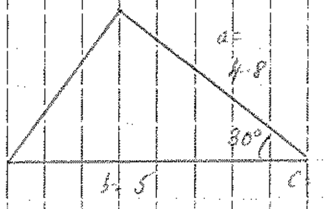
June 2006 - Core 2

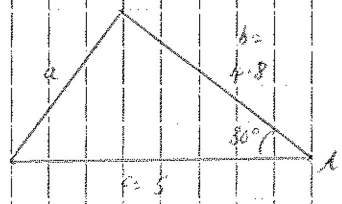
1.



(a) Area $= \frac{1}{2} ab \sin C$

$= \frac{1}{2} \times 4.8 \times 5 \times \sin 30$

$= 6 cm^2$



(b) Cosine Rule (from formula booklet)

$a^2 = b^2 + c^2 - 2bc \cos A$

$a^2 = 4.8^2 + 5^2 - 2 \times 4.8 \times 5 \times \cos 30$

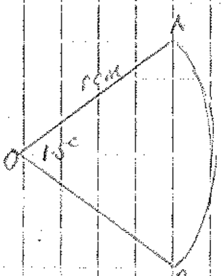
$a^2 = 23.04 + 25 - 41.569...$

$a^2 = 6.4707...$

$a = 2.5437...$

$= 2.54$ cm to 3sf.

2.



(a) Perimeter = Arc + Radius + Radius

$= r\theta + r + r$

$= r\theta + 2r$

$= r(\theta + 2)$

$r(\theta + 2) = 56$

$r(1.5 + 2) = 56$

$r \times 3.5 = 56$

$r = 16$ as required

(b) Area $= \frac{1}{2} r^2 \theta$

$= \frac{1}{2} \times 16^2 \times 1.5$

$= \frac{1}{2} \times 256 \times 1.5$

$= 192 cm^2$

3. (a) $u_n = 90 - 3n$

$n = 1 \quad u_1 = 90 - 3(1) = 87$

$n = 2 \quad u_2 = 90 - 3(2) = 84$

(b) Common Difference $= -3$

(c) $\sum_{n=1}^{k} u_n = 0$

The sum of an AP is:

$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$S_n = \frac{n}{2}\left[2(87) + (n-1) \times -3\right]$

$S_n = \frac{n}{2}\left[174 - 3n + 3\right]$

$\frac{n}{2}\left[177 - 3n\right] = 0$

$\frac{n}{2} = 0$ or $177 - 3n = 0$

$3n = 177$, $n = 59$

4. a) (i) $\sqrt{x} = x^{1/2}$

(ii) $\int \sqrt{x}\,dx$

$= \int x^{1/2}\,dx$

$= \frac{x^{3/2}}{3/2} + c$

$= \frac{2}{3}x^{3/2} + c$

(iii) $\int_1^4 \sqrt{x}\,dx$

$= \left[\frac{x^{3/2}}{3/2}\right]_1^4$

$= \left[\frac{4^{3/2}}{3/2}\right] - \left[\frac{1^{3/2}}{3/2}\right]$

$= \frac{8}{3/2} - \frac{1}{3/2}$

$= \frac{16}{3} - \frac{2}{3} = \frac{14}{3}$

b) (i) $y = x^{1/2}$ $\quad \frac{dy}{dx} = \frac{1}{2}x^{-1/2}$

At $x=4$ gradient $= \frac{1}{2}(4)^{-1/2} = \frac{1}{4}$

$y = x^{1/2}$

At $x=4$, $y = (4)^{1/2} = 2$

$y = mx + c$

$2 = \frac{1}{4}(4) + c$

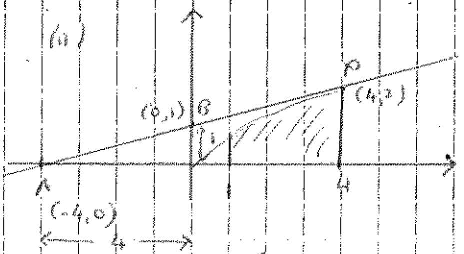
$c = 1$ $\qquad y = \frac{1}{4}x + 1$

OR $y - y_1 = m(x - x_1)$

$y - 2 = \frac{1}{4}(x - 4)$

$y = \frac{1}{4}x - 1 + 2$ $\qquad y = \frac{1}{4}x + 1$

(ii)



At B, $x = 0$

$y = \frac{1}{4}(0) + 1 = 1$

At A, $y = 0$

$0 = \frac{1}{4}x + 1$, $x = -4$

Triangle $= \frac{1}{2}bh$

$= \frac{1}{2} \times 4 \times 1 = 2$

c) $y = \sqrt{x} \rightarrow y = \sqrt{x-1}$

Translation of $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$f(x) \rightarrow f(x-a)$
Translation of 'a' units to the right

d) $\int_1^4 \sqrt{x-1}\,dx$

x	1	2	3	4
$y = \sqrt{x-1}$	0	1	1.41...	1.73...

Trapezium Rule
(from formula booklet)

$A = \frac{h}{2}\{y_0 + y_n + 2(y_1 + y_2 + \dots + y_{n-1})\}$ where $h = \frac{b-a}{n}$

$= \frac{1}{2} \times \{0 + 1.73 + 2(1 + 1.41)\}$ where $h = \frac{4-1}{4}$

$= \frac{1}{2} \times \{0 + 1.73 + 2(2.414\dots)\}$

$= \frac{1}{2} \times 6.560\dots$

$= 3.2802\dots$

$= 3.28$ (to 3sf)

5. (a) first term $= a$ $\quad S_\infty = \dfrac{a}{1-r}$

$\therefore \dfrac{a}{1-r} = 4a$

$\dfrac{a}{4a} = 1-r$

$\dfrac{1}{4} = 1-r$

$r = \dfrac{3}{4}$ as required

(b) first term $= 48$ $\quad S_n = \dfrac{a(1-r^n)}{r}$

ratio $= \dfrac{3}{4}$

$S_{10} = \dfrac{48(1-\frac{3}{4}^{10})}{\frac{3}{4}}$

$S_{10} = \dfrac{48(1-0.056\ldots)}{0.75}$

$= \dfrac{48 \times 0.9436\ldots}{0.25}$

$= 181.1878052$

$= 181.1878$ to 4dp

(c) (i) General term of a GP $\quad ar^{n-1}$

$u_n = 48 \times \dfrac{3}{4}^{n-1}$ $\qquad u_{2n} = 48 \times \dfrac{3}{4}^{2n-1}$

$= 48 \times \dfrac{\frac{3}{4}^{n}}{\frac{3}{4}}$ $\qquad = 48 \times \dfrac{\frac{3}{4}^{2n}}{\frac{3}{4}}$

$= 64 \times \dfrac{3}{4}^{n}$ $\qquad = 64 \times \dfrac{3}{4}^{2n}$

(ii) $\dfrac{u_n}{u_{2n}} = \dfrac{48 \times (\frac{3}{4})^{n-1}}{48 \times (\frac{3}{4})^{2n-1}}$ $\qquad$ note: $\log_{10}(u_n) - \log_{10}(u_{2n}) = \log_{10} \dfrac{u_n}{u_{2n}}$

$\log_{10} \dfrac{u_n}{u_{2n}} = \log_{10} \dfrac{(\frac{3}{4})^{n-1}}{(\frac{3}{4})^{2n-1}}$

$n-1-(2n-1)$
$= n-1-2n+1$
$= -n$

$= \log_{10} \dfrac{3}{4}^{-n}$

$= \log_{10} \dfrac{4}{3}^{n}$

$= n \times \log_{10} \dfrac{4}{3}$

(iii) $\log_{10} \dfrac{u_{100}}{u_{200}} = 100 \times \log_{10} \dfrac{4}{3}$

$= 12.4938\ldots$

$= 12.5$ to 3sf.

6 (a) $(1+x)^4 = \binom{4}{0}(1)^4(x)^0 + \binom{4}{1}(1)^3(x)^1 + \binom{4}{2}(1)^2(x)^2 + \binom{4}{3}(1)^1(x)^3 + \binom{4}{4}(1)^0(x)^4$

$= 1\times 1\times 1 + 4\times 1\times x + 6\times 1\times x^2 + 4\times 1\times x^3 + 1\times 1\times x^4$

$= 1 + 4x + 6x^2 + 4x^3 + x^4$

$(1+x)^4 = {}^4C_0 \times x^0 \times 1^4 = 1\times 1\times 1 = 1$

$+$

${}^4C_1 \times x^1 \times 1^3 = 4\times x\times 1 = 4x$

$+$

${}^4C_2 \times x^2 \times 1^2 = 6\times x^2\times 1 = 6x^2$

$+$

${}^4C_3 \times x^3 \times 1^1 = 4\times x^3\times 1 = 4x^3$

$+$

${}^4C_4 \times x^4 \times 1^0 = 1\times x^4\times 1 = x^4 = 1+4x+6x^2+4x^3+x^4$

(b) (i) $(1+\sqrt{5})^4 = 1 + 4x + 6x^2 + 4x^3 + x^4$

$(1+\sqrt{5})^4 = 1 + 4(\sqrt{5}) + 6(\sqrt{5})^2 + 4(\sqrt{5})^3 + (\sqrt{5})^4$

$= 1 + 4\sqrt{5} + 6\times 5 + 4\times 5\times\sqrt{5} + 5\times 5$

$= 1 + 4\sqrt{5} + 30 + 20\sqrt{5} + 25$

$= 56 + 24\sqrt{5}$

(ii) $\log_2 (1+\sqrt{5})^4 = \log_2 (56 + 24\sqrt{5})$

$= \log_2 8(7 + 3\sqrt{5})$

$= \log_2 8 + \log_2 (7+3\sqrt{5})$

$= 3 + \log_2 (7+3\sqrt{5})$

$\log_2 8 \Rightarrow 2^x = 8$

$x = 3$

$1 = 3$

7 (a) $f(x) = \frac{x^8 - 1}{x^3}$

$= x^{-3}(x^8 - 1)$

$= x^5 - x^{-3}$ $p = 5$, $q = -3$

(b) (i) $f(x) = x^5 - x^{-3}$

$f'(x) = 5x^4 + 3x^{-4}$

(ii) Since $x > 0$, $f'(x) > 0$

Therefore, f is increasing.

(c) $f'(x) = 5x^4 + 3x^{-4}$

at $x = 1$ $5(1)^4 + 3(1)^{-4}$

$5 \times 1 + 3 \times 1$

$5 + 3 = 8$

If gradient of the tangent at $x = 1$ is 8,

the gradient of the normal at $x = 1$ is $-\frac{1}{8}$ $(m_1 m_2 = -1)$

8 (a) (i) $4 \tan\theta \sin\theta = 15$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$4\left(\frac{\sin\theta}{\cos\theta}\right)\sin\theta = 15$

$4 \frac{\sin^2\theta}{\cos\theta} = 15$

$4\sin^2\theta = 15\cos\theta$ as required

(ii) $4\sin^2\theta = 15\cos\theta$

$\sin^2\theta = 1 - \cos^2\theta$

$4(1 - \cos^2\theta) = 15\cos\theta$

$4 - 4\cos^2\theta = 15\cos\theta$

$4\cos^2\theta + 15\cos\theta - 4 = 0$ as required

(b) (i) $4c^2 + 15c - 4 = 0$

$(4c - 1)(c + 4) = 0$

$c = 1/4$ or $c = -4$

(ii) $c = \cos\theta$

$\cos\theta = 1/4$ or $\cos\theta = -4$

Since $-1 \le \cos\theta \le 1$ (from graph)

$\cos\theta$ cannot be -4.

Therefore, the only solution is $\cos\theta = 1/4$

(iii) Solution of $4\tan\theta\sin\theta = 15$

$\rightarrow$ $\cos\theta = 1/4$

$\theta = 75.52\ldots$

$= 75.5°$ to 1dp

or $\theta = 360 - 75.52\ldots$

$= 284.5°$ to 1dp

(c) $\theta = 4x$ $\cos 4x = 1/4$ $4x = 75.5 \rightarrow x = 19°$

or $4x = 284.5 \rightarrow x = 71°$