Centre Number						Candidate Number				
Surname										
Other Names	ANSWERS									
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
TOTAL	

General Certificate of Education
Advanced Subsidiary Examination
June 2013

Mathematics MPC2

Unit Pure Core 2

Monday 13 May 2013 1.30 pm to 3.00 pm

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

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Answer **all** questions.

Answer each question in the space provided for that question.

1 A geometric series has first term 80 and common ratio $\frac{1}{2}$.

(a) Find the third term of the series. *(1 mark)*

(b) Find the sum to infinity of the series. *(2 marks)*

(c) Find the sum of the first 12 terms of the series, giving your answer to two decimal places. *(2 marks)*

QUESTION PART REFERENCE

Answer space for question 1

1(a) $a = 80 \quad r = \frac{1}{2}$

$U_3 = ar^{n-1}$

$= 80 \times \frac{1}{2}^2$

$= \underline{20}$

b) $S_\infty = \frac{a}{1-r}$

$= \frac{80}{1-\frac{1}{2}} = \underline{160}$

c) $S_n = \cancel{\frac{n}{2}(2a+(n-1)d)}$ $\quad S_{12} = \frac{a(1-r^n)}{1-r}$

$= \cancel{\frac{12}{2}(2(80)+(11)(\frac{1}{2}))}$ $\quad = \frac{80(1-\frac{1}{2}^{11})}{1-\frac{1}{2}}$

$= \cancel{993}$ $\quad = 159.960937..$

$= \underline{159.96}$

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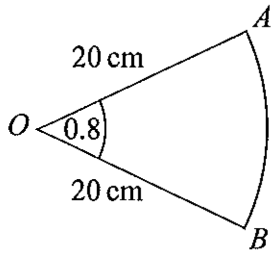
QUESTION PART REFERENCE

Answer space for question 1

2 The diagram shows a sector OAB of a circle with centre O.

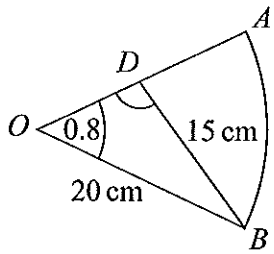


The radius of the circle is 20 cm and the angle $AOB = 0.8$ radians.

(a) Find the length of the arc AB. *(2 marks)*

(b) Find the area of the sector OAB. *(2 marks)*

(c) A line from B meets the radius OA at the point D, as shown in the diagram below.



The length of BD is 15 cm. Find the size of the **obtuse** angle ODB, in **radians**, giving your answer to three significant figures. *(4 marks)*

QUESTION PART REFERENCE

Answer space for question 2

2a) arc length $= r\theta$

$= 20 \times 0.8$

$= \underline{16 \text{cm}}$

b) area $= \frac{1}{2} r^2 \theta$

$= \frac{1}{2} \times 20^2 \times 0.8$

$= \underline{160 \text{cm}^2}$

Do not write outside the box

QUESTION PART REFERENCE	Answer space for question 2
c)	$\frac{\sin 0.8}{15} = \frac{\sin \theta}{20}$
	$\sin A = \frac{20 \times \sin 0.8}{15}$
	$= 1.27467\ldots$ (acute)
	obtuse $= \pi - 1.27467\ldots$
	$= 1.86692\ldots$
	$= \underline{1.87}$ (3sf)

Turn over ▶

Do not write outside the box

3 (a) (i) Using the binomial expansion, or otherwise, express $(2+y)^3$ in the form $a+by+cy^2+y^3$, where a, b and c are integers. *(2 marks)*

(ii) Hence show that $(2+x^{-2})^3+(2-x^{-2})^3$ can be expressed in the form $p+qx^{-4}$, where p and q are integers. *(3 marks)*

(b) (i) Hence find $\int\left[(2+x^{-2})^3+(2-x^{-2})^3\right]\mathrm{d}x$. *(2 marks)*

(ii) Hence find the value of $\int_1^2\left[(2+x^{-2})^3+(2-x^{-2})^3\right]\mathrm{d}x$. *(2 marks)*

QUESTION PART REFERENCE

Answer space for question 3

3ai) $(2+y)^3 = (1)(2)^3(y)^0 + (3)(2)^2(y)^1 + (3)(2)^1(y)^2 + (1)(2)^0(y)^3$

$= 8+12y+6y^2+y^3$

ii) $(2+x^{-2})^3 = 8+12(x^{-2})+6(x^{-2})^2+(x^{-2})^3$

$= 8+12x^{-2}+6x^{-4}+x^{-6}$

$(2+x^{-2})^3 = 8+12(-x^{-2})+6(-x^{-2})^2+(-x^{-2})^3$

$= 8-12x^{-2}+6x^{-4}-x^{-6}$

$(8+12x^{-2}+6x^{-4}+x^{-6})+(8-12x^{-2}+6x^{-4}-x^{-6})$

$\underline{16+12x^{-4}}$

bi) $\int 16+12x^{-4}\,dx = 16x+\frac{12x^{-3}}{-3}+c$

$= \underline{16x-4x^{-3}+c}$

Do not write outside the box

QUESTION PART REFERENCE

Answer space for question 3

ii) $\int_1^2 16 + 12x^{-4}\,dx = \left[16x - 4x^{-3}\right]_1^2$

$= \left(16(2) - 4(2)^{-3}\right) - \left(16(1) - 4(1)^{-3}\right)$

$= \left(32 - \frac{1}{2}\right) - \left(16 - 4\right)$

$= 31.5 - 12$

$= \underline{19.5}$

Turn over ▶

Do not write outside the box

4 (a) Sketch the graph of $y = 9^x$, indicating the value of the intercept on the y-axis. *(2 marks)*

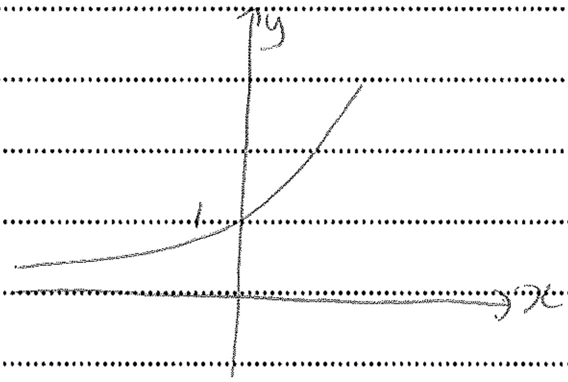
(b) Use logarithms to solve the equation $9^x = 15$, giving your value of x to three significant figures. *(2 marks)*

(c) The curve $y = 9^x$ is reflected in the y-axis to give the curve with equation $y = f(x)$. Write down an expression for $f(x)$. *(1 mark)*

QUESTION PART REFERENCE

Answer space for question 4

4a)



b) $9^x = 15$

$x \log 9 = \log 15$

$x = \dfrac{\log 15}{\log 9}$

$= 1.232486\ldots$

$= 1.23$ (3sf)

c) $9^x = g(x) \rightarrow g(-x) = 9^{-x}$

$f(x) = 9^{-x}$

Do not write outside the box

QUESTION PART REFERENCE

Answer space for question 4

Turn over ▶

Do not write outside the box

5 (a) Use the trapezium rule with five ordinates (four strips) to find an approximate value for $\int_0^2 \sqrt{8x^3+1}\,dx$, giving your answer to three significant figures. *(4 marks)*

(b) Describe the single transformation that maps the graph of $y = \sqrt{8x^3+1}$ onto the graph of $y = \sqrt{x^3+1}$. *(2 marks)*

(c) The curve with equation $y = \sqrt{x^3+1}$ is translated by $\begin{bmatrix} 2 \\ -0.7 \end{bmatrix}$ to give the curve with equation $y = g(x)$. Find the value of g(4). *(3 marks)*

QUESTION PART REFERENCE	**Answer space for question 5**

5a) $h = \frac{2-0}{4} = 1/2$

x	0	0.5	1	1.5	2
y	1	$\sqrt{2}$	$\sqrt{9}$	$\sqrt{28}$	$\sqrt{65}$

$\int_0^2 \sqrt{8x^3+1}\,dx = \frac{1/2}{2}\left(1+\sqrt{65}+2(\sqrt{2}+\sqrt{9}+\sqrt{28})\right)$

$= 7.118\ldots$

$= 7.12$ (3s.f.)

b) $y = \sqrt{8x^3+1} = f(x)$ $\quad 8x^3 = (2x)^3$

$\sqrt{8x^3+1} \rightarrow \sqrt{x^3+1}$

$\sqrt{(2x)^3+1} \rightarrow \sqrt{x^3+1}$

$f(x) \rightarrow f\left(\frac{x}{2}\right)$, stretch in x direction s.f 2

Do not write outside the box

QUESTION PART REFERENCE

Answer space for question 5

c) $y = \sqrt{x^3+1}$

$g(x) = \sqrt{(x-2)^3+1} - 0.7$

$g(4) = \sqrt{(4-2)^3+1} - 0.7$

$= \sqrt{9} - 0.7$

$= \underline{2.3}$

Turn over ▶

Do not write outside the box

6 A curve has the equation

$$y = \frac{12 + x^2\sqrt{x}}{x}, \quad x > 0$$

(a) Express $\dfrac{12 + x^2\sqrt{x}}{x}$ in the form $12x^p + x^q$. *(3 marks)*

(b) (i) Hence find $\dfrac{dy}{dx}$. *(2 marks)*

(ii) Find an equation of the normal to the curve at the point on the curve where $x = 4$. *(4 marks)*

(iii) The curve has a stationary point P. Show that the x-coordinate of P can be written in the form 2^k, where k is a rational number. *(3 marks)*

QUESTION PART REFERENCE

Answer space for question 6

6)a) $y = \dfrac{12 + x^2\sqrt{x}}{x}$

$= \dfrac{12}{x} + \dfrac{x^2\sqrt{x}}{x}$

$= 12x^{-1} + x\sqrt{x}$

$= 12x^{-1} + x^{3/2}$

b i) $\dfrac{dy}{dx} = -12x^{-2} + \dfrac{3}{2}x^{1/2}$

ii) when $x=4$, $\dfrac{dy}{dx} = -12(4)^{-2} + \dfrac{3}{2}(4)^{1/2}$

$= 9/4 \Rightarrow$ normal $= -\dfrac{4}{9}$

$y = \dfrac{12 + 4^2\sqrt{4}}{4} = 11$

$y - 11 = -\dfrac{4}{9}(x - 4)$

$9y - 99 = -4x + 16$

$4x + 9y = 115$

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QUESTION PART REFERENCE

Answer space for question 6

iii) $\frac{dy}{dx} = 0$

$-12x^{-2} + \frac{3}{2}x^{1/2} = 0$

$\frac{3}{2}x^{1/2} = \frac{12}{x^2}$

$x^{5/2} = 8$

$x = 8^{2/5}$

$x = ((2)^3)^{2/5}$

$x = 2^{6/5}$

Turn over ▶

Do not write outside the box

7 The nth term of a sequence is u_n. The sequence is defined by

$$u_{n+1} = pu_n + q$$

where p and q are constants.

The first two terms of the sequence are given by $u_1 = 96$ and $u_2 = 72$.

The limit of u_n as n tends to infinity is 24.

(a) Show that $p = \frac{2}{3}$. *(4 marks)*

(b) Find the value of u_3. *(2 marks)*

QUESTION PART REFERENCE

Answer space for question 7

7a) $U_2 = pU_1 + q$ $\quad U_{n+1} \to 24$

$72 = 96p + q$ $\quad U_n \to 24$

$24 = 24p + q$

$48 = 72p$

$p = \frac{48}{72} = \frac{2}{3}$ (as rq)

b) $q + 24\left(\frac{2}{3}\right) = 24$

$q + 16 = 24$

$q = 8$

$U_3 = \frac{2}{3}(72) + 8$

$= 56$

QUESTION PART REFERENCE	Answer space for question 7

Do not write outside the box

8 (a) Given that $\log_a b = c$, express b in terms of a and c. *(1 mark)*

(b) By forming a quadratic equation, show that there is only one value of x which satisfies the equation $2\log_2(x+7) - \log_2(x+5) = 3$. *(6 marks)*

QUESTION PART REFERENCE

Answer space for question 8

8a) $\log_a b = c$

$\underline{b = a^c}$

b) $2\log_2(x+7) - \log_2(x+5) = 3$

$\log_2(x+7)^2 - \log_2(x+5) = 3$

$\log_2 \frac{(x+7)^2}{(x+5)} = 3$

$\frac{(x+7)^2}{(x+5)} = 2^3$

$(x+7)^2 = 8(x+5)$

$x^2 + 14x + 49 = 8x + 40$

$x^2 + 6x + 9 = 0$

$(x+3)^2 = 0$

$\underline{x = -3}$ $\therefore$ only one solution

OR $6^2 - 4(1)(9) = 0$

$36 - 36 = 0$

$\therefore$ only one solution

Do not write outside the box

QUESTION PART REFERENCE

Answer space for question 8

9 (a) (i) On the axes given below, sketch the graph of $y = \tan x$ for $0° \leqslant x \leqslant 360°$. *(3 marks)*

(ii) Solve the equation $\tan x = -1$, giving all values of x in the interval $0° \leqslant x \leqslant 360°$. *(2 marks)*

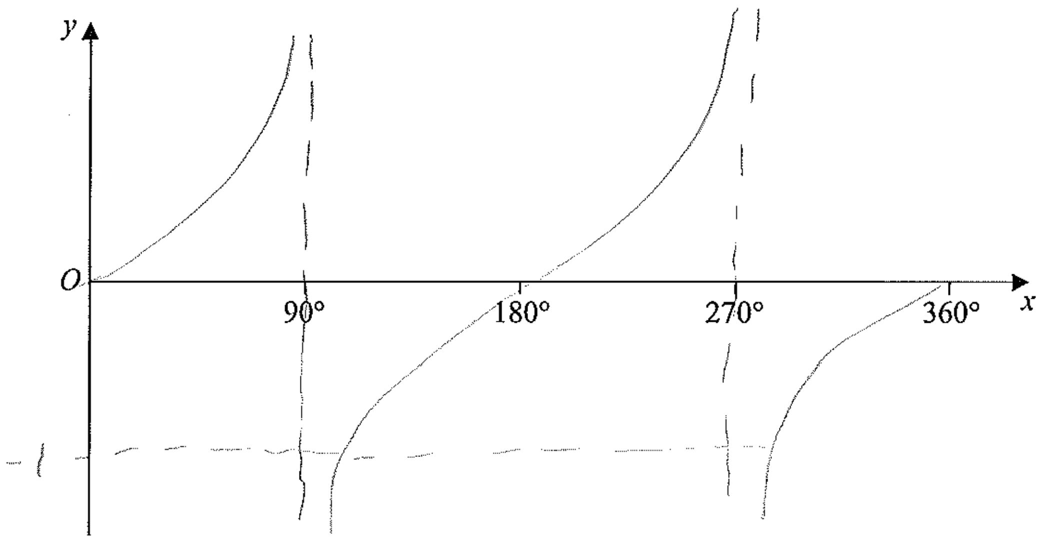
(b) (i) Given that $6 \tan\theta \sin\theta = 5$, show that $6\cos^2\theta + 5\cos\theta - 6 = 0$. *(3 marks)*

(ii) **Hence** solve the equation $6 \tan 3x \sin 3x = 5$, giving all values of x to the nearest degree in the interval $0° \leqslant x \leqslant 180°$. *(6 marks)*

QUESTION PART REFERENCE

Answer space for question 9

(a)(i)



ii) $\tan x = -1$

$x = \tan^{-1}(-1)$ $\quad 0° \leqslant x \leqslant 360$

$x = \underline{135°}, \underline{315°}$

bi) $6 \tan\theta \sin\theta = 5$

$6\left(\frac{\sin\theta}{\cos\theta}\right)\sin\theta = 5$

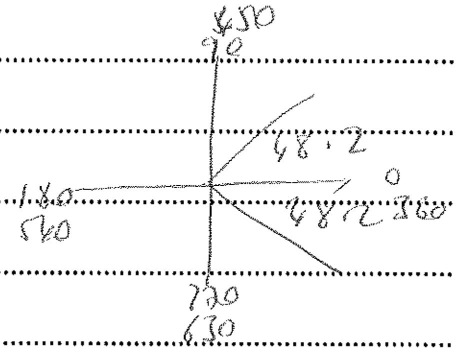
$\frac{6\sin^2\theta}{\cos\theta} = 5$

$6\sin^2\theta = 5\cos\theta$

$6(1 - \cos^2\theta) = 5\cos\theta$

$6 - 6\cos^2\theta = 5\cos\theta \rightarrow 6\cos^2\theta - 5\cos\theta - 6 = 0$ (as req)

QUESTION PART REFERENCE	Answer space for question 9
ii)	$6\cos^2\theta + 5\cos\theta - 6 = 0$
	$(3\cos\theta - 2)(2\cos\theta + 3) = 0$
	$\cos\theta = \frac{2}{3}$ or $\cos\theta = -\frac{3}{2}$ ✗
	$\theta = 3x$
	$\cos 3x = \frac{2}{3}$ $\quad 0 \leq 3x \leq 540$
	$3x = \cos^{-1}\left(\frac{2}{3}\right)$
	$3x = 48.189, 311.\ldots, 408.189\ldots$
	$x = 16°, 104°, 136°$



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QUESTION PART REFERENCE

Answer space for question 9

END OF QUESTIONS