 14}{9x^2 - 4} \times \dfrac{3x^2 + 2x}{x + 2}$

$= \dfrac{(3x - 7)(x + 2)}{(3x - 2)(3x + 2)} \times \dfrac{x(3x + 2)}{x + 2}$

Diff of 2 squares $\longrightarrow$

$= \dfrac{(3x - 7)(x + 2)(x)(3x + 2)}{(3x - 2)(3x + 2)(x + 2)} = \dfrac{x(3x - 7)}{3x - 2}$

Answer .... *(5 marks)*

T

11 Show that the tangents to the curve $y = x^3 + 3x^2 + 3x + 1$ at $x = 1$ and $x = -3$ are parallel.

Need to find gradients of tangents:

$dy/dx = 3x^2 + 6x + 3$

when $x = 1 \rightarrow dy/dx = 3(1)^2 + 6(1) + 3 = 12$

when $x = -3 \rightarrow dy/dx = 3(-3)^2 + 6(-3) + 3$

$= 27 - 18 + 3 = 12$

dy/dx gives gradient, both gradients $= 12$

$\therefore$ the lines are Parallel

(5 marks)

Turn over for the next question

10

12 Make x the subject of $\frac{12}{y} = \frac{4}{x} - \frac{1}{3}$

① Make common denominator on right

$\rightarrow \frac{12}{y} = \frac{12}{3x} - \frac{x}{3x}$

$\rightarrow \frac{12}{y} = \frac{12 - x}{3x}$

$(\times 3x) \rightarrow \frac{36x}{y} = 12 - x$

$(\times y) \rightarrow 36x = 12y - xy$

$(+xy) \rightarrow 36x + xy = 12y$

(FACT) $\rightarrow x(36 + y) = 12y$

$\div (36+y) \rightarrow x = \frac{12}{36+y}$

Answer $x = \frac{12}{36+y}$ *(5 marks)*

13 $x^3 + 2x^2 - 9x - 18 \equiv (x^2 - a^2)(x + b)$ where a and b are integers.

Work out the three linear factors of $x^3 + 2x^2 - 9x - 18$

You get -18 by $-a^2 \times b$

Only number that works for a is 3

$-(3^2) \times b = 18 \rightarrow b = 2$

Answer $\therefore (x^2 - 3^2)(x + 2)$ *(3 marks)*

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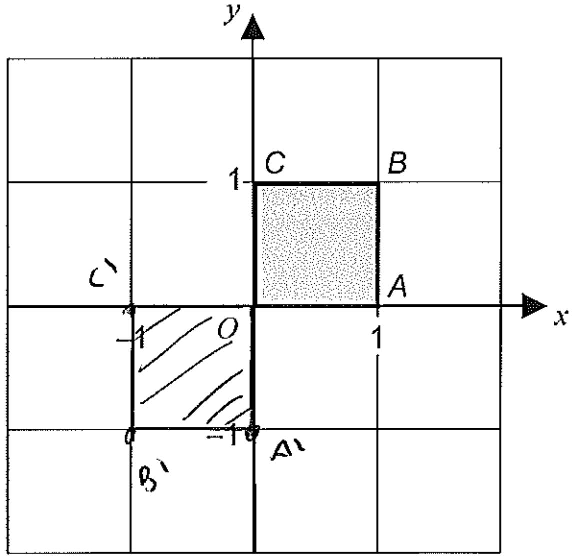
14 The diagram shows the unit square *OABC*.



Reflection in $y = -x$

14 (a) The image of *OABC* after transformation by the matrix $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ is $OA'B'C'$.

Draw and label $OA'B'C'$.

(2 marks)

$T_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

14 (b) The unit square *OABC* is transformed by reflection in the line $y = x$ followed by enlargement about the origin with scale factor 2. $T_2 = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$

What is the matrix of the combined transformation?

Order : $T_2 \times T_1 \times$ Object

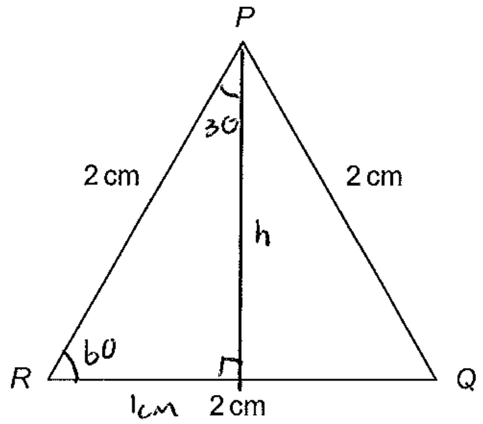
$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix}$$

Answer $\begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix}$ (4 marks)

14

15 (a) Use the equilateral triangle *PQR* to show that $\cos 60° = \frac{1}{2}$



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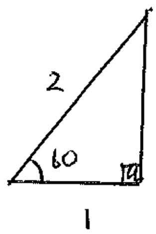
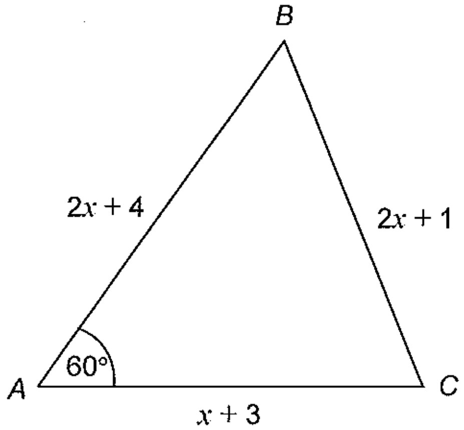
use Pythag for h: $h^2 =$ ← Don't need! :)

$\cos 60 = \frac{adj}{hyp} = \frac{1}{2}$

(2 marks)



15 (b) In triangle *ABC*, angle *BAC* = 60°



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Use the cosine rule to show that $x = 4 + 2\sqrt{7}$

By cosine rule: $(2x+1)^2 = (x+3)^2 + (2x+4)^2 -$

$\boxed{\cos 60 = \frac{1}{2}}$

$2(x+3)(2x+4)\cos(60)$

$\rightarrow 4x^2 + 4x + 1 = x^2 + 6x + 9 + 4x^2 + 16x + 16 -$

$2[2x^2 + 4x + 6x + 12] \times \frac{1}{2}$

$= (2x^2 + 10x + 12)$

$\rightarrow 4x^2 + 4x + 1 = 3x^2 + 12x + 13$

$\rightarrow x^2 + 4x + 1 = 12x + 13$

$\rightarrow x^2 - 8x - 12 = 0$

Complete square (or use formula)

$\rightarrow (x-4)^2 - 28 = 0$

(6 marks)

$\rightarrow (x-4)^2 = 28$

$\rightarrow x - 4 = \pm\sqrt{28}$ $= \sqrt{4} \times \sqrt{7}$

END OF QUESTIONS

$\rightarrow x - 4 = \pm 2\sqrt{7}$

$\rightarrow x = 4 \pm 2\sqrt{7}$

8

But x cannot be negative,

so $x = 4 + 2\sqrt{7}$