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## General Certificate of Education

# Mathematics 6360

## MPC1 Pure Core 1

# Mark Scheme

*2009 examination - January series*

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

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## Key to mark scheme and abbreviations used in marking

| | |
|---|---|
| M | mark is for method |
| m or dM | mark is dependent on one or more M marks and is for method |
| A | mark is dependent on M or m marks and is for accuracy |
| B | mark is independent of M or m marks and is for method and accuracy |
| E | mark is for explanation |

| | | | |
|---|---|---|---|
| √or ft or F | follow through from previous incorrect result | MC | mis-copy |
| CAO | correct answer only | MR | mis-read |
| CSO | correct solution only | RA | required accuracy |
| AWFW | anything which falls within | FW | further work |
| AWRT | anything which rounds to | ISW | ignore subsequent work |
| ACF | any correct form | FIW | from incorrect work |
| AG | answer given | BOD | given benefit of doubt |
| SC | special case | WR | work replaced by candidate |
| OE | or equivalent | FB | formulae book |
| A2,1 | 2 or 1 (or 0) accuracy marks | NOS | not on scheme |
| $-x$ EE | deduct $x$ marks for each error | G | graph |
| NMS | no method shown | c | candidate |
| PI | possibly implied | sf | significant figure(s) |
| SCA | substantially correct approach | dp | decimal place(s) |

### No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

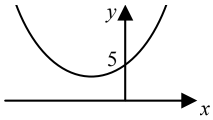
Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

**Otherwise we require evidence of a correct method for any marks to be awarded.**

## MPC1

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **1(a)** | $M\,(3, 2)$ | B1 B1 | 2 | B1 for each coordinate |
| **(b)** | Gradient $AB = \dfrac{-2-6}{5-1} = \left(\dfrac{-8}{4}\right)$ | M1 | | May use coords of $M$ instead of $A$ or $B$ - condone one slip |
| | $= -2$ | A1 | 2 | CSO Answer must be simplified to –2 |
| **(c) (i)** | Gradient of perpendicular = $\dfrac{1}{2}$ | B1√ | | ft "their" –1/gradient $AB$ |
| | $\Rightarrow\; y-2=\dfrac{1}{2}(x-3)$ | M1 | | attempt at perp to $AB$; ft their $M$ coords |
| | $\Rightarrow\; 2y-4=x-3 \;\Rightarrow\; x-2y+1=0$ **AG** | A1 | 3 | CSO Must write down the printed answer |
| **(ii)** | $k-2(k+5)+1=0$ or $\dfrac{(k+5)-2}{k-3}=\dfrac{1}{2}$ | M1 | | Sub into given line equation or correct expression involving gradients<br>Condone omission of brackets or use of $x$ |
| | $\Rightarrow k=-9$ | A1 | 2 | Condone $x=-9$<br>*(Full marks for correct answer without working)* |
| | **Total** | | **9** | |
| **2(a)** | $(x-1)(2x-3)$ | B1 | 1 | $(1-x)(3-2x)$ or $2(x-1)(x-1.5)$ etc |
| **(b)** | Critical values are 1, $1\frac{1}{2}$ | B1√ | | Correct or ft their factors from (a) |
| | Sign diagram or sketch | M1 | | $+ \quad \mid \quad - \quad \mid \quad +$<br>$1 \qquad 1\frac{1}{2}$ |
| | $\Rightarrow 1<x<1\frac{1}{2}$ | A1 | 3 | *Full marks for correct inequality without working* |
| | **Total** | | **4** | |
| **3(a)** | $\dfrac{7+\sqrt{5}}{3+\sqrt{5}} \times \dfrac{3-\sqrt{5}}{3-\sqrt{5}}$ | M1 | | Multiply by $\dfrac{3-\sqrt{5}}{3-\sqrt{5}}$ or $\dfrac{\sqrt{5}-3}{\sqrt{5}-3}$ |
| | Numerator $= 21+3\sqrt{5}-7\sqrt{5}-\left(\sqrt{5}\right)^2$ | m1 | | Condone one slip $\quad 16-4\sqrt{5}$ |
| | Denominator = 9 – 5 = 4 | B1 | | ( Or 5 – 9 = – 4 from other conjugate) |
| | *Answer* $= 4-\sqrt{5}$ | A1 | 4 | CSO |
| **(b)** | $\sqrt{45}=3\sqrt{5}$ | B1 | | |
| | $\dfrac{20}{\sqrt{5}}=\dfrac{20\sqrt{5}}{5}$ | M1 | | May score if combined as one expression<br>Must have 5 in denominator |
| | Sum $= 7\sqrt{5}$ | A1 | 3 | |
| | **Total** | | **7** | |

**MPC1 (cont)**

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **4(a)(i)** | $(x+1)^2$ | B1 | | $p = 1$ |
| | $+\ 4$ | B1 | 2 | $q = 4$ |
| **(ii)** | $(x+1)^2 \geqslant 0 \Rightarrow (x+1)^2 + 4 > 0$<br>($\Rightarrow x^2 + 2x + 5 > 0$ for all values of $x$) | E1 | 1 | Condone if they say $(x+1)^2$ *positive*<br>and adding 4 so always positive |
| **(b)(i)** | $x = -1$ **or** $y = 4$ | M1 | | ft their $x = -p$ or $y = q$ |
| | Minimum point is (–1, 4) | A1 | 2 | |
| **(ii)** |  | B1 | | Sketch roughly as shown |
| | | B1 | 2 | $y$-intercept 5 or (0, 5) marked or stated |
| **(c)** | Translation (not shift, move etc) | E1 | | and NO other transformation stated |
| | through $\begin{bmatrix}-1\\4\end{bmatrix}$ (or 1 left, 4 up etc) | M1 | | either component correct or ft their $-p$, $q$ |
| | | A1 | 3 | correct translation<br>M1, A1 independent of E mark |
| | **Total** | | **10** | |
| **5(a)(i)** | $\frac{dx}{dt} = 2t^3 - 40t + 66$ | M1 | | one term correct |
| | | A1 | | another term correct |
| | | A1 | 3 | all correct unsimplified (no + $c$ etc) |
| **(ii)** | $\frac{d^2x}{d\,t^2} = 6t^2 - 40$ | M1 | | ft one term correct |
| | | A1√ | 2 | ft all "correct", 2 terms equivalent |
| **(b)** | $\frac{dx}{dt} = 54 - 120 + 66$ | M1 | | substitute $t = 3$ into their $\frac{dx}{dt}$ |
| | $= 0 \Rightarrow$ stationary value | A1 | | CSO<br>shown = 0 ( 54 or $2 \times 27$ seen )<br>and statement |
| | Substitute $t = 3$ into $\frac{d^2x}{dt^2}$ ( = 14 ) | M1 | | |
| | $\frac{d^2x}{dt^2} > 0 \Rightarrow$ minimum value | A1 | 4 | CSO; all values (if stated) must be correct |
| **(c)** | Substitute $t = 1$ into their $\frac{dx}{dt}$ | M1 | | must be their $\frac{dx}{d\,t}$ NOT $\frac{d^2x}{dt^2}$ etc |
| | $\frac{dx}{dt} = 28$ | A1√ | 2 | ft their $\frac{dx}{dt}$ when $t = 1$ |
| **(d)** | Substitute $t = 2$ into their $\frac{dx}{dt}$ | M1 | | must be their $\frac{dx}{d\,t}$ NOT $\frac{d^2x}{dt^2}$ or $x$ |
| | $= 16 - 80 + 66 = 2 \quad (> 0)$ | | | Interpreting their value of $\frac{dx}{dt}$ |
| | $\Rightarrow$ increasing when $t = 2$ | E1√ | 2 | Allow decreasing if their $\frac{dx}{dt} < 0$ |
| | **Total** | | **13** | |

**MPC1 (cont)**

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **6(a)(i)** | $\text{p}(2) = 8 + 2 - 10$ | M1 | | Must find p(2)   NOT long division |
| | $\Rightarrow \text{p}(2) = 0 \Rightarrow (x-2)$ is factor | A1 | 2 | Shown = 0   plus a statement |
| **(ii)** | Attempt at long division (generous) | M1 | | Obtaining a quotient $x^2 + cx + d$ or equating coefficients (full method) |
| | $\text{p}(x) = (x-2)(x^2 + 2x + 5)$ | A1 | 2 | $a = 2,\ b = 5$ by inspection B1, B1 |
| **(b)(i)** | $\frac{\text{d}y}{\text{d}x} = 3x^2 + 1$ | M1<br>A1 | | One term correct<br>All correct – no +*c* etc |
| | When $x = 2$ $\frac{\text{d}y}{\text{d}x} = 3 \times 4 + 1$ | m1 | | Sub $x = 2$ into their $\frac{\text{d}y}{\text{d}x}$ |
| | Therefore gradient at *Q* is 13 | A1 | 4 | CSO |
| **(ii)** | $y = 13(x-2)$ | M1 | | Tangent (NOT normal) attempted<br>ft their gradient answer from (b)(i) |
| | | A1 | 2 | CSO; correct in any form |
| **(iii)** | $\int \ldots \,\text{d}x = \frac{x^4}{4} + \frac{x^2}{2} - 10x\ (+c)$ | M1<br>A1<br>A1 | 3 | one term correct<br>second term correct<br>all correct (condone no +c) |
| **(iv)** | $[4 + 2 - 20] - [0] = -14$ | M1 | | $\text{F}(2)$ attempted and possibly F(0)<br>Must have earned M1 in (b)(iii) |
| | Area of shaded region = 14 | A1 | 2 | CSO; separate statement following correct evaluation of limits |
| | **Total** | | **15** | |

**MPC1 (cont)**

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **7(a)(i)** | $(x-3)^2+(y+5)^2$ | B1 | | One term correct |
| | | B1 | | LHS correct with + and squares |
| | $= 25 - 9 + 9 = 25 \quad ( = 5^2 )$ | B1 | 3 | Condone RHS = 25 |
| **(b)(i)** | $C\ (3, -5)$ | B1√ | | |
| **(ii)** | Radius = 5 | B1√ | 2 | Correct or ft their RHS provided > 0 |
| **(c)(i)** | $(7-3)^2+(-2+5)^2=16+9=25$<br>$\Rightarrow D$ lies on circle<br>*Must see statement* | B1 | 1 | Or sub'n of (7, –2) in original equation<br>$7^2+(-2)^2-42-20+9=0$<br>Or sub $x$=7 into eqn & showing $y$ = –2 etc |
| **(ii)** | Attempt at gradient of *CD* as normal | M1 | | withhold if subsequently uses $m_1m_2=-1$ |
| | grad *CD* = $\dfrac{-2-(-5)}{7-3}=\dfrac{3}{4}$ | | | $\dfrac{\Delta y}{\Delta x}$ (condone one slip) FT their centre *C* |
| | $y+2=\frac{3}{4}(x-7)$ or $y+5=\frac{3}{4}(x-3)$ | A1 | | Correct equation in any form $y=\frac{3}{4}x-\frac{29}{4}$ |
| | $\Rightarrow 3x-4y=29$ | A1 | 3 | CSO ***Integer*** coefficients<br>Condone $4y-3x+29=0$ etc |
| **(d)(i)** | $y=kx$ sub'd into original circle equation | M1 | | or using their completed square form and |
| | $x^2+(kx)^2-6x+10kx+9=0$ | | | multiplying out |
| | $\Rightarrow (k^2+1)x^2+2(5k-3)x+9=0$ **AG** | A1 | 2 | CSO<br>must see at least previous line for A1<br>any error such as $kx^2=...=k^2x^2$ gets A0 |
| **(ii)** | $4(5k-3)^2-36(k^2+1)$ | M1 | | Discriminant in *k* (can be seen in quad formula)<br>Condone one slip |
| | $= 64k^2-120k$ | A1 | | or $8k^2-15k=0$ OE |
| | Equal roots: $4(5k-3)^2-36(k^2+1)=0$ | B1 | | $b^2-4ac=0$ clearly stated or evident by an equation in *k* with at most 2 slips. |
| | $8k^2-15k=0$ | | | |
| | | m1 | | Attempt to solve ***their*** quadratic or linear equation if *k* has been cancelled |
| | $\Rightarrow k=0, \quad k=\dfrac{15}{8}$ | A1 | 5 | OE but must have *k*=0<br>If "=0" is not seen but correct values of *k* are found, candidate will lose B1 mark but may earn all other marks |
| **(iii)** | (Line is a) **tangent** (to the circle) | E1 | 1 | Line **touches** circle at one point |
| | **Total** | | **17** | |
| | **TOTAL** | | **75** | |