Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature	WRITTEN SOLUTIONS									

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
TOTAL	

AQA

General Certificate of Education
Advanced Subsidiary Examination
January 2013

Mathematics MPC1

Unit Pure Core 1

Monday 14 January 2013 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You must **not** use a calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The use of calculators is **not** permitted.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

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Answer **all** questions.

Answer each question in the space provided for that question.

1 The point A has coordinates $(-3, 2)$ and the point B has coordinates $(7, k)$.

The line AB has equation $3x + 5y = 1$.

(a) (i) Show that $k = -4$. *(1 mark)*

(ii) Hence find the coordinates of the midpoint of AB. *(2 marks)*

(b) Find the gradient of AB. *(2 marks)*

(c) A line which passes through the point A is perpendicular to the line AB. Find an equation of this line, giving your answer in the form $px + qy + r = 0$, where p, q and r are integers. *(3 marks)*

(d) The line AB, with equation $3x + 5y = 1$, intersects the line $5x + 8y = 4$ at the point C. Find the coordinates of C. *(3 marks)*

QUESTION PART REFERENCE	Answer space for question 1
	$A(-3, 2) \quad B(7, k)$
a i)	$3x + 5y = 1 \qquad x = 7, \ y = k$
	$3(7) + 5k = 1$
	$21 + 5k = 1 \quad (-21)$
	$5k = -20 \quad (\div 5)$
	$\underline{\underline{k = -4}}$ (as req)
	OR $3(7) + 5(-4) = 1$
	$21 + -20 = 1$ ✓
ii)	Midpoint $= \left(\frac{-3+7}{2}, \frac{2+-4}{2}\right)$
	$= \left(\frac{4}{2}, \frac{-2}{2}\right)$
	$= \underline{\underline{(2, -1)}}$

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QUESTION PART REFERENCE

Answer space for question 1

$A\,(\overset{x_1}{-3},\overset{y_1}{2})$ $\quad B\,(\overset{x_2}{7},\overset{y_2}{-4})$

b) gradient $= \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{-4 - 2}{7 - -3}$

$= \dfrac{-6}{10} = -\dfrac{3}{5}$

c) perpendicular gradient is $5/3$

point $A(-3, 2)$

$y - 2 = \dfrac{5}{3}(x + 3)$

$3y - 6 = 5x + 15$

$5x - 3y + 21 = 0$

d) $3x + 5y = 1$ ① $(\times 8)$

$5x + 8y = 4$ ② $(\times 5)$

$24x + 40y = 8$ ③ $\quad$ ④ − ③

$25x + 40y = 20$ ④

$-\;24x + 40y = 8$

$x = 12$ → sub in ①

$3(12) + 5y = 1$

$36 + 5y = 1$

$5y = -35$ $\qquad C = (12, -7)$

$y = -7$

Turn over ▶

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2 A bird flies from a tree. At time t seconds, the bird's height, y metres, above the horizontal ground is given by

$$y = \frac{1}{8}t^4 - t^2 + 5, \quad 0 \leqslant t \leqslant 4$$

(a) Find $\dfrac{dy}{dt}$. *(2 marks)*

(b) (i) Find the rate of change of height of the bird in metres per second when $t = 1$. *(2 marks)*

(ii) Determine, with a reason, whether the bird's height above the horizontal ground is increasing or decreasing when $t = 1$. *(1 mark)*

(c) (i) Find the value of $\dfrac{d^2y}{dt^2}$ when $t = 2$. *(2 marks)*

(ii) Given that y has a stationary value when $t = 2$, state whether this is a maximum value or a minimum value. *(1 mark)*

QUESTION PART REFERENCE	Answer space for question 2
2)	$y = \frac{1}{8}t^4 - t^2 + 5$
a)	$\frac{dy}{dt} = \frac{4}{8}t^3 - 2t$
bi)	when $t = 1$
	$\frac{dy}{dt} = \frac{4}{8}(1)^3 - 2(1)$
	$= -1\frac{1}{2}$
ii)	$\frac{dy}{dt} < 0 \therefore$ height is decreasing at $t = 1$

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QUESTION PART REFERENCE

Answer space for question 2

c i) $\frac{dy}{dt} = \frac{1}{2}t^3 - 2t$

$\frac{d^2y}{d\cancel{x}t^2} = \underline{\frac{3}{2}t^2 - 2}$ when $t=2$, $\frac{d^2y}{dx^2} = \frac{3}{2}(2)^2 - 2$

$= \underline{4}$

ii) $\frac{d^2y}{d\cancel{x}t^2} > 0$ $\therefore$ a minimum value at $t=2$

Turn over ▶

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3 (a) (i) Express $\sqrt{18}$ in the form $k\sqrt{2}$, where k is an integer. *(1 mark)*

(ii) Simplify $\dfrac{\sqrt{8}}{\sqrt{18}+\sqrt{32}}$. *(3 marks)*

(b) Express $\dfrac{7\sqrt{2}-\sqrt{3}}{2\sqrt{2}-\sqrt{3}}$ in the form $m+\sqrt{n}$, where m and n are integers. *(4 marks)*

QUESTION PART REFERENCE

Answer space for question 3

3ai) $\sqrt{18} = \sqrt{9} \times \sqrt{2}$

$= 3\sqrt{2}$

ii) $\dfrac{\sqrt{8}}{\sqrt{18}+\sqrt{32}} = \dfrac{\sqrt{4}\times\sqrt{2}}{3\sqrt{2}+\sqrt{16}\times\sqrt{2}}$

$= \dfrac{2\sqrt{2}}{3\sqrt{2}+4\sqrt{2}}$

$= \dfrac{2\cancel{\sqrt{2}}}{7\cancel{\sqrt{2}}} = \dfrac{2}{7}$

b) $\dfrac{(7\sqrt{2}-\sqrt{3})}{(2\sqrt{2}-\sqrt{3})} \times \dfrac{(2\sqrt{2}+\sqrt{3})}{(2\sqrt{2}+\sqrt{3})} = \dfrac{14\sqrt{4}+7\sqrt{6}-2\sqrt{6}-\sqrt{9}}{4\sqrt{4}-2\sqrt{6}+2\sqrt{6}-\sqrt{9}}$

$= \dfrac{14(2)+5\sqrt{6}-3}{4(2)-3}$

$= \dfrac{28+5\sqrt{6}-3}{8-3}$

$= \dfrac{25+5\sqrt{6}}{5}$

$= 5+\sqrt{6}$

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QUESTION PART REFERENCE

Answer space for question 3

Turn over ▶

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4 (a) (i) Express $x^2 - 6x + 11$ in the form $(x-p)^2 + q$. *(2 marks)*

(ii) **Use the result from part (a)(i)** to show that the equation $x^2 - 6x + 11 = 0$ has no real solutions. *(2 marks)*

(b) A curve has equation $y = x^2 - 6x + 11$.

(i) Find the coordinates of the vertex of the curve. *(2 marks)*

(ii) Sketch the curve, indicating the value of y where the curve crosses the y-axis. *(3 marks)*

(iii) Describe the geometrical transformation that maps the curve with equation $y = x^2 - 6x + 11$ onto the curve with equation $y = x^2$. *(3 marks)*

QUESTION PART REFERENCE

Answer space for question 4

4ai) $x^2 - 6x + 11 = (x-3)^2 - 9 + 11$

$= \underline{\underline{(x-3)^2 + 2}}$

ii) $(x-3)^2 + 2 = 0 \quad (-2)$

$(x-3)^2 = -2 \quad (\sqrt{\ })$

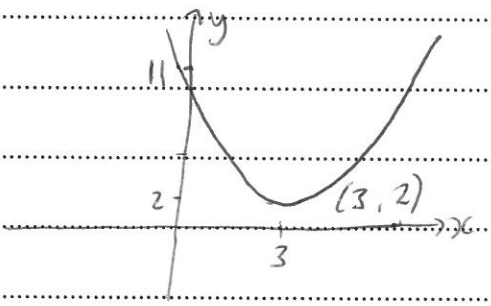
$x - 3 = \pm\sqrt{-2}$

can't $\sqrt{\ }$ a negative number $\therefore$ no real solution

bi) $(x-3)^2 + 2$

coordinate of vertex is $\underline{\underline{(3, 2)}}$

ii)



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QUESTION PART REFERENCE	**Answer space for question 4**
iii)	from $y = (x-3)^2 + 2$ to $y = x^2$
	Translation $\begin{pmatrix} -3 \\ -2 \end{pmatrix}$

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5 The polynomial $p(x)$ is given by

$$p(x) = x^3 - 4x^2 - 3x + 18$$

(a) Use the Remainder Theorem to find the remainder when $p(x)$ is divided by $x + 1$. *(2 marks)*

(b) (i) Use the Factor Theorem to show that $x - 3$ is a factor of $p(x)$. *(2 marks)*

(ii) Express $p(x)$ as a product of linear factors. *(3 marks)*

(c) Sketch the curve with equation $y = x^3 - 4x^2 - 3x + 18$, stating the values of x where the curve meets the x-axis. *(3 marks)*

QUESTION PART REFERENCE

Answer space for question 5

5) $p(x) = x^3 - 4x^2 - 3x + 18$

a) $p(-1) = (-1)^3 - 4(-1)^2 - 3(-1) + 18$

$= -1 - 4 + 3 + 18$

$= 16$ remainder is $\underline{16}$

bi) $p(3) = (3)^3 - 4(3)^2 - 3(3) + 18$

$= 27 - 36 - 9 + 18$

$= 0$ $\therefore$ $(x-3)$ is a factor of $p(x)$

ii)

$$
\begin{array}{r}
x^2 - x - 6 \\
x-3 \,\overline{)\, x^3 - 4x^2 - 3x + 18} \\
-\underline{x^3 - 3x^2} \quad\quad\quad \\
0 - x^2 - 3x \quad\quad \\
-\underline{-x^2 + 3x} \quad\quad \\
0 - 6x + 18 \\
-\underline{-6x + 18} \\
0 + 0
\end{array}
$$

$(x-3)(x^2 - x - 6)$

$\underline{(x-3)(x-3)(x+2)}$

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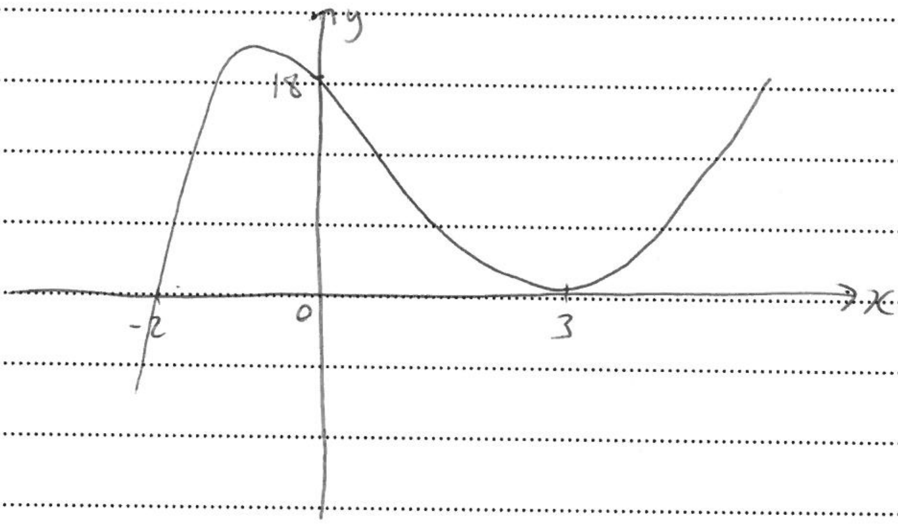
QUESTION PART REFERENCE

Answer space for question 5

c) x intersects at $(3, 0)$ - touches
$(-2, 0)$
y intersects at $(0, 18)$



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6 The gradient, $\frac{dy}{dx}$, of a curve at the point (x, y) is given by

$$\frac{dy}{dx} = 10x^4 - 6x^2 + 5$$

The curve passes through the point $P(1, 4)$.

(a) Find the equation of the tangent to the curve at the point P, giving your answer in the form $y = mx + c$. *(3 marks)*

(b) Find the equation of the curve. *(5 marks)*

QUESTION PART REFERENCE	**Answer space for question 6**
6)	$\frac{dy}{dx} = 10x^4 - 6x^2 + 5$ $\quad P(1, 4)$
a)	$y = 4, \quad x = 1$
	when $x = 1$, $\frac{dy}{dx} = 10(1)^4 - 6(1)^2 + 5$
	$= 10 - 6 + 5$
	$= 9$ (gradient)
	$y - 4 = 9(x - 1)$
	$y - 4 = 9x - 9$
	$y = 9x - 5$
b)	$y = \frac{10x^5}{5} - \frac{6x^3}{3} + 5x + c$ $\quad x = 1, y = 4$
	$y = 2x^5 - 2x^3 + 5x + c$
	$4 = 2(1)^5 - 2(1)^3 + 5(1) + c$
	$4 = 2 - 2 + 5 + c$
	$c = -1$ $\quad$ so, $y = 2x^5 - 2x^3 + 5x - 1$

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QUESTION PART REFERENCE

Answer space for question 6

Turn over ▶

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7 A circle with centre $C(-3, 2)$ has equation

$$x^2 + y^2 + 6x - 4y = 12$$

(a) Find the y-coordinates of the points where the circle crosses the y-axis. *(3 marks)*

(b) Find the radius of the circle. *(3 marks)*

(c) The point $P(2, 5)$ lies outside the circle.

(i) Find the length of CP, giving your answer in the form $\sqrt{n}$, where n is an integer. *(2 marks)*

(ii) The point Q lies on the circle so that PQ is a tangent to the circle. Find the length of PQ. *(2 marks)*

QUESTION PART REFERENCE

Answer space for question 7

7) $x^2 + y^2 + 6x - 4y = 12$

a) $x = 0$ when crosses y axis

$y^2 - 4y = 12$

$y^2 - 4y - 12 = 0$

$(y - 6)(y + 2) = 0$

$\underline{y = 6}$ or $\underline{y = -2}$

b) $x^2 + 6x + y^2 - 4y = 12$

$(x + 3)^2 - 9 + (y - 2)^2 - 4 = 12$

$(x + 3)^2 + (y - 2)^2 = 25$

$r^2 = 25$

$\underline{r = 5}$

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QUESTION PART REFERENCE	Answer space for question 7

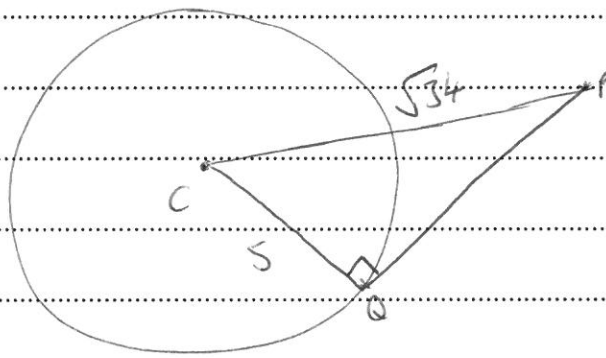
ci) $C(-3,2) \qquad P(2,5)$

$CP = \sqrt{(-3-2)^2 + (2-5)^2}$

$= \sqrt{25+9}$

$= \underline{\underline{\sqrt{34}}}$

ii)



$PQ^2 = (\sqrt{34})^2 - 5^2$

$= 34 - 25$

$PQ = \sqrt{9}$

$\underline{PQ = 3}$

Turn over ▶

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8 A curve has equation $y = 2x^2 - x - 1$ and a line has equation $y = k(2x - 3)$, where k is a constant.

(a) Show that the x-coordinate of any point of intersection of the curve and the line satisfies the equation

$$2x^2 - (2k + 1)x + 3k - 1 = 0$$ *(1 mark)*

(b) The curve and the line intersect at two distinct points.

(i) Show that $4k^2 - 20k + 9 > 0$. *(3 marks)*

(ii) Find the possible values of k. *(4 marks)*

QUESTION PART REFERENCE

Answer space for question 8

8) $y = 2x^2 - x - 1$ $\quad y = k(2x-3)$

a) $2x^2 - x - 1 = k(2x-3)$

$2x^2 - x - 1 = 2kx - 3k$

$2x^2 - 2kx - x + 3k - 1 = 0$

$2x^2 - (2k+1)x + 3k - 1 = 0$ (as required)

bi) $b^2 - 4ac > 0$ as two distinct solutions

$a = 2,\ b = -(2k+1) \quad c = 3k - 1$

$= -2k - 1$

$(-2k-1)^2 - 4(2)(3k-1) > 0$

$(2k+1)^2 - 8(3k-1) > 0$

$4k^2 + 4k + 1 - 24k + 8 > 0$ $\quad$ $(2k+1)(2k+1)$

$4k^2 - 20k + 9 > 0$ $\quad$ $4k^2 + 2k + 2k + 1$

(as required) $\quad$ $4k^2 + 4k + 1$

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QUESTION PART REFERENCE

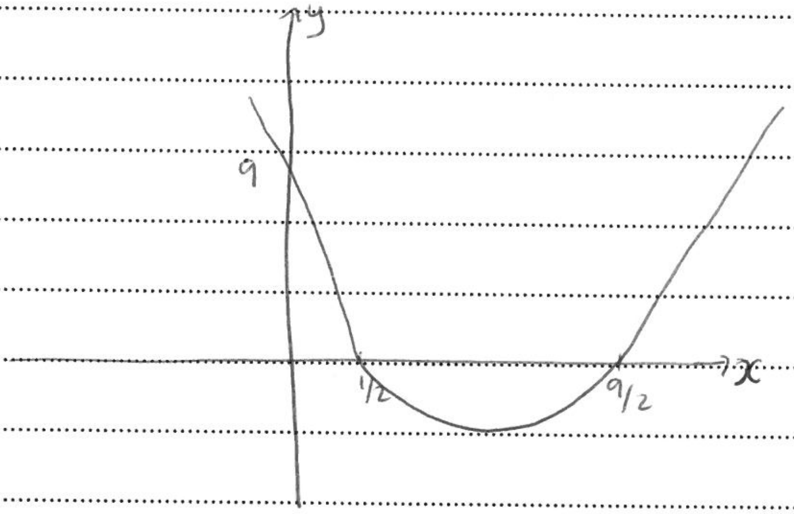
Answer space for question 8

ii) $4k^2 - 20k + 9 > 0$

$(2k - 1)(2k - 9)$

$2k - 1 = 0$ OR $2k - 9 = 0$

$k = 1/2$ $k = 9/2$



$4k^2 - 20k + 9 > 0$

so graph is <u>above</u> x axis (greater than 0)

$\underline{k < 1/2}$, $\underline{k > 9/2}$

END OF QUESTIONS

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