### What is your favourite number and why?

Skewes number (or more precisely, the first Skewes number). It arises in analytic number theory, from attempts to describe the distribution of prime numbers. In 1896, Jaques Hadamard, and Charles Jean de la Vallée-Poussin proved independently that the number of primes less than a given number tended to the value of the offset logarithmic integral of that number:

In the 1900s it was known that this approach yielded an over-estimate. In fact we now know that this an over-estimate up to at least 1022 and many mathematicians assumed that this would always be the case.

However, in 1914, J.E. Littlewood proved that at some point, there is a “crossover” beyond which this approach yields an *under*estimate of the number of primes, and, even more bizarrely, beyond that there are an infinite number of such crossovers between overestimates and underestimates (Littlewood, 1914).

In 1933, Stanley Skewes proved that the first crossover had to occur for a value of x less than eee79 or 10101034 (Skewes, 1933). Commenting on this number, G.H. Hardy said: “I think that this is the largest number which has served any definite purpose in mathematics.” (Hardy, 1937 p. 152)

In an article on the work of his friend and colleague Srinivasa Ramanujan, Hardy tried to describe how big this number was in the following way:

The number of protons in the universe is about 1080. The number of chess games in much bigger, perhaps 101050 (in any case a second order exponential). If the universe were the chessboard, and the protons the chessmen, and any interchange in the position of two protons, a move, then the number of possible games would be something like the Skewes number. However much the number may be reduced by refinements on Skewes' argument, it does not seem at all likely that we shall *ever* know a single instance of the truth of Littlewood's theorem. (loc cit., my emphasis)

In fact, Skewes proved that this number is an upper bound for the first crossover only if the Riemann hypothesis is true. The Riemann hypothesis, put forward by Bernhard Riemann in 1859, concerns the Riemann zeta function, which is defined for values greater than 1 as follows:

However, it can also be defined in other ways that allow for negative and complex values. The Riemann hypothesis is that all the zeroes of the zeta function are negative even numbers (-2, -4, -6, …) or are complex numbers with real component of ½. In 1900, the German mathematician David Hilbert included the status of the Riemann hypothesis in his list of the most important unsolved problems in mathematics (Hilbert, 1902). The majority of mathematicians appear to believe that the hypothesis is true, although it is worth noting that J. E. Littlewood believed strongly that it was likely to be false (Littlewood, 1962).

Twenty years later, Skewes came up with an upper bound for the first crossover if the Riemann hypothesis is false, which a lot larger: eeee7.7 (Skewes, 1955). More recently, Bays and Hudson (2000) showed that, if the Riemann hypothesis is true, there is at least one crossover before 1.39822 x 10316—a much lower estimate than that found by Skewes in 1933, but still an incredible large number.

If you want to look at even larger numbers, you could look at Graham’s number which is way bigger than the two Skewes’ numbers discussed above. It’s actually listed in the Guinness Book of Records as the largest number ever used in a mathematical proof (Wilkins, 2011). If that’s not big enough for you, could try to get your head around the number known as TREE(3).

And, of course, it is important to remember that these are all *finite* numbers, which begins to provide some insight into why no human really understands things that are infinite.