

Core 1 May 2007

① A (6, -1)

B (2, 5)

ai) gradient AB = $\frac{5 - (-1)}{2 - 6} = \frac{6}{-4} = -\frac{3}{2}$ (as required)

ii) M = $-\frac{3}{2}$ B(2, 5)

$$y - 5 = -\frac{3}{2}(x - 2)$$

$$2y - 10 = -3x + 6$$

$$\underline{3x + 2y = 16}$$

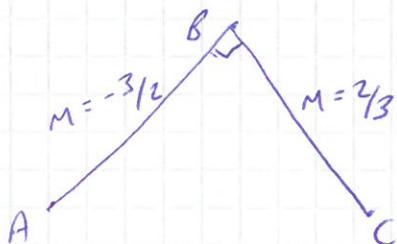
bi) M = $\frac{2}{3}$ (perpendicular) B(2, 5)

$$y - 5 = \frac{2}{3}(x - 2)$$

$$3y - 15 = 2x - 4$$

$$\underline{3y = 2x + 11}$$

ii)



B (2, 5) C (k, 7)

$$\frac{7 - 5}{k - 2} = \frac{2}{3}$$

$$k - 2 = 3$$

$$\underline{k = 5}$$

② a) $\frac{\sqrt{63}}{3} + \frac{14}{\sqrt{7}}$

$$\sqrt{7} + 2\sqrt{7} = \underline{\underline{3\sqrt{7}}}$$

$$\sqrt{63} = \sqrt{9 \times 7} = 3\sqrt{7}$$

$$\frac{\sqrt{63}}{3} = \frac{3\sqrt{7}}{3} = \sqrt{7}$$

$$\frac{14}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} = \frac{14\sqrt{7}}{7} = 2\sqrt{7}$$

Core 1 May 2007

$$\begin{aligned} \textcircled{2} \text{ b) } \frac{(\sqrt{7}+1)}{(\sqrt{7}-2)} \times \frac{(\sqrt{7}+2)}{(\sqrt{7}+2)} &= \frac{\sqrt{49} + 2\sqrt{7} + \sqrt{7} + 2}{\sqrt{49} + 2\sqrt{7} - 2\sqrt{7} - 4} \\ &= \frac{7 + 3\sqrt{7} + 2}{7 - 4} \\ &= \frac{9 + 3\sqrt{7}}{3} = \underline{\underline{3 + \sqrt{7}}} \\ &= \underline{\underline{\sqrt{7} + 3}} \end{aligned}$$

$$\begin{aligned} \textcircled{3} \text{ ai) } x^2 + 10x + 19 &= (x+5)^2 - 25 + 19 \\ &= \underline{\underline{(x+5)^2 - 6}} \end{aligned}$$

ii) vertex $(-5, -6)$

iii) line of symmetry $x = -5$

iv) $y = x^2 \rightarrow y = (x+5)^2 - 6$

Translation $\begin{pmatrix} -5 \\ -6 \end{pmatrix}$

b) $y = x + 11$, $y = x^2 + 10x + 19$

$$x^2 + 10x + 19 = x + 11$$

$$x^2 + 9x + 8 = 0$$

$$(x+8)(x+1) = 0$$

$$x = -8$$

$$, \quad x = -1$$

$$y = -8 + 11$$

$$y = -1 + 11$$

$$= \underline{\underline{3}}$$

$$= 10$$

$$\underline{\underline{(-8, 3)}}$$

$$\underline{\underline{(-1, 10)}}$$

Core 1 May 2007

$$\textcircled{4} \quad y = \frac{1}{4}t^4 - 26t^2 + 96t \quad 0 \leq t \leq 4$$

$$\text{ai) } \frac{dy}{dt} = t^3 - 52t + 96$$

$$\text{ii) } \frac{d^2y}{dt^2} = 3t^2 - 52$$

$$\begin{aligned} \text{b) when } t=2, \quad \frac{dy}{dt} &= (2)^3 - 52(2) + 96 \\ &= 8 - 104 + 96 \\ &= 0 \quad \therefore \text{stationary point when } t=2 \end{aligned}$$

$$\begin{aligned} \text{When } t=2, \quad \frac{d^2y}{dt^2} &= 3(2)^2 - 52 \\ &= 12 - 52 \\ &= -40 \end{aligned}$$

$$\frac{d^2y}{dt^2} < 0 \quad \therefore \text{maximum point at } t=2$$

$$\begin{aligned} \text{c) when } t=1, \quad \frac{dy}{dt} &= 1(1)^3 - 52(1) + 96 \\ &= 1 - 52 + 96 \\ &= 45 \end{aligned}$$

$$\frac{dy}{dt} > 0 \quad \therefore \text{rate of change is } \underline{45 \text{ cm s}^{-1}}$$

$$\begin{aligned} \text{d) when } t=3, \quad \frac{dy}{dt} &= (3)^3 - 52(3) + 96 \\ &= 27 - 156 + 96 \\ &= -33 \end{aligned}$$

$$\frac{dy}{dt} < 0 \quad \therefore \text{decreasing when } t=3$$

Core 1 May 2007

⑤ $(x+3)^2 + (y-2)^2 = 25$

ai) $C = \underline{(-3, 2)}$

ii) $r^2 = 25$

radius = 5

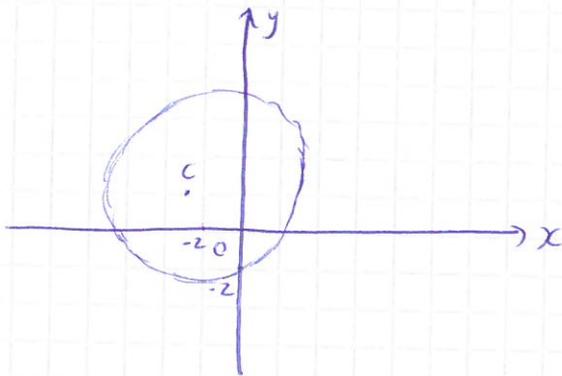
bi) $N(0, -2) \rightarrow$ sub in

$$(0+3)^2 + (-2-2)^2 = 25$$

$$9 + 16 = 25 \checkmark$$

$\therefore N$ lies on the circle

ii)



iii) $C(-3, 2)$ $N(0, -2)$

gradient $CN = \frac{-2-2}{0-(-3)} = \frac{-4}{3}$ (normal)

$$y+2 = -\frac{4}{3}x$$

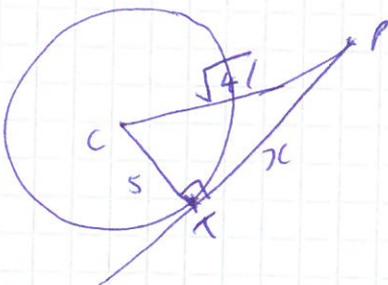
$$3y+6 = -4x$$

$$\underline{4x+3y+6=0}$$

ci) $P(2, 6)$ $C(-3, 2)$

$$\begin{aligned} \text{distance } PC &= \sqrt{(-3-2)^2 + (6-2)^2} = \sqrt{5^2 + 4^2} \\ &= \underline{\underline{\sqrt{41}}} \end{aligned}$$

ii)



$$x^2 = (\sqrt{41})^2 - 5^2$$

$$x^2 = 41 - 25$$

$$x^2 = 16$$

length tangent = 4 (PT)

Core 1 May 2007

⑥ $f(x) = x^3 + 4x - 5$

ai) $f(1) = (1)^3 + 4(1) - 5$

$= 1 + 4 - 5 = 0 \quad \therefore (x-1) \text{ is a factor of } f(x)$

ii)

$$\begin{array}{r} x^2 + x + 5 \\ x-1 \overline{) x^3 + 0x^2 + 4x - 5} \\ \underline{-x^3 - x^2} \\ 0 + x^2 + 4x \\ \underline{-x^2 - x} \\ 0 + 5x - 5 \\ \underline{-5x - 5} \\ 0 \end{array}$$

$(x-1)(x^2 + x + 5)$

iii) $f(x) = (x-1)(x^2 + x + 5)$

$(x-1)(x^2 + x + 5) = 0$

$x-1=0$

$x=1$

$x^2 + x + 5 = 0 \quad a=1, b=1, c=5$

$b^2 - 4ac \rightarrow (1)^2 - 4(1)(5)$

$1 - 20 = -19$

$b^2 - 4ac < 0 \quad \therefore \text{no real solutions}$

b) $y = x^3 + 4x - 5 \quad A(1,0) \quad B(2,11)$

i) $\int (x^3 + 4x - 5) dx = \frac{x^4}{4} + \frac{4x^2}{2} - 5x + C$
 $= \frac{x^4}{4} + 2x^2 - 5x + C$

ii) shaded region = area of triangle - area under curve

area of $\Delta = \frac{11 \times 1}{2} = \frac{11}{2} = 5\frac{1}{2}$

area under curve = $\left[\frac{x^4}{4} + 2x^2 - 5x \right]_1^2$

$= \left(\frac{2^4}{4} + 2(2)^2 - 5(2) \right) - \left(\frac{1}{4} + 2 - 5 \right)$

$= (4 + 8 - 10) - (-2\frac{3}{4}) = 4\frac{3}{4}$

shaded = $5\frac{1}{2} - 4\frac{3}{4}$

$= \frac{3}{4}$

Core 1 May 2007

$$(7) (2k-3)x^2 + 2x + (k-1) = 0$$

a) $b^2 - 4ac \geq 0$ if real roots

$$a = 2k-3, \quad b = 2, \quad c = k-1$$

$$2^2 - 4(2k-3)(k-1) \geq 0$$

$$4 - 4(2k^2 - 2k - 3k + 3) \geq 0$$

$$4 - 4(2k^2 - 5k + 3) \geq 0$$

$$4 - 8k^2 + 20k - 12 \geq 0$$

$$-8 - 8k^2 + 20k \geq 0$$

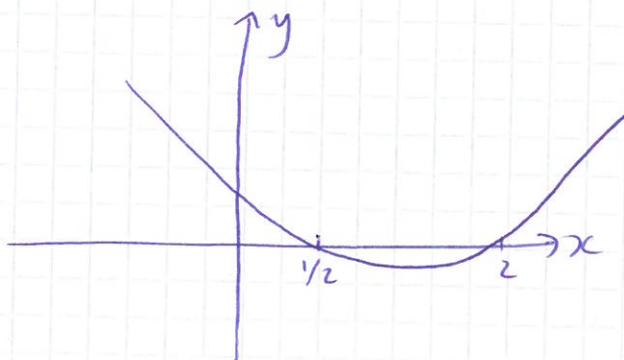
$$8k^2 - 20k + 8 \leq 0 \quad (\div 4)$$

$$\underline{2k^2 - 5k + 2 \leq 0} \quad (\text{as required})$$

b) $2k^2 - 5k + 2$

$$\underline{(2k-1)(k-2)}$$

ii) $(2k-1)(k-2) \rightarrow$ critical values at $\frac{1}{2}$ and 2



$$2k^2 - 5k + 2 \leq 0$$

graph below x axis

$$\therefore \underline{\underline{\frac{1}{2} \leq k \leq 2}}$$