

General Certificate of Education
January 2007
Advanced Level Examination



MATHEMATICS
Unit Pure Core 4

MPC4

Thursday 25 January 2007 9.00 am to 10.30 am

For this paper you must have:

- an 8-page answer book
 - the **blue** AQA booklet of formulae and statistical tables.
- You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC4.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

Information

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 A curve is defined by the parametric equations

$$x = 1 + 2t, \quad y = 1 - 4t^2$$

(a) (i) Find $\frac{dx}{dt}$ and $\frac{dy}{dt}$. (2 marks)

(ii) Hence find $\frac{dy}{dx}$ in terms of t . (2 marks)

(b) Find an equation of the normal to the curve at the point where $t = 1$. (4 marks)

(c) Find a cartesian equation of the curve. (3 marks)

2 The polynomial $f(x)$ is defined by $f(x) = 2x^3 - 7x^2 + 13$.

(a) Use the Remainder Theorem to find the remainder when $f(x)$ is divided by $(2x - 3)$. (2 marks)

(b) The polynomial $g(x)$ is defined by $g(x) = 2x^3 - 7x^2 + 13 + d$, where d is a constant.

Given that $(2x - 3)$ is a factor of $g(x)$, show that $d = -4$. (2 marks)

(c) Express $g(x)$ in the form $(2x - 3)(x^2 + ax + b)$. (2 marks)

3 (a) Express $\cos 2x$ in terms of $\sin x$. (1 mark)

(b) (i) Hence show that $3 \sin x - \cos 2x = 2 \sin^2 x + 3 \sin x - 1$ for all values of x . (2 marks)

(ii) Solve the equation $3 \sin x - \cos 2x = 1$ for $0^\circ < x < 360^\circ$. (4 marks)

(c) Use your answer from part (a) to find $\int \sin^2 x \, dx$. (2 marks)

- 4 (a) (i) Express $\frac{3x-5}{x-3}$ in the form $A + \frac{B}{x-3}$, where A and B are integers. (2 marks)
- (ii) Hence find $\int \frac{3x-5}{x-3} dx$. (2 marks)
- (b) (i) Express $\frac{6x-5}{4x^2-25}$ in the form $\frac{P}{2x+5} + \frac{Q}{2x-5}$, where P and Q are integers. (3 marks)
- (ii) Hence find $\int \frac{6x-5}{4x^2-25} dx$. (3 marks)
- 5 (a) Find the binomial expansion of $(1+x)^{\frac{1}{3}}$ up to the term in x^2 . (2 marks)
- (b) (i) Show that $(8+3x)^{\frac{1}{3}} \approx 2 + \frac{1}{4}x - \frac{1}{32}x^2$ for small values of x . (3 marks)
- (ii) Hence show that $\sqrt[3]{9} \approx \frac{599}{288}$. (2 marks)
- 6 The points A , B and C have coordinates $(3, -2, 4)$, $(5, 4, 0)$ and $(11, 6, -4)$ respectively.
- (a) (i) Find the vector \overrightarrow{BA} . (2 marks)
- (ii) Show that the size of angle ABC is $\cos^{-1}\left(-\frac{5}{7}\right)$. (5 marks)
- (b) The line l has equation $\mathbf{r} = \begin{bmatrix} 8 \\ -3 \\ 2 \end{bmatrix} + \lambda \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix}$.
- (i) Verify that C lies on l . (2 marks)
- (ii) Show that AB is parallel to l . (1 mark)
- (c) The quadrilateral $ABCD$ is a parallelogram. Find the coordinates of D . (3 marks)

Turn over for the next question

Turn over ►

- 7 (a) Use the identity

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

to express $\tan 2x$ in terms of $\tan x$.

(2 marks)

- (b) Show that

$$2 - 2 \tan x - \frac{2 \tan x}{\tan 2x} = (1 - \tan x)^2$$

for all values of x , $\tan 2x \neq 0$.

(4 marks)

- 8 (a) (i) Solve the differential equation $\frac{dy}{dt} = y \sin t$ to obtain y in terms of t . (4 marks)

- (ii) Given that $y = 50$ when $t = \pi$, show that $y = 50e^{-(1+\cos t)}$. (3 marks)

- (b) A wave machine at a leisure pool produces waves. The height of the water, y cm, above a fixed point at time t seconds is given by the differential equation

$$\frac{dy}{dt} = y \sin t$$

- (i) Given that this height is 50 cm after π seconds, find, to the nearest centimetre, the height of the water after 6 seconds. (2 marks)

- (ii) Find $\frac{d^2y}{dt^2}$ and hence verify that the water reaches a maximum height after π seconds. (4 marks)

END OF QUESTIONS