Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
TOTAL	

AQA

General Certificate of Education
Advanced Subsidiary Examination
June 2011

Mathematics **MPC2**

Unit Pure Core 2

Wednesday 18 May 2011 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

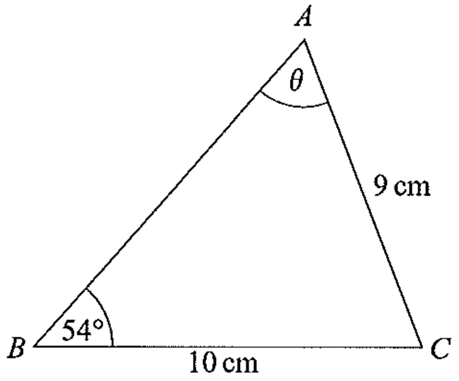
Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions in the spaces provided.

1 The triangle ABC, shown in the diagram, is such that $AC = 9\,\text{cm}$, $BC = 10\,\text{cm}$, angle $ABC = 54°$ and the acute angle $BAC = \theta$.



(a) Show that $\theta = 64°$, correct to the nearest degree. *(3 marks)*

(b) Calculate the area of triangle ABC, giving your answer to the nearest square centimetre. *(3 marks)*

QUESTION PART REFERENCE

(a) Sine Rule $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

$\dfrac{10}{\sin A} = \dfrac{9}{\sin 54}$

$\sin A = \dfrac{10 \times \sin 54}{9}$

$= 0.8989077715$

$A = \sin^{-1}(0.898\ldots)$

$= 64.0148\ldots\ldots$

$= 64°$ to the nearest degree as required

Do not write outside the box

QUESTION PART REFERENCE

(b) Area of a Triangle = $\frac{1}{2}ab\sin C$

$= \frac{1}{2} \times 9 \times 10 \times \sin 62$

$= 39.7326 \ldots$

$= 40\,cm^2$ to the nearest cm^2

Turn over ▶

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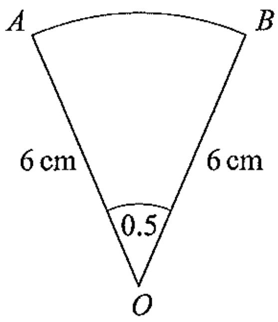
2 The diagram shows a sector OAB of a circle with centre O.



The radius of the circle is 6 cm and the angle $AOB = 0.5$ radians.

(a) Find the area of the sector OAB. *(2 marks)*

(b) (i) Find the length of the arc AB. *(2 marks)*

(ii) Hence show that

the perimeter of the sector $OAB = k \times$ the length of the arc AB

where k is an integer. *(2 marks)*

QUESTION PART REFERENCE	
(a)	$A = \frac{1}{2}r^2\theta$ (θ in radians)
	$= \frac{1}{2} \times 6^2 \times 0.5$
	$= 9 \text{ cm}^2$
(b)	(i) Arc length $= r\theta$
	$= 6 \times 0.5$
	$= 3$ cm
	(ii) Perimeter of Arc $= 6 + 6 + 3$
	$= 15$ cm
	$= 5 \times 3$ $\therefore$ Perimeter $= 5 \times$ (length of) arc

Do not write outside the box

QUESTION PART REFERENCE

Do not write outside the box

3 (a) The expression $(2+x^2)^3$ can be written in the form

$$8+px^2+qx^4+x^6$$

Show that $p=12$ and find the value of the integer q. *(3 marks)*

(b) (i) Hence find $\int \frac{(2+x^2)^3}{x^4}\,dx$. *(5 marks)*

(ii) Hence find the exact value of $\int_1^2 \frac{(2+x^2)^3}{x^4}\,dx$. *(2 marks)*

QUESTION PART REFERENCE

(a) $(2+x^2)^3 = \binom{3}{0}(2)^3 + \binom{3}{1}(2)^2(x^2)^1 + \binom{3}{2}(2)^1(x^2)^2 + \binom{3}{3}(x^2)^3$

$= 8 + 3\times4\times x^2 + 3\times2\times x^4 + x^6$

$= 8 + 12x^2 + 6x^4 + x^6$

OR $\;{}^3C_3 \times (2)^3 \times (x^2)^0$

$1 \times 8 \times 1 = 8$

${}^3C_2 \times (2)^2 \times (x^2)^1 \quad +$

$3 \times 4 \times x^2 = 12x^2$

${}^3C_1 \times (2)^1 \times (x^2)^2 \quad +$

$3 \times 2 \times x^4 = 6x^4$

${}^3C_0 \times (2)^0 \times (x^2)^3 \quad +$

$1 \times 1 \times x^6 = x^6$

$8 + 12x^2 + 6x^4 + x^6$

$p \qquad q$

$p = 12 \qquad q = 6$

Do not write outside the box

QUESTION PART REFERENCE

(b) (i) $\int \frac{(2+x^2)^3}{x^4} \, dx$

$\rightarrow \int x^{-4}\left[8 + 12x^2 + 6x^4 + x^6\right] dx$

$\rightarrow \int 8x^{-4} + 12x^{-2} + 6 + x^2 \, dx$

$\rightarrow \frac{8x^{-3}}{-3} + \frac{12x^{-1}}{-1} + \frac{6x}{1} + \frac{x^3}{3} + c$

$= -\frac{8x^{-3}}{3} - \frac{12x^{-1}}{1} + 6x + \frac{x^3}{3} + c$

(ii) $\int_1^2 \frac{(2+x^2)^3}{x^4} \, dx$

$= \left[-\frac{8x^{-3}}{3} - 12x^{-1} + 6x + \frac{x^3}{3}\right]_1^2$

$= \left[\frac{-8(2)^{-3}}{3} - 12(2)^{-1} + 6(2) + \frac{(2)^3}{3}\right] - \left[\frac{-8(1)^{-3}}{1} - 12(1)^{-1} + 6(1) + \frac{(1)^3}{3}\right]$

$= \left[-\frac{1}{3} - 6 + 12 + \frac{8}{3}\right] - \left[-\frac{8}{3} - 12 + 6 + \frac{1}{3}\right]$

$= \frac{25}{3} - \left(-\frac{25}{3}\right)$

$= \frac{50}{3}$

$= 16\frac{2}{3}$

Turn over ▶

4 (a) Sketch the curve with equation $y = 4^x$, indicating the coordinates of any point where the curve intersects the coordinate axes. *(2 marks)*

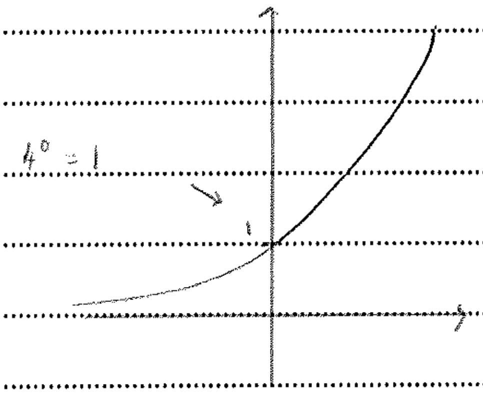
(b) Describe the geometrical transformation that maps the graph of $y = 4^x$ onto the graph of $y = 4^x - 5$. *(2 marks)*

(c) (i) Use the substitution $Y = 2^x$ to show that the equation $4^x - 2^{x+2} - 5 = 0$ can be written as $Y^2 - 4Y - 5 = 0$. *(2 marks)*

(ii) Hence show that the equation $4^x - 2^{x+2} - 5 = 0$ has only one real solution. Use logarithms to find this solution, giving your answer to three decimal places. *(4 marks)*

QUESTION PART REFERENCE

(a)



(b) $y = 4^x \rightarrow y = 4^x - 5$

$f(x) \rightarrow f(x) - a$

Translation of 'a' down

Translation of $\begin{bmatrix} 0 \\ -5 \end{bmatrix}$

(c) (i) $4^x - 2^{x+2} - 5 = 0$

$Y = 2^x$ $(2^x)^2 - 2^2(2^x) - 5 = 0$

$Y^2 - 4Y - 5 = 0$

Note: $4^x = (2^2)^x$

$= 2^{2x}$

or $(2^x)^2$

(ii)

Do not write outside the box

QUESTION PART REFERENCE

(ii) $y^2 - 4y - 5 = 0$

$(y-5)(y+1) = 0$

$y = 5$ or $y = -1$

Since $y = 2^x$ and $2^x > 0$ for all real values of x

$2^x = 5$ is the only one real solution

$\log 2^x = \log 5$

$x \log 2 = \log 5$

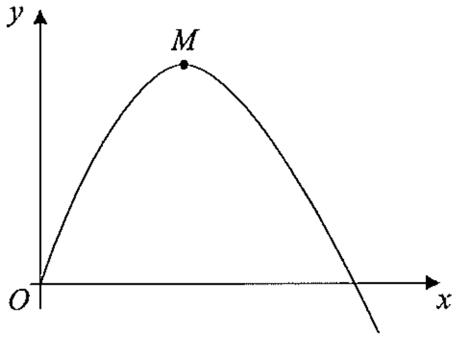
$x = \frac{\log 5}{\log 2}$

$x = 2.3219\ldots$

$= 2.322$ to 3dp

Do not write outside the box

5 The diagram shows part of a curve with a maximum point M.



The curve is defined for $x \geqslant 0$ by the equation

$$y = 6x - 2x^{\frac{3}{2}}$$

(a) Find $\dfrac{dy}{dx}$. *(3 marks)*

(b) (i) Hence find the coordinates of the maximum point M. *(3 marks)*

(ii) Write down the equation of the normal to the curve at M. *(1 mark)*

(c) The point $P\left(\frac{9}{4}, \frac{27}{4}\right)$ lies on the curve.

(i) Find an equation of the normal to the curve at the point P, giving your answer in the form $ax + by = c$, where a, b and c are positive integers. *(4 marks)*

(ii) The normals to the curve at the points M and P intersect at the point R. Find the coordinates of R. *(2 marks)*

QUESTION PART REFERENCE

(a) $y = 6x - 2x^{3/2}$

$\frac{dy}{dx} = 6 - \left(\frac{3}{2}\right) \times 2x^{1/2}$

$= 6 - 3x^{1/2}$

Do not write outside the box

QUESTION PART REFERENCE

(b) (i) At maximum point, $\frac{dy}{dx} = 0$

$6 - 3x^{1/2} = 0$

$3x^{1/2} = 6$

$x^{1/2} = 2$

$x = 2^2 = 4$

At $x = 4$, $y = 6x - 2x^{3/2}$

$y = 6(4) - 2(4)^{3/2}$

$= 24 - 2 \times 8$

$= 24 - 16$

$= 8$

$(4, 8)$

(ii) Eqn of normal $x = 4$

(c) (i) At $x = \frac{9}{4}$, $\frac{dy}{dx} = 6 - 3\left(\frac{9}{4}\right)^{1/2}$

$= 6 - 3\left(\frac{3}{2}\right) = \frac{3}{2}$ $\therefore$ gradient of normal $= -\frac{2}{3}$

since $m_1 m_2 = -1$

$y = mx + c$

$\frac{27}{4} = \left(-\frac{2}{3}\right)\left(\frac{9}{4}\right) + c$

$\frac{27}{4} = -\frac{18}{12} + c$

$c = \frac{33}{4}$ $\therefore$ $y = -\frac{2}{3}x + \frac{33}{4}$

$\times 12$ $8x + 12y = 99$ as required

Turn over ▶

(ii) $x = 4$ and $8x + 12y = 99$

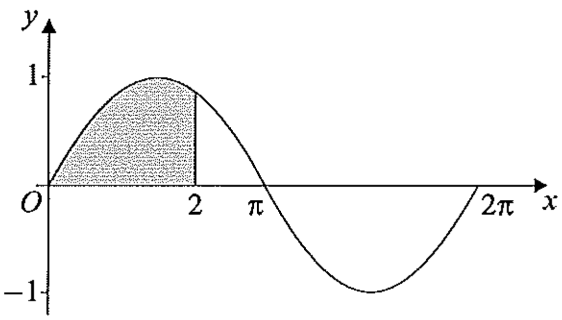
$8(4) + 12y = 99$

$12y = 67$

$y = \frac{67}{12}$ Intersect at $\left(4, \frac{67}{12}\right)$

Do not write outside the box

6 A curve C, defined for $0 \leqslant x \leqslant 2\pi$ by the equation $y = \sin x$, where x is in radians, is sketched below. The region bounded by the curve C, the x-axis from 0 to 2 and the line $x = 2$ is shaded.



(a) The area of the shaded region is given by $\int_0^2 \sin x \, dx$, where x is in radians.

Use the trapezium rule with five ordinates (four strips) to find an approximate value for the area of the shaded region, giving your answer to three significant figures. *(4 marks)*

(b) Describe the geometrical transformation that maps the graph of $y = \sin x$ onto the graph of $y = 2\sin x$. *(2 marks)*

(c) Use a trigonometrical identity to solve the equation

$$2\sin x = \cos x$$

in the interval $0 \leqslant x \leqslant 2\pi$, giving your solutions in radians to three significant figures. *(4 marks)*

QUESTION PART REFERENCE

(a) $\int_0^2 \sin x \, dx$

x	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2
$y = \sin x$	0	0.479...	0.841...	0.997...	0.909...

Trapezium Rule (from formula booklet) $\frac{1}{2}h\{y_0 + y_n + 2(y_1 + y_2 + \dots + y_{n-1})\}$ where $h = \frac{b-a}{n-1}$

$= \frac{1}{2} \times \frac{1}{2} \times \{0 + 0.909\ldots + 2(0.479\ldots + 0.841\ldots + 0.997\ldots)\}$ $h = \frac{2-0}{4} = \frac{1}{2}$

$= \frac{1}{4} \times 5.543$

$= 1.38575$

$= 1.39$ to 3sf

QUESTION PART REFERENCE

(b) $y = \sin x \rightarrow y = 2\sin x$

$y = f(x) \rightarrow y = af(x)$
Stretch of scale factor a in the y-direction

Stretch of scale factor 2
in the y-direction

(c) $2\sin x = \cos x$

$\div \cos x$

$\frac{2\sin x}{\cos x} = \frac{\cos x}{\cos x}$

$2\tan x = 1$

$\tan x = \frac{1}{2}$

In Radians

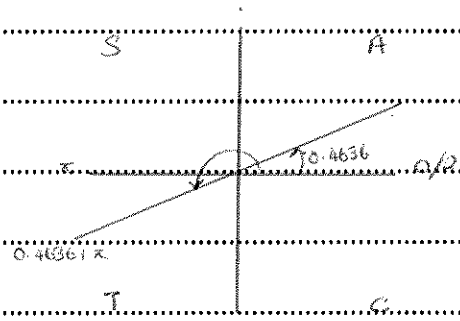
$x = \tan^{-1}\frac{1}{2}$

$= 0.4636\ldots$
$= 0.464$ to 3sf

$+\pi$

$= 3.605\ldots$

$= 3.61$ to 3sf



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7 The nth term of a sequence is u_n. The sequence is defined by

$$u_{n+1} = pu_n + q$$

where p and q are constants.

The first two terms of the sequence are given by $u_1 = 60$ and $u_2 = 48$.

The limit of u_n as n tends to infinity is 12.

(a) Show that $p = \frac{3}{4}$ and find the value of q. *(5 marks)*

(b) Find the value of u_3. *(1 mark)*

QUESTION PART REFERENCE

(a) $u_1 = 60$

$u_2 = p \times 60 + q = 48$ $\qquad 60p + q = 48$

$u_\infty = p \times 12 + q = 12$ $\qquad -\ \underline{12p + q = 12}$

$48p = 36$

$p = \frac{36}{48} = \frac{3}{4}$

$12p + q = 12$

$12(\frac{3}{4}) + q = 12$

$9 + q = 12$

$q = 3$

(b) $u_{n+1} = \frac{3}{4}u_n + 3$

$u_3 = \frac{3}{4}(48) + 3$

$= 36 + 3$

$= 39$

Do not write outside the box

QUESTION PART REFERENCE

8 Prove that, for all values of x, the value of the expression

$$(3\sin x + \cos x)^2 + (\sin x - 3\cos x)^2$$

is an integer and state its value. *(4 marks)*

QUESTION PART REFERENCE

$(3\sin x + \cos x)^2 + (\sin x - 3\cos x)^2$

$9\sin^2 x + \cancel{3\sin x\cos x} + \cancel{3\sin x\cos x} + \cos^2 x + \sin^2 x - \cancel{3\sin x\cos x} - \cancel{3\sin x\cos x} + 9\cos^2 x$

$10\sin^2 x + 10\cos^2 x$

$10(\sin^2 x + \cos^2 x)$

Since $\sin^2 x + \cos^2 x = 1$

$10 \times 1 = 10$

Do not write outside the box

QUESTION PART REFERENCE

Do not write outside the box

9 The first term of a geometric series is 12 and the common ratio of the series is $\frac{3}{8}$.

(a) Find the sum to infinity of the series. *(2 marks)*

(b) Show that the sixth term of the series can be written in the form $\frac{3^6}{2^{13}}$. *(3 marks)*

(c) The nth term of the series is u_n.

(i) Write down an expression for u_n in terms of n. *(1 mark)*

(ii) Hence show that

$$\log_a u_n = n \log_a 3 - (3n - 5)\log_a 2$$ *(4 marks)*

QUESTION PART REFERENCE

(a) $S_\infty = \frac{a}{1-r}$ (from formula booklet)

$= \frac{12}{1 - 3/8}$

$= \frac{12}{5/8}$

$= \frac{96}{5}$

$= 19.2$

(b) $u_n = a r^{n-1}$ (from formula booklet)

$u_6 = 12 \times \frac{3}{8}^{6-1}$

$= 12 \times \frac{3}{8}^{5}$

$= 12 \times \frac{243}{32768} = \frac{729}{8192}$

$u_6 = 12 \times \frac{3}{8}^{6-1}$

$= (2 \times 2 \times 3) \times \left(\frac{3^5}{2^3}\right)$

$= \frac{2 \times 2 \times 3 \times 3^5}{2^{15}} = \frac{3^6}{2^{13}}$

Do not write outside the box

QUESTION PART REFERENCE

(c) $u_n = ar^{n-1}$

$= 12 \times \left(\frac{3}{8}\right)^{n-1}$

(i) $\log u_n = \log 12 \times \left(\frac{3}{8}\right)^{n-1}$

$= \log 12 + \log \frac{3}{8}^{n-1}$

$= \log 12 + (n-1) \log \frac{3}{8}$

$= \log \underset{2\times2\times3}{\underset{\downarrow}{12}} + (n-1)\left[\log 3 - \log \underset{2^3}{\underset{\downarrow}{8}}\right]$

$= \log 2^2 \times 3 + (n-1)\left[\log 3 - \log 2^3\right]$

$= \log 2^2 + \log 3 + (n-1)\left[\log 3 - 3\log 2\right]$

$= 2\log 2 + \cancel{\log 3} + n\log 3 - 3n\log 2 - \cancel{\log 3} + 3\log 2$

$= n\log 3 - 3n\log 2 + 5\log 2$

$= n\log 3 - (3n-5)\log 2$ as required

Turn over ▶

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QUESTION PART REFERENCE

END OF QUESTIONS