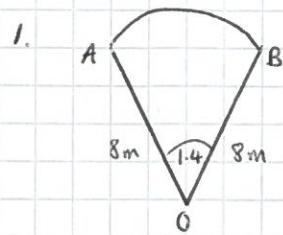


June 2010



$$(a) \text{ Area of a Sector} = \frac{1}{2} r^2 \theta \quad (1)$$

$$= \frac{1}{2} \times 8^2 \times 1.4$$

$$= 44.8 \text{ m}^2 \quad (1)$$

$$(b) (i) \text{ Perimeter of the Sector} = \text{radius} + \text{radius} + \text{length of arc} \quad (1)$$

$$= 8 + 8 + 8(1.4) \quad (1)$$

$$= 27.2 \text{ m} \quad (1)$$

$$(ii) \text{ Perimeter} = \text{Circumference}$$

$$27.2 = x \theta$$

$$27.2 = x \times 2\pi \quad (1)$$

$$x = \frac{27.2}{2\pi} = 4.329014452$$

$$= 4.33 \text{ to } 3 \text{ sf.} \quad (1)$$

$$2. \quad n^{\text{th}} \text{ term} = u_n \quad u_{n+1} = 6 + \frac{2}{5} u_n \quad u_1 = 2.$$

$$(a) \quad u_1 = 2$$

$$u_2 = 6 + \frac{2}{5}(2) = 6.8 \quad (1)$$

$$u_3 = 6 + \frac{2}{5}(6.8) = 8.72 \quad (1)$$

$$(b) \quad \text{As } n \rightarrow \infty$$

$$L = 6 + \frac{2}{5} L \quad (1)$$

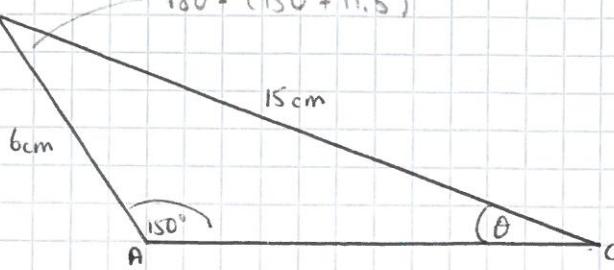
$$L - \frac{2}{5} L = 6$$

$$\frac{3}{5} L = 6$$

$$L = 6 \times \frac{5}{3} \quad (1)$$

$$L = 10 \quad (1)$$

$$3. \quad B$$



(a) Sine Rule

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin \theta}{6} = \frac{\sin 150}{15} \quad (i)$$

$$\sin \theta = 6 \times \frac{\sin 150}{15} \quad (i)$$

$$\sin \theta = \frac{1}{5}$$

$$\theta = \sin^{-1} \frac{1}{5} = 11.53695903^\circ$$

= 11.5° to the nearest 0.1° (i)

$$(b) \text{ Area of the Triangle} = \frac{1}{2} ab \sin C$$

(i) area of a triangle
(i) angle ABC

$$= \frac{1}{2} \times 6 \times 15 \times \sin 18.5^\circ$$

$$= 14.27870954$$

$$= 14.3 \text{ cm}^2 \text{ to } 3sf \quad (i)$$

$$4.(a) \quad \left(1 + \frac{1}{x^2}\right)^3 = {}^3C_0 (1)^3 \left(\frac{1}{x^2}\right)^0 + {}^3C_1 (1)^2 \left(\frac{1}{x^2}\right)^1 + {}^3C_2 (1)^1 \left(\frac{1}{x^2}\right)^2 + {}^3C_3 (1)^0 \left(\frac{1}{x^2}\right)^3$$

$$= 1 \times 1 \times 1 + 3 \times 1 \times -\frac{1}{x^2} + 3 \times 1 \times \frac{1}{x^4} + 1 \times 1 \times -\frac{1}{x^6}$$

$$= 1 - \frac{3}{x^2} + \frac{3}{x^4} - \frac{1}{x^6}$$

$$= 1 + \frac{p}{x^2} + \frac{q}{x^4} - \frac{r}{x^6}$$

$p = -3$ (i) $q = 3$ (i)

$$(b) \text{ (i)} \int \left(1 - \frac{1}{x^2} \right)^3 dx$$

$$= \int \left(1 - \frac{3}{x^2} + \frac{3}{x^4} - \frac{1}{x^6} \right) dx$$

$$= \int \left(1 - 3x^{-2} + 3x^{-4} - x^{-6} \right) dx \quad (1)$$

$$= x - \frac{3x^{-1}}{-1} + \frac{3x^{-3}}{-3} - \frac{x^{-5}}{-5} + c$$

$$= x + 3x^{-1} - x^{-3} + \frac{x^{-5}}{5} + c \quad (3) \text{ All terms correct}$$

$$= x + \frac{3}{x} - \frac{1}{x^3} + \frac{1}{5(x^5)} + c$$

$$\text{(ii)} \int_{\frac{1}{2}}^1 \left(1 - \frac{1}{x^2} \right)^3 dx$$

$$= \left[x + \frac{3}{x} - \frac{1}{x^3} + \frac{1}{5x^5} \right]_{\frac{1}{2}}^1$$

$$= \left(1 + \frac{3}{1} - \frac{1}{1^3} + \frac{1}{5(1)^5} \right) - \left(\left(\frac{1}{2}\right) + \frac{3}{\left(\frac{1}{2}\right)} - \frac{1}{\left(\frac{1}{2}\right)^3} + \frac{1}{5\left(\frac{1}{2}\right)^5} \right)$$

$$= \left(1 + 3 - 1 + \frac{1}{5} \right) - \left(\frac{1}{2} + 6 - 8 + \frac{32}{5} \right) \quad (1)$$

$$= 3 \frac{1}{5} - \left(4 \frac{9}{10} \right)$$

$$= 3 \frac{2}{10} - 4 \frac{9}{10}$$

$$= -1 \frac{7}{10} \quad (1)$$

5. (a) Geometric Series

$$a = 10 \quad S_{\infty} = 50$$

$$(i) \quad S_{\infty} = \frac{a}{1-r}$$

$$50 = \frac{10}{1-r} \quad (i) \quad 1-r = \frac{10}{50}$$

$$1-r = \frac{1}{5} \quad r = \frac{4}{5} \quad \text{as required.}$$

$$(ii) \quad u_n = ar^{n-1} \quad (i)$$

$$u_2 = 10 \times \left(\frac{4}{5}\right)^{2-1}$$

$$= 10 \times \frac{4}{5}$$

$$= 8 \quad (i)$$

(b) Arithmetic Series

$$u_4 = 10 \quad u_8 = 8$$

$$u_n = a + (n-1)d$$

$$u_8 = a + (8-1)d$$

$$= a + 7d = 8 \quad (i) \quad \rightarrow \quad a = 8 - 7d$$

$$u_4 = a + (4-1)d$$

$$= a + 3d = 10 \quad (i) \quad \rightarrow \quad a = 10 - 3d$$

$$\begin{array}{rcl} 10 - 3d & = & 8 - 7d \\ -8 & & \\ \hline 2 - 3d & = & -7d \end{array}$$

$$\begin{array}{rcl} +3d & & \\ \hline 2 & = & -4d \end{array}$$

$$d = \frac{2}{-4} = -\frac{1}{2} \quad (i)$$

$$\begin{aligned} a &= 8 - 7d \\ &= 8 - 7\left(-\frac{1}{2}\right) \quad (i) \end{aligned}$$

$$= 8 + \frac{7}{2}$$

$$= 11\frac{1}{2} \quad (i)$$

$$(ii) \sum_{n=1}^{40} u_n$$

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$S_{40} = \frac{40}{2} [2(11.5) + (40-1)(-0.5)] \quad (1)$$

$$= 20 [23 + 39(-0.5)]$$

$$= 20 [23 - 19.5]$$

$$= 20 \times 3.5$$

$$= 70 \quad (1)$$

6 (a)

$$y = \frac{x^3 + \sqrt{x}}{x}, \quad x > 0$$

$$= x^{-1} (x^3 + x^{1/2}) \quad (1) \quad \sqrt{x} = x^{1/2}$$

$$= x^2 + x^{-1/2} \quad (1) \quad p=2, \quad q=-\frac{1}{2}$$

$$(b) (i) \quad \frac{dy}{dx} = 2x^1 - \frac{1}{2} x^{-3/2}$$

$$= 2x - \frac{1}{2x^{3/2}} \quad (2)$$

(ii) Eqn of the normal at $x=1$

$$\text{Gradient of tangent } \frac{dy}{dx} \Rightarrow 2(1) - \frac{1}{2(1)^{3/2}} = 2 - \frac{1}{2} = \frac{1}{2} = \frac{3}{2} \quad (1)$$

$$\therefore \text{Gradient of normal} = -\frac{2}{3} \quad (m_1 m_2 = -1)$$

$$\text{At } x=1 \quad y = \frac{1^3 + \sqrt{1}}{1} = \frac{1 + \sqrt{1}}{1} = 2. \quad (1)$$

$$\text{Eqn, } y - y_1 = m(x - x_1)$$

$$y - 2 = -\frac{2}{3}(x - 1) \rightarrow y - 2 = -\frac{2}{3}x + \frac{2}{3} \rightarrow y = -\frac{2}{3}x + \frac{8}{3}$$

$$(c) \quad (i) \quad \frac{dy}{dx} = 2x - \frac{1}{2x^{3/2}}$$

$$= 2x - \frac{1}{2} x^{-3/2}$$

$$\frac{d^2y}{dx^2} = 2 + \left(\frac{3}{2}\right)\left(\frac{1}{2}\right)x^{-5/2}$$

$$= 2 + \frac{3}{4}x^{-5/2} \quad (2)$$

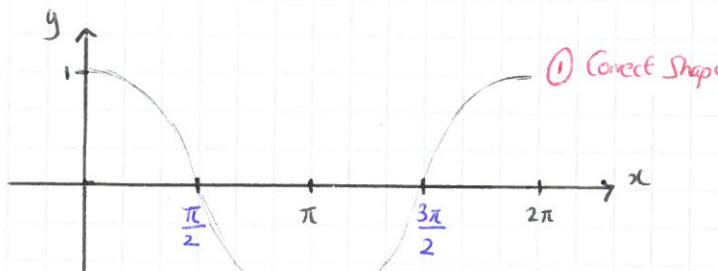
$$= 2 + \frac{3}{4x^{5/2}}$$

(ii) Curve, C has no maximum points.

if $\frac{d^2y}{dx^2} > 0$ for all values of x . (1)

Since $x > 0$, $\frac{d^2y}{dx^2}$ must always be positive. (1)

7. (a)



(1) Correct shape

(1) Three correct intercepts stated
at $(0, 2)$, $(\frac{\pi}{2}, 0)$ and $(\frac{3\pi}{2}, 0)$

$$(b) \quad (i) \quad \sin^2 \theta = \cos \theta (2 - \cos \theta)$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\sin^2 \theta = 1 - \cos^2 \theta$$

$$1 - \cos^2 \theta = \cos \theta (2 - \cos \theta)$$

$$1 - \cos^2 \theta = 2\cos \theta - \cos^2 \theta$$

$$= 2\cos \theta$$

(1)

=

$$(i) \sin^2 2x = \cos 2x (2 - \cos 2x)$$

From part (i) $\cos \theta = \frac{1}{2}$

$$\therefore \cos 2x = \frac{1}{2} \quad (1)$$

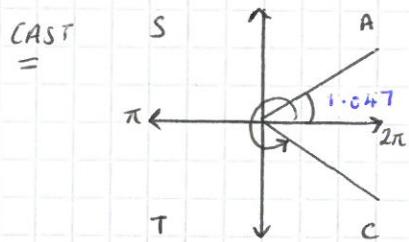
$$2x = \cos^{-1} \frac{1}{2}$$

$$2x = \frac{1}{3}\pi = 1.047197551$$

OR $2\pi - \frac{1}{3}\pi = 5.235987758$

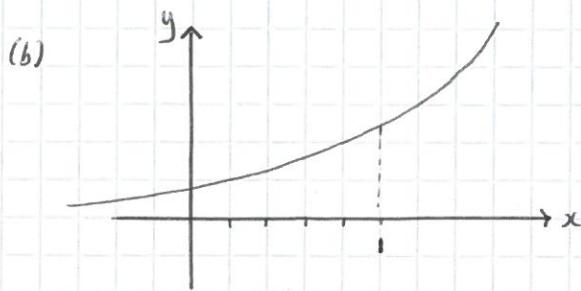
$$x = \frac{1}{6}\pi = 0.5235987756.$$

OR $= 2.617993878$



Solutions to 3sf $x = 0.524$ or 2.62

8. (a) $y = 2^{4x}$ at A (since if $x=0$, $y=2^{4(0)}=1$)



$$\int_0^1 2^{4x} dx \quad \text{six ordinates}$$

x	0.	0.2	0.4	0.6	0.8	1.0
y	1	1.74	3.03	5.278	9.189	16

Trapezium Rule, $A = \frac{h}{2} \{ y_0 + y_5 + 2(y_1 + y_2 + y_3 + y_4) \}$

$$= \frac{0.2}{2} \{ 1 + 16 + 2(1.74 + 3.03 + 5.278 + 9.189) \}$$

$$= 5.5474$$

$$= 5.55 \text{ to } 2 \text{ dp.}$$

(c) $y = 2^{4x} \rightarrow y = 2^{4x-3}$

$$= 2^{-3} 2^{4x}$$

$$= \frac{1}{8} \times 2^{4x}$$

↑

(a) $f(x)$ Stretch in the y -direction, of scale factor $\frac{1}{8}$ (i)

(d) $y = 2^{4x}$ Translated by $\left(-\frac{1}{2}\right)$ 1 right, $\frac{1}{2}$ down

$$g(x) = 2^{4(x-1)} - \frac{1}{2}$$

At 0, $y=0$ (along x -axis)

$$2^{4(x-1)} - \frac{1}{2} = 0$$

$$2^{4(x-1)} = \frac{1}{2}$$

$$2^{4(x-1)} = 2^{-1}$$

$$\text{so, } 4(x-1) = -1$$

$$4x - 4 = -1$$

$$4x - 3 = 0$$

$$4x = 3, \quad x = \frac{3}{4}$$

$$(e) (i) \log_a k = 3\log_2 2 + \log_2 5 - \log_2 4$$

Log Rules:

$$\log m + \log n = \log mn$$

$$\log_a k = \log_a 2^3 + \log_a 5 - \log_a 4 \quad (1) \text{ One law used}$$

$$\log m - \log n = \log \frac{m}{n}$$

$$\log_a k = \log_a (2^3 \times 5) - \log_a 4 \quad (2) \text{ Second law used}$$

$$\log m^n = n \log m.$$

$$\log_a k = \log_a \frac{2^3 \times 5}{4} \quad (1)$$

$$k = \frac{2^3 \times 5}{4} = \frac{40}{4} = 10 \quad (1)$$

as required

$$(ii) y = \frac{5}{4} \quad \text{integers} \quad y = 2^{4x-3}$$

$$\therefore 2^{4x-3} = \frac{5}{4}$$

$$\log 2^{4x-3} = \log \frac{5}{4}$$

$$(4x-3) \log 2 = \log \frac{5}{4} \quad (2)$$

$$4x \log 2 - 3 \log 2 = \log \frac{5}{4}$$

$$x \times 4 \log 2 = 3 \log 2 + \log \frac{5}{4}$$

$$x = \frac{\log 2^3 + \log \frac{5}{4}}{4 \log 2} \quad (1)$$

$$= \frac{\log (2^3)(\frac{5}{4})}{4 \log 2}$$

$$= \frac{\log 10}{4 \log 2}$$

$$\log_{10} 10 = 1$$

Since $10^1 = 10$

$$= \frac{1}{4 \log_{10} 2} \quad (1)$$