January 2007

1. (a) Rectangle = 2 × Sector

$bh = 2 \times \frac{1}{2} r^2 \theta$

$3 \times 6 = 2 \times \frac{1}{2} \times 6^2 \times \theta$

$18 = 2 \times 18 \times \theta$

$\theta = \frac{18}{36} = 0.5$ as required

(b) Length of Arc $= r\theta$
$= 6 \times 0.5$
$= 3$

Perimeter of Sector $= 6 + 3 + 6 = 15 \text{ cm}$

2. (a) $d = 58 - 51$
$= 7$

(b) $u_n = a + (n-1)d$

$u_{101} = 51 + (101 - 1) \times 7$

$= 51 + 100 \times 7$

$= 751$

(c) $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

Last 100 $= S_{200} - S_{100}$

$= \frac{200}{2}\left[2(51) + (200-1) \times 7\right] - \frac{100}{2}\left[2(51) + (100-1) \times 7\right]$

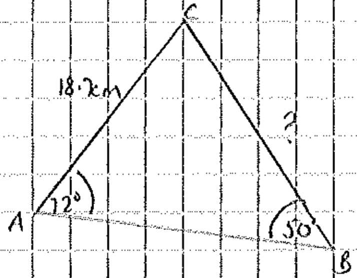
$= 100\left[102 + 1393\right] - 50\left[102 + 693\right]$

$= 100(1495) - 50(795)$

$= 149500 - 39750$

$= 109750$

3. (a)



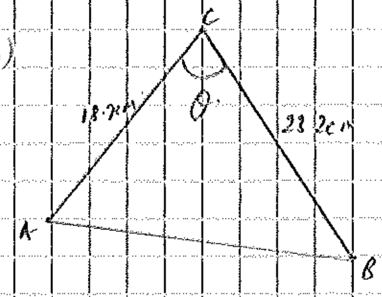
Sine Rule $\frac{18.7}{\sin 50} = \frac{BC}{\sin 72}$

$BC = \frac{18.7}{\sin 50} \times \sin 72$

$= 23.216\ldots$

$= 23.2\text{ cm}^2$ to the nearest 0.1 cm².

(b)



Area $= \frac{1}{2}ab\sin C$

$= \frac{1}{2} \times 18.7 \times 23.2 \times \sin(180-72-50)$

$= \frac{1}{2} \times 18.7 \times 23.2 \times \sin 58$

$= 183.958\ldots$

$= 184\text{ cm}^2$ to the nearest cm²

4. Trapezium Rule, 4 ordinates

$A = \frac{h}{2}\left\{y_0 + y_3 + 2(y_1 + y_2)\right\}$

x	0	1	2	3
y	$\sqrt{3}$	2	$\sqrt{7}$	$\sqrt{12}$

$h = 1$

$A = \frac{1}{2}\left\{\sqrt{3} + \sqrt{12} + 2(2 + \sqrt{7})\right\}$

$= \frac{1}{2}\left\{14.4876\ldots\right\}$

$= 7.2438\ldots$

$= 7.244$ to 3dp.

5. (a) $y = 4x^{1/2} - x^{3/2}$

(i) $\frac{dy}{dx} = \frac{1}{2} \times 4x^{-1/2} - \frac{3}{2}x^{1/2}$

$= 2x^{-1/2} - \frac{3}{2}x^{1/2}$

(ii) at $P(4,0)$ $\frac{dy}{dx} = 2(4)^{-1/2} - \frac{3}{2}(4)^{1/2}$

$= 2\left(\sqrt{\tfrac{1}{4}}\right) - \frac{3}{2}\left(\sqrt{4}\right)$

$= 2\left(\tfrac{1}{2}\right) - \frac{3}{2}(2)$

$= 1 - 3$

$= -2$ as required.

5. (a) (iii) Gradient of tangent at $P(4,0)$ $= -2$ (from (a)(ii))

Gradient of normal at $P(4,0)$ $= \frac{1}{2}$ since $m_1 m_2 = -1$

$y = mx + c$

$0 = \frac{1}{2}(4) + c$

$0 = 2 + c, \quad c = -2$

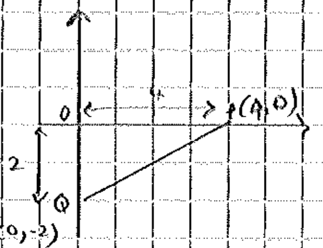
Eqn of Normal $y = \frac{1}{2}x - 2$.

(iv) Q lies on the y-axis $\therefore$ the x-coordinate is 0.

$y = \frac{1}{2}x - 2$

$y = \frac{1}{2}(0) - 2$

$y = -2$.



Area $= \frac{1}{2}bh$

$= \frac{1}{2}(2)(4)$

$= 4$

(v) At the max point, $\frac{dy}{dx} = 0$

$2x^{-\frac{1}{2}} - \frac{3}{2}x^{\frac{1}{2}} = 0$

$2x^{-\frac{1}{2}} = \frac{3}{2}x^{\frac{1}{2}}$.

$4x^{-\frac{1}{2}} = 3x^{\frac{1}{2}}$.

$\frac{4}{3}x^{-\frac{1}{2}} = x^{\frac{1}{2}}$

$\frac{4}{3} = \frac{x^{\frac{1}{2}}}{x^{-\frac{1}{2}}}$ $\leftarrow$ Index laws $x^a \div x^b = x^{a-b}$

$\frac{4}{3} = x^{\frac{1}{2} - -\frac{1}{2}}$

$\frac{4}{3} = x$

x-coordinate $= \frac{4}{3}$

5. (b) (i) $\int \left(4x^{1/2} - x^{3/2}\right) dx$

$$\frac{4x^{3/2}}{3/2} - \frac{x^{5/2}}{5/2} + c$$

$$\frac{2}{3} \times 4x^{3/2} - \frac{2}{5} x^{5/2} + c$$

$$\frac{8}{3} x^{3/2} - \frac{2}{5} x^{5/2} + c$$

~~(ii)~~ $\cancel{\int_0^4 \frac{8}{3} x^{3/2} - \frac{2}{5} x^{5/2}}$

(ii) $\int_0^4 \left(4x^{1/2} - x^{3/2}\right) dx \quad + \quad$ Area of Triangle

$$= \left[\frac{8}{3} x^{3/2} - \frac{2}{5} x^{5/2}\right]_0^4 + 4$$

$$= \left[\left(\frac{8}{3}(4)^{3/2} - \frac{2}{5}(4)^{5/2}\right) - \left(\frac{8}{3}(0)^{3/2} - \frac{2}{5}(0)^{5/2}\right)\right] + 4$$

$$= \left[\left(\frac{8}{3} \times 8 - \frac{2}{5} \times 32\right) - (0)\right] + 4$$

$$= \left[\left(\frac{64}{3} - \frac{64}{5}\right)\right] + 4$$

$$= \left[\frac{320 - 192}{15}\right] + 4$$

$$= \frac{128}{15} + 4 = \frac{188}{15} = 12.53\ldots$$

6. (b) (i) $(1 + x)^3 = {}^3C_0 \times (1)^3 \times (x)^0 = 1 \times 1 \times 1 = 1$

$${}^3C_1 \times (1)^2 \times (x)^1 = 3 \times 1 \times x = 3x$$

$${}^3C_2 \times (1)^1 \times (x)^2 = 3 \times 1 \times x^2 = 3x^2$$

$${}^3C_3 \times (1)^0 \times (x)^3 = 1 \times 1 \times x^3 = x^3$$

$$1 + 3x + 3x^2 + x^3$$

6. (a) (ii) $(1+x)^4 = {}^4C_0 \times (1)^4 \times (x)^0 = 1 \times 1 \times 1 = 1$

${}^4C_1 \times (1)^3 \times (x)^1 = 4 \times 1 \times x = 4x.$

${}^4C_2 \times (1)^2 \times (x)^2 = 6 \times 1 \times x^2 = 6x^2$

${}^4C_3 \times (1)^1 \times (x)^3 = 4 \times 1 \times x^3 = 4x^3$

${}^4C_4 \times (1)^0 \times (x)^4 = 1 \times 1 \times x^4 = x^4$

$1 + 4x + 6x^2 + 4x^3 + x^4.$

(b) (i) $(1+4x)^3 = 1 + 3(4x) + 3(4x)^2 + (4x)^3$ from part (a)(i)

$= 1 + 12x + 3(16x^2) + 64x^3$

$= 1 + 12x + 48x^2 + 64x^3$

(ii) $(1+3x)^4 = 1 + 4(3x) + 6(3x)^2 + 4(3x)^3 + (3x)^4$ from part (a)(ii)

$= 1 + 12x + 6(9x^2) + 4(27x^3) + 81x^4$

$= 1 + 12x + 54x^2 + 108x^3 + 81x^4$

(c) $(1+3x)^4 - (1+4x)^3$

$1 + 12x + 54x^2 + 108x^3 + 81x^4 - (1 + 12x + 48x^2 + 64x^3)$

$1 - 1 + 12x - 12x + 54x^2 - 48x^2 + 108x^3 - 64x^3 + 81x^4$

$6x^2 + 44x^3 + 81x^4$

$p = 6,\ q = 44,\ r = 81$

7. (a) $\log_a x = \log_a 16 - \log_a 2$

$\log_a x = \log_a \frac{16}{2}$

$\log_a x = \log_a 8$

$x = 8$

(b) $\log_a y = 2\log_a 3 + \log_a 4 + 1$

$\log_a y = \log_a 3^2 + \log_a 4 + \log_a a$

$\log_a y = \log_a (9 \times 4 \times a)$

$\log_a y = \log_a 36a$

$y = 36a$

4. (a) $\dfrac{3 + \sin^2\theta}{\cos\theta - 2} = 3\cos\theta$

$3 + \sin^2\theta = 3\cos\theta(\cos\theta - 2)$

$3 + \sin^2\theta = 3\cos^2\theta - 6\cos\theta$

$3 + (1 - \cos^2\theta) = 3\cos^2\theta - 6\cos\theta$

$4 - \cos^2\theta = 3\cos^2\theta - 6\cos\theta$

$0 = 4\cos^2\theta - 6\cos\theta - 4$

$\div 2 \quad 0 = 2\cos^2\theta - 3\cos\theta - 2$

$0 = (2\cos\theta + 1)(\cos\theta - 2)$

$\cos\theta = -\frac{1}{2}$ or $\cos\theta \neq 2$

$\cos\theta = -\frac{1}{2}$ is the only solution

(b) from (a) $\cos 3x = -\frac{1}{2}$

$3x = \cos^{-1} \frac{1}{2}$

$3x = 120°, (360 - 120 =) 240°, (360 + 120 =) 480°, (720 - 120 =) 600°$, etc.

$x = 40°, 80°, 160°$ for $0 \leq x \leq 180°$