

Core 2 - May 2006

1a) area of sector = $\frac{1}{2} r^2 \theta$

$$8.1 = \frac{1}{2} \times 5^2 \times \theta$$

$$8.1 = 12.5 \theta$$

$$\theta = \frac{8.1}{12.5}$$

$$\theta = \underline{0.648} \text{ (as req)}$$

b) perimeter = arc length + r + r

$$\text{Arc length} = r\theta$$

$$= 5 \times 0.648$$

$$= 3.24 \text{ cm}$$

$$\text{perimeter} = 3.24 + 5 + 5$$

$$= \underline{13.24 \text{ cm}}$$

2a) $\frac{\sin B}{b} = \frac{\sin A}{a}$

$$\frac{\sin B}{4.8} = \frac{\sin 100}{12}$$

$$\sin B = \frac{4.8 \times \sin 100}{12}$$

$$\sin B = 0.39392 \dots$$

$$B = \sin^{-1}(0.39392 \dots)$$

$$B = 23.178 \dots$$

$$B = \underline{23.2} \text{ (1dp) (as req)}$$

b) Area of triangle = $\frac{1}{2} ab \sin C$

$$\hat{C} = 180 - (100 + 23.2)$$
$$= 56.8$$

$$\text{Area} = \frac{1}{2} \times 4.8 \times 12 \times \sin 56.8$$

$$= 24.09 \dots = \underline{24.1 \text{ cm}^2} \text{ (3sf)}$$

$$3a) a=1, d=6$$

$$\begin{aligned}U_{10} &= a + (n-1)d \\ &= 1 + (10-1)6 \\ &= 1 + 9(6) \\ &= \underline{\underline{55}}\end{aligned}$$

$$b) S_n = \frac{n}{2} (2a + (n-1)d)$$

$$7400 = \frac{n}{2} (2(1) + (n-1)6)$$

$$7400 = \frac{n}{2} (2 + 6n - 6)$$

$$7400 = \frac{n}{2} (6n - 4)$$

$$7400 = n(3n - 2)$$

$$7400 = 3n^2 - 2n$$

$$\underline{3n^2 - 2n - 7400 = 0} \quad (n \text{ is req})$$

$$ii) 3n^2 - 2n - 7400 = 0$$

$$(3n + 148)(n - 50) = 0$$

$$n = \frac{-148}{3} \times$$

$$\underline{\underline{n = 50}}$$

$$\begin{aligned}4a) (1-2x)^4 &= (1)(1)^4(-2x)^4 + (4)(1)^3(-2x)^1 + (6)(1)^2(-2x)^2 + \\ &\quad (4)(1)^1(-2x)^3 + (1)(1)^0(-2x)^4 \\ &= \underline{1 - 8x + 24x^2 - 32x^3 + 16x^4} \quad p = -8, q = 24\end{aligned}$$

$$\begin{aligned}b) (2+x)^9 &= (9)(2)^8(x) \quad \text{OR} \quad ({}^9C_1)(2)^8(x) \\ &= \underline{2304x}\end{aligned}$$

$$c) (1-8x)(2^9 + 2304x)$$

$$(1-8x)(512 + 2304x) = -4096x + 2304x$$

$$= \underline{\underline{-1792x}}$$

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$$\text{5a) } \log_a x = 2 \log_a 6 - \log_a 3$$

$$\log_a x = \log_a 6^2 - \log_a 3$$

$$\log_a x = \log_a \frac{6^2}{3}$$

$$\log_a x = \log_a \frac{36}{3}$$

$$x = \frac{36}{3} = \underline{\underline{12}} \text{ (at req.)}$$

$$\text{b) } \log_a y + \log_a 5 = 7$$

$$\log_a 5y = 7 \quad (\text{eliminate logs})$$

$$5y = a^7$$

$$\underline{\underline{y = \frac{1}{5} a^7}}$$

$$\text{6ai) } x=0, \quad y = 27 - 3^0$$

$$\underline{\underline{y = 26}}$$

$$\text{ii) } y=0, \quad 27 - 3^x = 0$$

$$27 - 3^x = 0 \quad \therefore \underline{\underline{x = 3}}$$

$$\text{b) } \int_0^3 (27 - 3^x) dx$$

$$h = \frac{3-0}{3} = 1$$

x	0	1	2	3
y	26	24	18	0

$$\text{Area} = \frac{1}{2} (26 + 2(24 + 18) + 0)$$

$$= \underline{\underline{55}}$$

$$\text{ci) } 3^x = 13$$

$$\log 3^x = \log 13$$

$$x \log 3 = \log 13$$

$$x = \frac{\log 13}{\log 3} = 2.334717 = \underline{\underline{2.3347}} \text{ (4dp)}$$

$$i) 3^x = 18$$

$$y = 27 - 3^x$$

intersects with $y = k$

$$k = 27 - 18$$

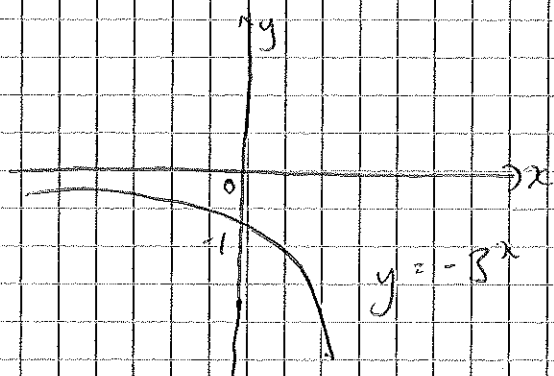
$$k = \underline{9}$$

$$di) y = -3^x \leftarrow y = 27 - 3^x$$

$$f(x) \leftarrow 27 + f(x) \text{ OR } f(x) + 27$$

$$\text{Translation } \begin{pmatrix} 0 \\ -27 \end{pmatrix}$$

ii)



reflect $y = 3^x$ in x axis

$$Pa) \frac{dy}{dx} = 3x^{1/2} + 16x^{-2} - 7$$

$$i) \text{ when } x = k, \frac{dy}{dx} = 3(k)^{1/2} + 16(k)^{-2} - 7 = \underline{0} \text{ (as req)}$$

$$ii) \frac{16}{x^2} = \underline{16x^{-2}}$$

$$iii) \frac{d^2y}{dx^2} = \frac{3}{2}x^{-1/2} - 32x^{-3}$$

$$iv) \text{ when } x = k, \frac{d^2y}{dx^2} = \frac{3}{2}(k)^{-1/2} - 32(k)^{-3} = \underline{\frac{1}{4}}$$

$$\frac{d^2y}{dx^2} > 0 \therefore \text{minimum}$$

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76i) $P(1, 8)$

$$\text{When } x=1, \quad \frac{dy}{dx} = 3(1)^{1/2} + \frac{16}{1^2} = 7$$
$$= \underline{12} \text{ (a.i. r.e.g.)}$$

ii) grad of normal is $-\frac{1}{12}$

$$y - 8 = -\frac{1}{12}(x - 1)$$

$$12y - 96 = -1(x - 1)$$

$$12y - 96 = -x + 1$$

$$\underline{12y + x = 97}$$

ci) $\int 3x^{1/2} + 16x^{-2} - 7 dx$

$$= \frac{3x^{3/2}}{3/2} + \frac{16x^{-1}}{-1} - 7x + C$$

$$= 2x^{3/2} - 16x^{-1} - 7x + C$$

ii) $y = 2x^{3/2} - 16x^{-1} - 7x + C$

$x=1, y=8$

$$8 = 2(1)^{3/2} - 16(1)^{-1} - 7(1) + C$$

$$8 = 2 - 16 - 7 + C$$

$$C = 29$$

$$\underline{y = 2x^{3/2} - 16x^{-1} - 7x + 29}$$

8a) $y = \tan\left(\frac{1}{2}x\right)$ from $y = \tan x$
 $f\left(\frac{x}{2}\right)$ $y = f(x)$

stretch s.f. 2 in x direction

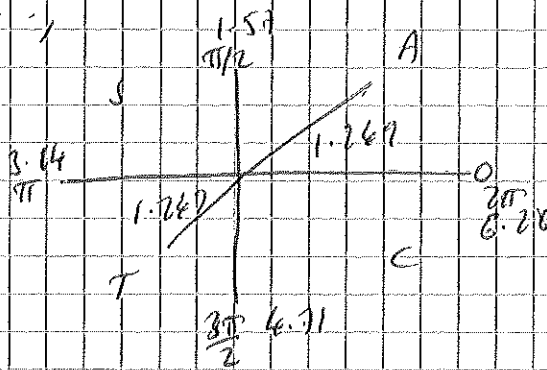
b) $\tan \frac{1}{2}x = 3$

$\frac{1}{2}x = \tan^{-1}(3)$

$0 < \frac{x}{2} < 2\pi$

$\frac{1}{2}x = 1.249, \dots$
 4.3906

$x = \frac{2.498}{8.781}$



c) $\cos \theta (\sin \theta - 3 \cos \theta) = 0$

$\cos \theta = 0$

$\sin \theta - 3 \cos \theta = 0$

$\theta = 1.5707, \dots$

$\sin \theta = 3 \cos \theta$

$4.7123, \dots$

$\frac{\sin \theta}{\cos \theta} = 3$

$\tan \theta = 3$

$\theta = 1.249, \dots, 4.3906, \dots$

$\theta = \underline{1.57}, \underline{4.71}, \underline{1.25}, \underline{4.39} \quad (3, 2)$