Centre Number						Candidate Number				
Surname										
Other Names	WRITTEN									
Candidate Signature	SOLUTIONS									

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
TOTAL	

General Certificate of Education
Advanced Subsidiary Examination
January 2013

Mathematics MPC2

Unit Pure Core 2

Monday 14 January 2013 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

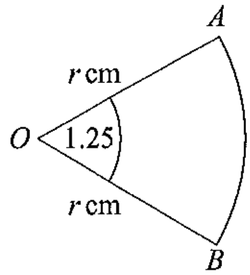
Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

Answer **all** questions.

Answer each question in the space provided for that question.

1 The diagram shows a sector OAB of a circle with centre O and radius r cm.



The angle AOB is 1.25 radians. The perimeter of the sector is 39 cm.

(a) Show that $r = 12$. *(3 marks)*

(b) Calculate the area of the sector OAB. *(2 marks)*

QUESTION PART REFERENCE

Answer space for question 1

1a) perimeter $= r + r + r\theta$

$39 = r + r + 1.25r$

$39 = 3.25r$

$r = \dfrac{39}{3.25}$

$r = 12$ (as req)

b) area $= \frac{1}{2} r^2 \theta$

$= \frac{1}{2} \times 12^2 \times 1.25$

$= 90 \text{ cm}^2$

Do not write outside the box

2 (a) Use the trapezium rule with five ordinates (four strips) to find an approximate value for

$$\int_1^5 \frac{1}{x^2+1}\,dx$$

giving your answer to three significant figures. *(4 marks)*

(b) (i) Find $\int \left(x^{-\frac{3}{2}} + 6x^{\frac{1}{2}}\right) dx$, giving the coefficient of each term in its simplest form. *(3 marks)*

(ii) Hence find the value of $\int_1^4 \left(x^{-\frac{3}{2}} + 6x^{\frac{1}{2}}\right) dx$. *(2 marks)*

QUESTION PART REFERENCE

Answer space for question 2

2a) $h = \frac{5-1}{4} = 1$

x	1	2	3	4	5
y	$1/2$	$1/5$	$1/10$	$1/17$	$1/26$

$$\int_1^5 \frac{1}{x^2+1}\,dx \approx \frac{1}{2}\left(1/2 + 2\left(1/5 + 1/10 + 1/17\right) + 1/26\right)$$

$= 0.6280542\ldots$

$= 0.628$ (3sf)

bi) $\int \left(x^{-3/2} + 6x^{1/2}\right) dx = \frac{x^{-1/2}}{-1/2} + \frac{6x^{3/2}}{3/2} + c$

$= -2x^{-1/2} + 4x^{3/2} + c$

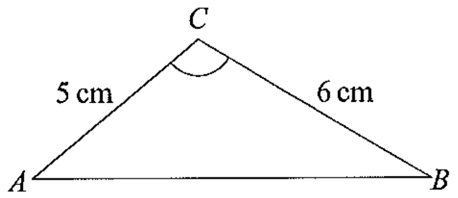
ii) $\left[-2x^{-1/2} + 4x^{3/2}\right]_1^4 = \left(-2(4)^{-1/2} + 4(4)^{3/2}\right) - \left(-2(1)^{-1/2} + 4(1)^{3/2}\right)$

$= 31 - 2$

$= 29$

Do not write outside the box

3 The diagram shows a triangle ABC.



The lengths of AC and BC are 5 cm and 6 cm respectively.

The area of triangle ABC is $12.5\,\text{cm}^2$, and angle ACB is **obtuse**.

(a) Find the size of angle ACB, giving your answer to the nearest 0.1°. *(3 marks)*

(b) Find the length of AB, giving your answer to two significant figures. *(3 marks)*

QUESTION PART REFERENCE

Answer space for question 3

3a) $\text{Area} = \frac{1}{2}ab\sin C$

$12.5 = \frac{1}{2}(5)(6)\sin C$

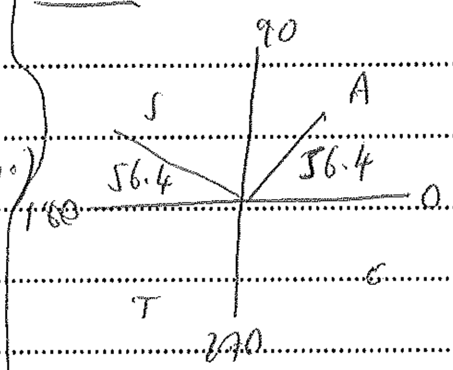
$12.5 = 15\sin C \quad (\div 15)$

$\frac{12.5}{15} = \sin C$

$C = \sin^{-1}\left(\frac{12.5}{15}\right)$

$C = 56.44269\ldots$

obtuse, so



$A\hat{C}B = 180 - 56.44269\ldots$

$= \underline{\underline{123.6°}}$ (nearest 0.1°)

Do not write outside the box

QUESTION PART REFERENCE	Answer space for question 3
b)	$AB^2 = 5^2 + 6^2 - (2(5)(6)\cos 123.557\ldots)$
	$AB^2 = 94.166\ldots$
	$AB = \sqrt{94.166\ldots}$
	$AB = 9.703\ldots$
	$AB = 9.7\,cm$ (2sf)

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4 Given that

$$\log_a N - \log_a x = \frac{3}{2}$$

express x in terms of a and N, giving your answer in a form not involving logarithms. *(3 marks)*

QUESTION PART REFERENCE

Answer space for question 4

4) $\log_a N - \log_a x = \frac{3}{2}$

$\log_a\left(\frac{N}{x}\right) = \frac{3}{2}$

$\frac{N}{x} = a^{3/2}$

$x = \frac{N}{a^{3/2}}$ OR $x = Na^{-3/2}$

Do not write outside the box

5 The point $P(2, 8)$ lies on a curve, and the point M is the only stationary point of the curve.

The curve has equation $y = 6 + 2x - \dfrac{8}{x^2}$.

(a) Find $\dfrac{dy}{dx}$. *(3 marks)*

(b) Show that the normal to the curve at the point $P(2, 8)$ has equation $x + 4y = 34$. *(3 marks)*

(c) (i) Show that the stationary point M lies on the x-axis. *(3 marks)*

(ii) Hence **write down** the equation of the tangent to the curve at M. *(1 mark)*

(d) The tangent to the curve at M and the normal to the curve at P intersect at the point T. Find the coordinates of T. *(2 marks)*

QUESTION PART REFERENCE | **Answer space for question 5**

5a) $y = 6 + 2x - 8x^{-2}$

$\dfrac{dy}{dx} = \underline{2 + 16x^{-3}}$

b) grad of tangent at P, when $x = 2$:-

$\dfrac{dy}{dx} = 2 + 16(2)^{-3}$

$= 4$, $\therefore$ gradient of normal is $-\frac{1}{4}$

through $(2, 8)$

$y - 8 = -\frac{1}{4}(x - 2)$

$4y - 32 = -x + 2$

$\underline{x + 4y = 34}$ (as req)

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QUESTION PART REFERENCE	Answer space for question 5
c)i)	$\frac{dy}{dx} = 0$ if stationary point, so
	$2 + 16x^{-3} = 0$
	$16x^{-3} = -2$
	$\frac{16}{x^3} = -2$
	$x^3 = \frac{16}{-2}$
	$x^3 = -8$, when $x = -2$,
	$x = -2$ $\quad y = 6 + 2(-2) - \frac{8}{(-2)^2}$
	$= 6 - 4 - 2 = 0$
	$\therefore M(-2, 0)$ which lies on x axis.
ii)	$y = 0$
d)	$y = 0$ intersects with $x + 4y = 34$
	so, $x + 4(0) = 34$
	$x = 34$
	$T(34, 0)$

6 (a) A geometric series begins $420 + 294 + 205.8 + \ldots$

(i) Show that the common ratio of the series is 0.7. *(1 mark)*

(ii) Find the sum to infinity of the series. *(2 marks)*

(iii) Write the nth term of the series in the form $p \times q^n$, where p and q are constants. *(2 marks)*

(b) The first term of an arithmetic series is 240 and the common difference of the series is -8.

The nth term of the series is u_n.

(i) Write down an expression for u_n. *(1 mark)*

(ii) Given that $u_k = 0$, find the value of $\sum_{n=1}^{k} u_n$. *(4 marks)*

QUESTION PART REFERENCE

Answer space for question 6

6ai) $r = \frac{294}{420} = 0.7$ (as req)

ii) $S_\infty = \frac{a}{1-r}$ $\quad a = 420, \; r = 0.7$

$= \frac{420}{1-0.7} = 1400$

iii) n^{th} term $= ar^{n-1}$

$= 420 \times 0.7^{n-1}$

$= 420 \times 0.7^n \times 0.7^{-1}$

$= 600 \times 0.7^n$

Do not write outside the box

QUESTION PART REFERENCE

Answer space for question 6

b)i) $U_n = a + (n-1)d$ $a = 240,\ d = -8$

$= 240 - 8(n-1)$

$= 240 - 8n + 8$

$= 248 - 8n$

ii) $U_k = 0$

so, $248 - 8k = 0$

$8k = 248$

$k = 31$

$\sum_{n=1}^{k} U_n = 240 + 232 + \dots + 0$

$S_k = \frac{n}{2}(a + l)$ $a = 240,\ l = 0$

$n = 31$

$= \frac{31}{2}(240 + 0)$

$= 3720$

7 (a) Describe a geometrical transformation that maps the graph of $y = 4^x$ onto the graph of $y = 3 \times 4^x$. *(2 marks)*

(b) Sketch the curve with equation $y = 3 \times 4^x$, indicating the value of the intercept on the y-axis. *(2 marks)*

(c) The curve with equation $y = 4^{-x}$ intersects the curve $y = 3 \times 4^x$ at the point P. Use logarithms to find the x-coordinate of P, giving your answer to three significant figures. *(5 marks)*

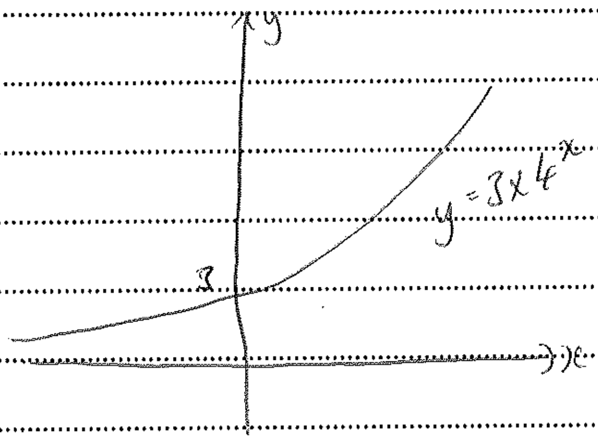
QUESTION PART REFERENCE

Answer space for question 7

7a) $f(x) = 4^x \rightarrow 3 \times 4^x = 3f(x)$

stretch, scale factor 3, in y axis

b)



c) $4^{-x} = 3 \times 4^x$

$\log 4^{-x} = \log 3 + \log 4^x$

$-x \log 4 = \log 3 + x \log 4$

$2x \log 4 = -\log 3$

$2x = \dfrac{-\log 3}{\log 4}$

$x = \dfrac{-\log 3}{2 \log 4} = -0.3962406..$

$= -0.396$ (3sf)

Do not write outside the box

8 (a) Expand $\left(1+\frac{4}{x}\right)^2$. *(1 mark)*

(b) The first four terms of the binomial expansion of $\left(1+\frac{x}{4}\right)^8$ in ascending powers of x are $1+ax+bx^2+cx^3$. Find the values of the constants a, b and c. *(4 marks)*

(c) Hence find the coefficient of x in the expansion of $\left(1+\frac{4}{x}\right)^2\left(1+\frac{x}{4}\right)^8$. *(4 marks)*

QUESTION PART REFERENCE

Answer space for question 8

8a) $\left(1+\frac{4}{x}\right)^2 = \left(1+\frac{4}{x}\right)\left(1+\frac{4}{x}\right)$

$= 1+\frac{4}{x}+\frac{4}{x}+\frac{16}{x^2}$

$= 1+\frac{8}{x}+\frac{16}{x^2}$

b) $\left(1+\frac{x}{4}\right)^8 = {}^8C_0(1)^8\left(\frac{x}{4}\right)^0 + {}^8C_1(1)^7\left(\frac{x}{4}\right)^1 + {}^8C_2(1)^6\left(\frac{x}{4}\right)^2 + {}^8C_3(1)^5\left(\frac{x}{4}\right)^3$

$= (1)(1)(1) + (8)(1)\left(\frac{x}{4}\right) + (28)(1)\left(\frac{x^2}{16}\right) + (56)(1)\left(\frac{x^3}{64}\right)$

$= 1+2x+\frac{28}{16}x^2+\frac{56}{64}x^3$

c) $\left(1+\frac{8}{x}+\frac{16}{x^2}\right)\left(1+2x+\frac{28}{16}x^2+\frac{56}{64}x^3\right)$

$2x + 14x^2 + 14x = 30x$

coefficient is 30

9 (a) Write down the two solutions of the equation $\tan(x + 30°) = \tan 79°$ in the interval $0° \leqslant x \leqslant 360°$. *(2 marks)*

(b) Describe a single geometrical transformation that maps the graph of $y = \tan x$ onto the graph of $y = \tan(x + 30°)$. *(2 marks)*

(c) (i) Given that $5 + \sin^2\theta = (5 + 3\cos\theta)\cos\theta$, show that $\cos\theta = \frac{3}{4}$. *(5 marks)*

(ii) **Hence** solve the equation $5 + \sin^2 2x = (5 + 3\cos 2x)\cos 2x$ in the interval $0 < x < 2\pi$, giving your values of x in radians to three significant figures. *(3 marks)*

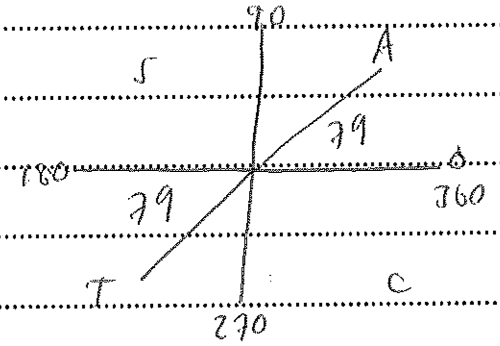
QUESTION PART REFERENCE

Answer space for question 9

9a) $\tan(x+30) = \tan 79 \qquad 0° \leq x \leq 360$

$x + 30 = 79, 259$

so, $x = 49°, 229°$



b) $y = \tan x \rightarrow y = \tan(x+30°)$

translation $\begin{pmatrix} -30 \\ 0 \end{pmatrix}$

ci) $5 + \sin^2\theta = (5 + 3\cos\theta)\cos\theta \qquad \boxed{\sin^2\theta = 1 - \cos^2\theta}$

$5 + (1 - \cos^2\theta) = 5\cos\theta + 3\cos^2\theta$

$6 - \cos^2\theta = 5\cos\theta + 3\cos^2\theta$

$4\cos^2\theta + 5\cos\theta - 6 = 0$

$(4\cos\theta - 3)(\cos\theta + 2) = 0$

$4\cos\theta - 3 = 0 \qquad$ OR $\qquad \cos\theta + 2 = 0$

$\cos\theta = \frac{3}{4}$ (as req.) $\qquad \cos\theta = -2$ X

not a solution

as $-1 \leq \cos\theta \leq 1$

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QUESTION PART REFERENCE

Answer space for question 9

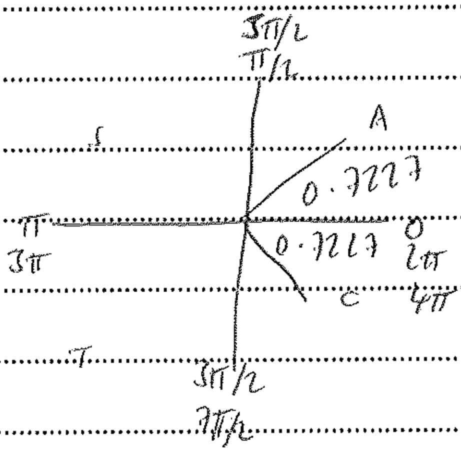
let $\theta = 2x$ $\qquad$ $0 < 2x < 4\pi$

$\cos\theta = \frac{3}{4}$

$\cos 2x = \frac{3}{4}$

$2x = \cos^{-1}\left(\frac{3}{4}\right)$

$2x = 0.7227\ldots,$
$5.5604\ldots,$
$7.005\ldots,$
$11.84367\ldots$



$x = 0.361, 2.78, 3.50, 5.92$

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