Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
TOTAL	

General Certificate of Education
Advanced Subsidiary Examination
January 2012

Mathematics MD01

Unit Decision 1

Monday 23 January 2012 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- You do not necessarily need to use all the space provided.

Do not write outside the box

Answer **all** questions in the spaces provided.

1 Use a Shell sort to rearrange the following numbers into ascending order, showing the new arrangement after each pass.

37 25 16 12 36 24 13 11 *(5 marks)*

QUESTION PART REFERENCE

n = 8, 4 sublists

37 37 36
25 25 24
16 16 13
12 12 11
36 ↙
24 swaps 36 37
13 24 25
11 13 16
11 12

36 now 2 36 13
24 sublists 24 11
13 13 → 16
11 → 11 12
37 37 36
25 25 24
16 16 37
12 12 25

13 11 16 12 36 24 37 25
11 12 13 16 24 25 36 37

Do not write outside the box

QUESTION PART REFERENCE

Do not write outside the box

2 (a) Draw a bipartite graph representing the following adjacency matrix.

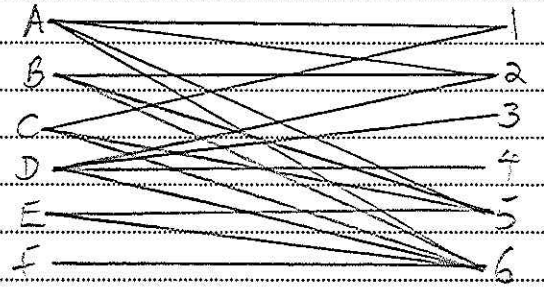
	1	2	3	4	5	6
A	1	1	0	0	1	1
B	0	1	0	0	1	1
C	1	0	0	0	1	1
D	0	1	1	1	0	1
E	0	0	0	0	1	1
F	0	0	0	0	0	1

(2 marks)

(b) Given that *A*, *B*, *C*, *D*, *E* and *F* represent six people and that 1, 2, 3, 4, 5 and 6 represent six tasks to which they may be assigned, explain why a complete matching is impossible. *(3 marks)*

QUESTION PART REFERENCE

(a)



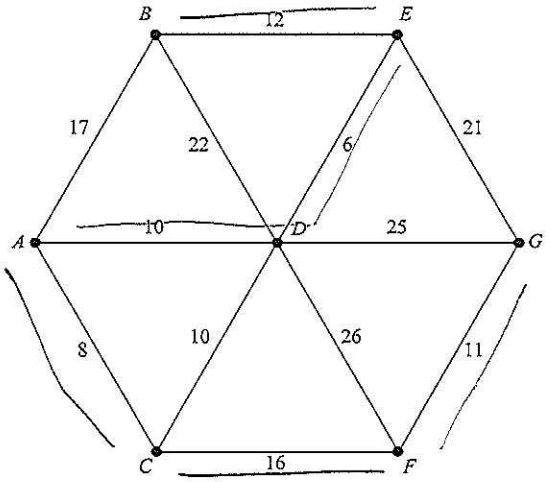
(b) Two people cannot be allocated the same task, therefore F must be with 6, E must be with 5, B must be with 2.

Do not write outside the box

QUESTION PART REFERENCE

Do not write outside the box

3 The following network shows the roads connecting seven villages, A, B, C, ..., G. The number on each edge represents the length, in miles, between a pair of villages.



(a) Use Kruskal's algorithm to find a minimum spanning tree for the network. *(5 marks)*

(b) State the length of your minimum spanning tree. *(1 mark)*

(c) There are two minimum spanning trees for this network. Draw both of these minimum spanning trees. *(3 marks)*

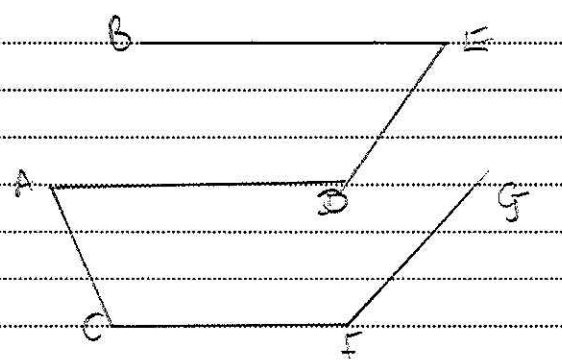
QUESTION PART REFERENCE

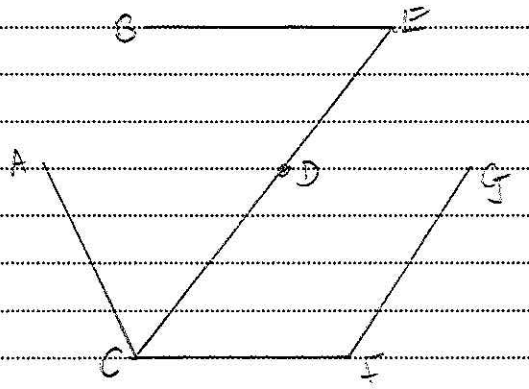
(a) ED 6
AC 8
AD 10
GF 11
BE 12
CF 16
(b) Total 63

Do not write outside the box

QUESTION PART REFERENCE

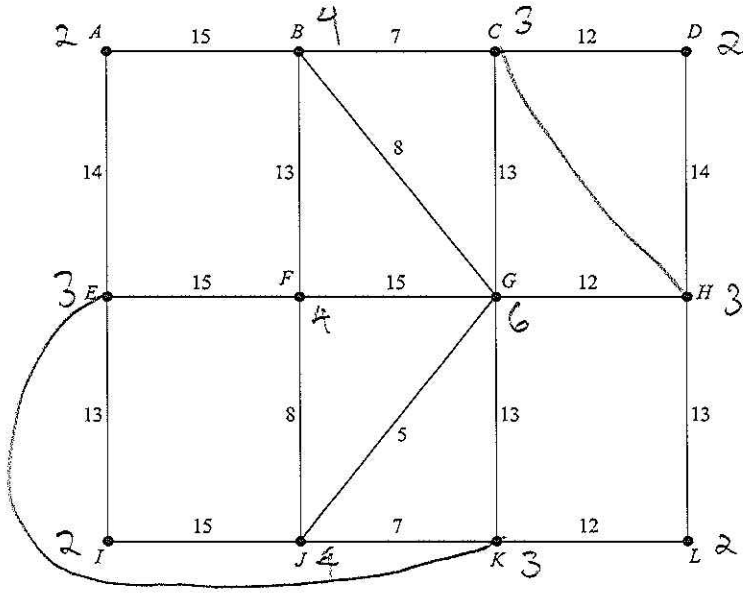
(c)





Do not write outside the box

4 The following network shows the times, in minutes, taken by a policeman to walk along roads connecting 12 places, A, B, ..., L, on his beat. Each day, the policeman has to walk along each road at least once, starting and finishing at A.



The total of all the times in the network is 224 minutes.

(a) Find the length of an optimal Chinese postman route for the policeman. *(5 marks)*

(b) State the number of times that the vertex J would appear in a route corresponding to the length found in part **(a)**. *(1 mark)*

QUESTION PART REFERENCE

(a) Odd C E K H

Pairings

CE + KH = 35 + 24 = 59

CK + EH = 25 + 40 = 65

CH + EK = 25 + 30 = 55 ✓

Total 224 + 55 = 279

Do not write outside the box

QUESTION PART REFERENCE

(b) 3

Turn over ▶

Do not write outside the box

5 The feasible region of a linear programming problem is determined by the following:

$$y \geqslant 20$$
$$x + y \geqslant 25$$
$$5x + 2y \leqslant 100$$
$$y \leqslant 4x$$
$$y \geqslant 2x$$

(a) On **Figure 1** opposite, draw a suitable diagram to represent the inequalities and indicate the feasible region. *(6 marks)*

(b) Use your diagram to find the minimum value of P, on the feasible region, in the case where:

(i) $P = x + 2y$;

(ii) $P = -x + y$.

In each case, state the corresponding values of x and y. *(4 marks)*

QUESTION PART REFERENCE

(b) minimum at $x = 5$, $y = 20$

(i) $P = 5 + (2 \times 20) = 45$

(ii) $P = -x + y$

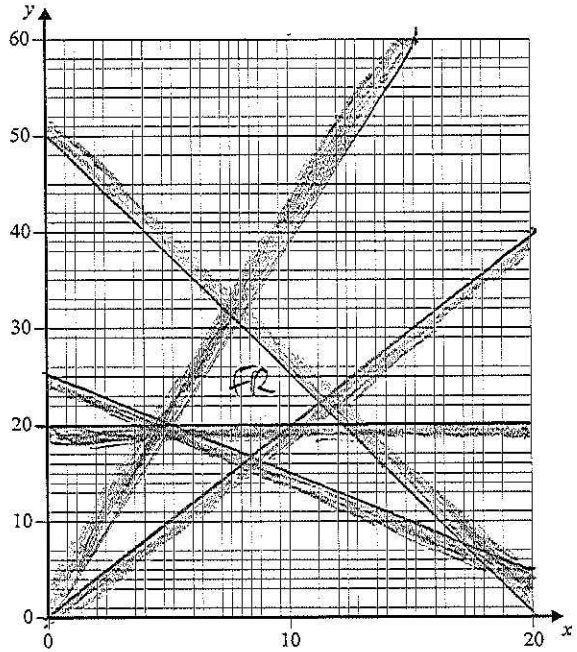
minimum at $x = 10$, $y = 20$

$P = -10 + 20 = 10$

Do not write outside the box

QUESTION PART REFERENCE

Figure 1



Turn over ▶

Do not write outside the box

6 The network below shows the lengths of roads, in miles, connecting 10 towns, $A, B, \ldots, J$.

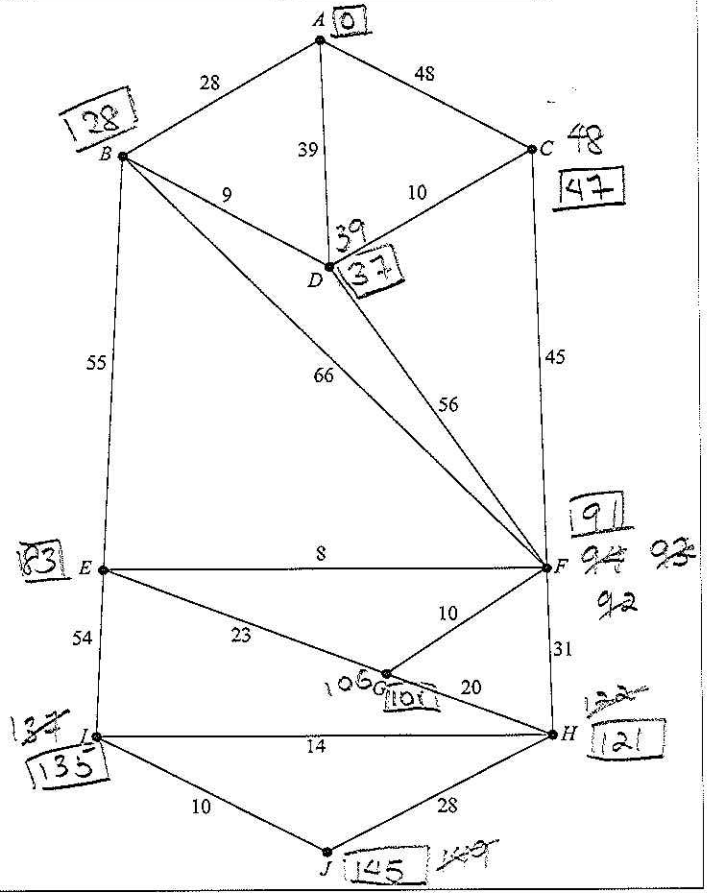
(a) Use Dijkstra's algorithm on the network to find the shortest distance from A to J. Show all your working at each vertex. *(7 marks)*

(b) Write down the corresponding route. *(1 mark)*

(c) A new road is to be constructed connecting B to G. Find the length of this new road if the shortest distance from A to J is reduced by 10 miles. State the new route. *(3 marks)*

QUESTION PART REFERENCE

(a)



Do not write outside the box

QUESTION PART REFERENCE

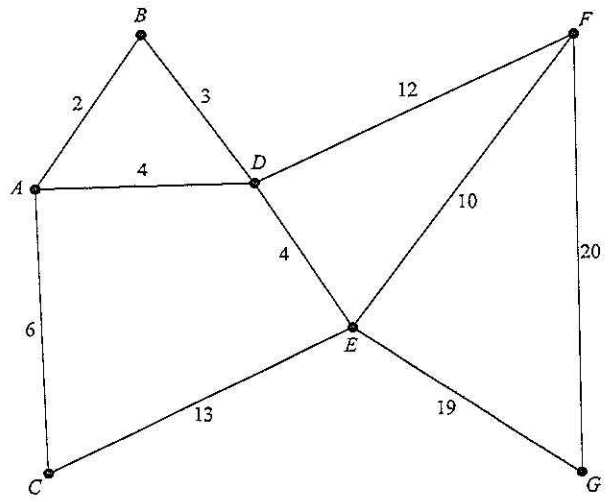
(b) Route A B E F G H I J

(c) 135 − (28 + GJ)
GJ = 44
= 135 − (28 + 44) = 63

Route A B G H I J

Do not write outside the box

7 The diagram shows the locations of some schools. The number on each edge shows the distance, in miles, between pairs of schools.



Sam, an adviser, intends to travel from one school to the next until he has visited all of the schools, before returning to his starting school. The shortest distances for Sam to travel between some of the schools are shown in **Table 1** opposite.

(a) Complete **Table 1**. *(2 marks)*

(b) (i) On the completed **Table 1**, use the nearest neighbour algorithm, starting from B, to find an upper bound for the length of Sam's tour. *(4 marks)*

(ii) Write down Sam's actual route if he were to follow the tour corresponding to the answer in part **(b)(i)**. *(2 marks)*

(iii) Using the nearest neighbour algorithm, starting from each of the other vertices in turn, the following upper bounds for the length of Sam's tour were obtained: 77, 77, 77, 76, 77 and 76.

Write down the best upper bound. *(1 mark)*

Do not write outside the box

QUESTION PART REFERENCE

Table 1

	A	B	C	D	E	F	G
A	–	2	6	4	8	16	27
B	2	–	8	3	7	15	26
C	6	8	–	10	13	22	32
D	4	3	10	–	4	12	23
E	8	7	13	4	–	10	19
F	16	15	22	12	10	–	20
G	27	26	32	23	19	20	–

b

(i) B A D E F G C B

total = 80 (2 + 4 + 4 + 10 + 20 + 32 + 8)

(ii) B A D E F G E C A B

(iii) 76 (lowest upper bound)

Question 7 continues on the next page

Do not write outside the box

7 (c) (i) On **Table 2** below, showing the order in which you select the edges, use Prim's algorithm, starting from *A*, to find a minimum spanning tree for the schools *A*, *B*, *C*, *D*, *F* and *G*. *(4 marks)*

(ii) Hence find a lower bound for the length of Sam's minimum tour. *(3 marks)*

(iii) By deleting each of the other vertices in turn, the following lower bounds for the length of a minimum tour were found: 50, 48, 52, 51, 56 and 64.

Write down the best lower bound. *(1 mark)*

(d) Given that the length of a minimum tour is *T* miles, use your answers to parts **(b)** and **(c)** to write down the smallest interval within which you know *T* must lie. *(2 marks)*

QUESTION PART REFERENCE

Table 2

	A (1)	B (2)	C (4)	D (3)	F (5)	G (6)
A	–	2	6	4	16	27
B	2	–	8	3	15	26
C	6	8	–	10	22	32
D	4	3	10	–	12	23
F	16	15	22	12	–	20
G	27	26	32	23	20	

(ii) lower bound

2 + 6 + 3 + 12 + 20 = 43

43 + (4 + 7) = 54

(iii) 64 (largest lower bound)

Do not write outside the box

QUESTION PART REFERENCE

(d) $64 \leq T \leq 76$

Turn over ▶

Do not write outside the box

8 Four **distinct positive integers** are $(3x-5)$, $(2x+3)$, $(x+1)$ and $(4x-13)$.

(a) Explain why $x \geqslant 4$. *(2 marks)*

(b) The four integers are to be sorted into ascending order using a **bubble sort**.

The original list is	$(3x-5)$	$(2x+3)$	$(x+1)$	$(4x-13)$
After the first pass, the list is	$(3x-5)$	$(x+1)$	$(4x-13)$	$(2x+3)$
After the second pass, the list is	$(x+1)$	$(4x-13)$	$(3x-5)$	$(2x+3)$
After the third pass, the list is	$(4x-13)$	$(x+1)$	$(3x-5)$	$(2x+3)$

(i) By considering the list after the first pass, write down **three** inequalities in terms of x. *(3 marks)*

(ii) By considering the list after the second pass, write down **two** further inequalities in terms of x. *(2 marks)*

(iii) By considering the list after the third pass, write down **one** further inequality in terms of x. *(1 mark)*

(c) Hence, by considering the results above, find the value of x. *(2 marks)*

QUESTION PART REFERENCE

(a) $3x - 5 > 0$

$2x + 3 > 0$

$x + 1 > 0$

$4x - 13 > 0$ $\quad x > 13/4 > 3\frac{1}{4}$

integer $x \geq 4$.

(b)

(i) $2x + 3 > 3x - 5 > x + 1 > 4x - 13$

(ii) $3x - 5 > 4x - 13$

(iii) $x + 1 > 4x - 13$

Do not write outside the box

QUESTION PART REFERENCE

6

(i) $\frac{13}{4} < x < \frac{14}{3}$

$3\frac{1}{4} < x < 4\frac{2}{3}$

$x = 4$

Turn over ▶

Do not write outside the box

QUESTION PART REFERENCE

END OF QUESTIONS