January 2007

1. (a) $A = \frac{1}{2} r^2 \theta$

$= \frac{1}{2} \times 6^2 \times 1.2$

$= \frac{1}{2} \times 36 \times 1.2$

$= 21.6 \quad cm^2$

(b) $P = r + r + r\theta$

$= 6 + 6 + 6 \times 1.2$

$= 19.2 \quad cm$

2. (a) $\int_0^3 \sqrt{2^x}\, dx$

x	0	1	2	3
y	1	$\sqrt{2}$	2	$\sqrt{8}$

$h = \frac{3-0}{3} = 1$

$A = \frac{1}{2} \left\{ 1 + \sqrt{8} + 2(\sqrt{2} + 2) \right\}$

$= \frac{1}{2} \times (10.656854\ldots)$

$= 5.3284\ldots$

$= 5.328$ to 3dp

3. (a) (i) $8^p = 64$

$8^2 = 64 \quad , \quad p = 2$

(ii) $8^q = \frac{1}{64}$

$8^{-2} = \frac{1}{64} \quad , \quad q = -2$

(iii) $\sqrt{8} = 8^r$

$\sqrt{8} = 8^{1/2} \quad , \quad r = 1/2$

(b) $\frac{8^x}{\sqrt{8}} = \frac{1}{64}$

$\frac{8^x}{8^{1/2}} = \frac{1}{8^2}$

$8^x = \frac{8^{1/2}}{8^2}$ $\qquad x = 1/2 - 2 = -3/2$

4. (a) Cosine Rule

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$6^2 = 4^2 + 5^2 - 2(4)(5)\cos\theta$$

$$36 = 16 + 25 - 40\cos\theta$$

$$36 = 41 - 40\cos\theta$$

$$-5 = -40\cos\theta$$

$$\cos\theta = \frac{-5}{-40} = \frac{1}{8}$$ as required

(b) $\sin^2\theta + \cos^2\theta = 1$

$$\sin^2\theta + \left(\frac{1}{8}\right)^2 = 1$$

$$\sin^2\theta + \frac{1}{64} = 1$$

$$\sin^2\theta = \frac{63}{64}$$

$$\sin\theta = \sqrt{\frac{63}{64}} = \frac{\sqrt{63}}{\sqrt{64}} = \frac{\sqrt{9}\sqrt{7}}{\sqrt{64}} = \frac{3\sqrt{7}}{8}$$ as required

(c) Area $= \frac{1}{2}ab\sin C$

$$= \frac{1}{2} \times 4 \times 5 \times \frac{3\sqrt{7}}{8}$$

$$= \frac{60\sqrt{7}}{16}$$

$$= \frac{30\sqrt{7}}{8}\ \text{cm}^2 \approx 9.9215\ \text{cm}^2.$$

5. (a) $u_n = a r^{n-1}$

$u_2 = a r^{2-1} = 48$ $\qquad ar = 48$

$u_4 = a r^{4-1} = 3$ $\qquad ar^3 = 3$

$$\frac{ar^3}{ar} = \frac{3}{48}$$

$$r^2 = \frac{1}{16}$$

$$r = \pm\sqrt{\frac{1}{16}} = \pm\frac{1}{4}$$

(b) If $r = -\frac{1}{4}$

(i) $u_1 = a \times r^{1-1}$
$= a \times 1$

$\underset{\uparrow}{48} \div -\frac{1}{4} = -192$

Second Term

(ii) $S_\infty = \frac{a}{1-r}$ $\qquad S_\infty = \frac{-192}{1--1/4} = \frac{-192}{5/4} = \frac{-192 \times 4}{5} = -153.6$

6. (a) (i) $y = x + 1 + 4x^{-2}$

$\frac{dy}{dx} = 1 - 8x^{-3}$

(ii) $\frac{dy}{dx} = 1 - \frac{8}{x^3} = 0$

$-\frac{8}{x^3} = -1$

$x^3 = 8$

$x = \sqrt[3]{8} = 2$

$y = (2) + 1 + 4(2)^{-2}$

$= 2 + 1 + \frac{4}{4}$

$= 4$ $(2, 4)$

(iii) Gradient at $x = 1$

$1 - \frac{8}{(1)^3} = 1 - 8 = -7$

Eqn of a line $y = mx + c$

$6 = -7(1) + c$

$c = 13$ $y = -7x + 13$

(b) (i) $\int (x + 1 + \frac{4}{x^2})\, dx$

$y = \frac{x^2}{2} + x + \frac{4x^{-1}}{-1} + c$

$= \frac{1}{2}x^2 + x - \frac{4}{x} + c$

(ii) $\int_1^4 (x + 1 + \frac{4}{x^2})\, dx$

$= \left[\frac{1}{2}x^2 + x - \frac{4}{x}\right]_1^4$

$= \left[\frac{(4)^2}{2} + (4) - \frac{4}{(4)}\right] - \left[\frac{(1)^2}{2} + (1) - \frac{4}{(1)}\right]$

$= \left[\frac{16}{2} + 4 - 1\right] - \left[\frac{1}{2} + 1 - 4\right]$

$= [11] - [-2\frac{1}{2}]$

$= 13\frac{1}{2}$

7. (a) $(1+2x)^8 = {}^8C_0 \times 1^8 \times (2x)^0 = 1 \times 1 \times 1 = 1$

$+ \; {}^8C_1 \times 1^7 \times (2x)^1 \quad 8 \times 1 \times 2x \quad + 16x$

$+ \; {}^8C_2 \times 1^6 \times (2x)^2 \quad 28 \times 1 \times 4x^2 \quad + 112x^2$

$+ \; {}^8C_3 \times 1^5 \times (2x)^3 \quad 56 \times 1 \times 8x^3 \quad + 448x^3$

$1 + 16x + 112x^2 + 448x^3 + \dots$

$a = 16, \quad b = 112, \quad c = 448$

(b) $(1+\frac{1}{2}x)(1+2x)^8$

$(1+\frac{1}{2}x)(1+16x+112x^2+448x^3+\dots)$

$1+16x+112x^2+448x^3+\frac{1}{2}x+(\frac{1}{2}x)(16x)+(\frac{1}{2}x)(112x^2)+\dots$

$1+16x+112x^2+448x^3+\frac{1}{2}x+8x^2+56x^3+\dots$

$1+16\frac{1}{2}x+120x^2+504x^3+\dots$

coefficient of x^3 is 504

8. (a) $\cos x = 0.3$ for $0 \le x \le 2\pi$ RADIANS!!

$x = \cos^{-1} 0.3 = 1.2661\dots$

$= 1.27$ to 3sf

$x = 2\pi - 1.2661\dots = 5.01708\dots$

$= 5.02$ to 3sf.

(b) (i) x-coordinate = π

y-coordinate $\Rightarrow y = \cos \pi$

$y = -1 \qquad (\pi, -1)$

(ii) $P(\alpha, k)$

$Q(2\pi-\alpha, k)$

(c) $y = \cos x \rightarrow y = \cos(2x)$

$f(x) \rightarrow f(2x)$

$\uparrow$

stretch in the x-direction
of scale factor $\frac{1}{2}$

(d) $\cos 2x = \cos \frac{4\pi}{5}$

$0 \le x \le 2\pi$
$0 \le 2x \le 4\pi$

$2x = \frac{4\pi}{5}$ $\quad$ $4\pi - \frac{4\pi}{5} = \frac{16\pi}{5}$

$x = \frac{4\pi}{10} = \frac{2\pi}{5}$ $\quad$ $x = \frac{8\pi}{5}$

7 (a) $3 \log_a x = \log_a 8$

$\log_a x^3 = \log_a 8$

$x^3 = 8$

$x = \sqrt[3]{8} = 2$

(b) $3 \log_a 6 - \log_a 8 = \log_a 27$

$\log_a 6^3 - \log_a 8 = \log_a 27$

$\log_a \frac{6^3}{8} = \log_a 27$

$\log_a \frac{216}{8} = \log_a 27$ as required

(c) (i) $P(3, p)$

$y = 3\log_{10} x - \log_{10} 8$

$y = \log_{10} x^3 - \log_{10} 8$

$y = \log_{10} \frac{x^3}{8}$

If $x = 3$, $y = \log_{10} \frac{3^3}{8}$

$y = \log_{10} \frac{27}{8}$

(ii) $Q(6, q)$

$y = 3\log_{10} x - \log_{10} 8$

$y = \log_{10} \frac{216}{8}$

Gradient $= \dfrac{\log_{10} \frac{216}{8} - \log_{10} \frac{27}{8}}{6 - 3}$

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